A TREATISE

ON

ELECTROMAGNETIC PHENOMENA

AND ON THE

COMPASS AND ITS DEVIATIONS ABOARD SHIP.

MATHEMATICAL, THEORETICAL, AND PRACTICAL.

BY

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PREFACE.

THE Chapters, Pages, Articles, Illustrations, and Tables are numbered consecutively throughout the Treatise. Vol. I is wholly occupied with PART FIRST, and ends with Chapter XII, Page 556, Art. 214, Plate *M*, Fig. 368, and Table 28: the regular sequence of all these follows in this volume.

The numbering of the equations begins a new series with each PART.

Messrs. Ritchie of Boston, who make compasses for the United States Navy, have afforded me every facility for acquiring knowledge of the practical performance of their work: it is that information that, with their consent, I have used in the following pages to illustrate Compass construction and supplement theoretical considerations.

NEW YORK, March 3, 1903.

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CHAPTER XIII.

HISTORICAL SKETCH.

215. Divers claims regarding the invention of the Compass.—Almost every country, ancient and medieval, has laid claim to the Compass as its own invention; to search out the evidence and facts of each—to confront them one with another and thrash out the few grains of truth that lie hidden in a stack of legendary chaff, is a task that does not enter into the plan of this work: it has been done by various writers, but with a result nearly always tinged with the bias of the author. With this one, the Chinese were the inventors; with another, the Northmen; with a third, the Italians; and with still more, the Arabs and the French. The truth is, that research, so far, has not indisputably established the fact when and where a magnetic needle first guided a ship.

The development of the compass was very slow, and indeed it is only within the last fifty years that such improvement has been made as to entitle it to entire confidence. Between the piece of steel of the twelfth century that was temporarily rubbed with a lodestone and set upon a float in water, and the refined compass of to-day—almost worthy of place among the instruments of precision—there is a gulf so wide and deep, that one can scarcely recognize in the former the prototype of the latter.

Such probable links, however, in the lineage as have

received general, though not universal, acceptance, will now be traced: they will be set down chronologically; and are not from original sources, but are quoted from such standard works as may be found in any large library.

216. The Chinese records.—The first use of the needle was not on board ship, but on land—to guide a cart: a pivoted manikin was set up in the front of the vehicle, its right arm extended, and in it was concealed a magnet which turned the little figure into the meridian, thus pointing out its direction.

Du Halde, who was a missionary in China and wrote a history of the country from data procured from Chinese books, states that "about the year 2634 B.C., the Emperor Hoang-te, being at war, an instrument was invented, which being placed in a car, *it pointed to the south* and enabled the imperial army to direct its march and surprise the enemy during a thick fog."

"About 1038 B.C., an embassy reached China from Cochin; the ambassadors had experienced great difficulty in finding their way to the imperial court; but on taking their final audience, the Emperor gave them an instrument of which one end pointed to the north and the other to the south, that they might find their way home with less embarrassment than they had experienced on their route to his dominions."

The instrument was then called *Tin-nan-ching*, or needle pointing to the south, and this is the name the Chinese give the mariner's compass to-day: they mark the south with a distinctive symbol and reckon from it as we do from the north.

Against this ancient origin, there are authors who assert that these carts were unknown before the fifth century, A.D.; and they further say that it was only toward the thirteenth century of our era that reliable records show them to have used the compass afloat.

While it may be true that the Chinese are not venturesome sailors, and had only small vessels in which they probably skirted the coasts, even in voyages to the Persian Gulf and during their varied communication with the Arabs, which occurred in the early centuries of our era; still, *if* they guided carts on land even as late as the fifth century, A.D. (which seems to be conceded), it is more than probable that a similar instrument was used about the same time for guiding ships at sea.

217. The Greeks and Romans.—In Art. 167, Part First, is quoted an extract from a poem by Lucretius, who flourished about 100 B.C.: in it the peculiar virtue of the lodestone is set forth; Lucretius even observed that it caused iron filings placed in a brass vessel to jump wildly about—"to rave"—under the varied movement of this natural magnet: but these matters had been known long previously; there is no mention, however, of the lodestone having been used to guide anything either afloat or ashore.

218. Records during the Christian era.—In a Chinese dictionary of date 121 A.D., the lodestone is defined as "A stone with which an attraction can be given to the needle"; and in another work compiled about 400 A.D., there are these passages: "They had ships which directed their course to the south by the magnetized needle." . . . "The fortune-tellers rub the points of a needle with the *stone of love* for rendering it proper to indicate the south."

And in the eleventh century A.D., the following is found in a Chinese work: "Take a single filament from a piece of new cotton and attach it exactly to the middle of the needle by a bit of wax as large as a mustard seed. Hang it up in a place where there is no wind. Then the needle constantly shows the south; but among such needles there are some which, being rubbed, indicate the north. Our soothsayers have some which show south and some which show north. Of this property of the magnet to indicate

the south, like that of the cypress to show the west, no one can tell the origin." There is no statement, however, that the needle thus prepared, was used for any *guiding* purpose.

The first mention in European annals of the use of the needle at sea seems to be contained in a poem by Guyot de Province, written in 1100 A.D.; its manuscript is in the Library of Paris, and the portion relating to the magnet is as follows:

(Old French.)

Voisisse qu'il semblas l'estocle
Qui ne se meut. Bien la voyent
Li mariniers qui si avoient,
Et lor seu, et lor voie tiennent,
Ils l'appellent la tresmaintaigne.
Icelle estaiche est moult certaine:
Toutes les autres se remouent
Et rechangent lor lieus, et tornent,
Mai celle estoile ne se meut,
Un Art font, qui mentir ne puet.
Par la vertu de la mariniere,
Une pierre laide et bruniere,
Ou li fers volontiers se joint
Ont, si esgardent le droit point,
Puisqu' une aiguille ont touchié
Et en un festu l'ont couchié,
En l'eve lemettent san plus
Et li festus la tiennent dessus.
Puis se tourne la pointe toute
Contre l'estoile, si sans doute
Que ja nus hom n'en doutera
Ne ja por rien ne faussera.
Quand la mer est obscure et brune
Quand ne voit estoile ne lune,
Dont font à l'aguille allumer,
Puis n'ont ils garde d'esgarer
Contre l'estoile va la pointe.

(The preceding in modern French.)

Ils voient qui ressemble à l'etoile, laquelle ne remue jamais. Les mariniers, guidés par elle, la connaissent assez bien; et par son moyen, ils vont et reviennent, marquent le cours, et poursuivent leur route: ils l'appellent la tramontaine (etoile polaire). Cette etoile est fixe; toutes les autres se meuvent, changent leur position, et retournent; mais celle-ci ne bouge point. Ils font une expérience qui ne peut pas les tromper. Ils ont une pierre brute et brune à laquelle, par la vertu de l'instrument appelé marinière, le fer s'unit volontiers; et, par ce moyen, ils s'apercoivent de la droiture du point. Lorsqu'une aiguille l'a touchée, et que l'on l'a mise sur un petit morceau de bois, et posée sur l'eau, le bois la tient sur la surface. C'est alors que la pointe de l'aiguille se tourne entièrement vers l'etoile, et avec une telle exactitude, que personne en saurait douter; et il n'y a pas à craindre que rien au monde puisse la détourner de cette situation. Lorsqu'il ne parait point d'étoiles, ni de lune, ils regardent l'aiguille avec une lumière, et ne peuvent pas s'égarer, car la pointe se dirige vers l'étoile.

Cardinal Jacques de Vitry, a native of France, who had engaged in the Crusades, wrote about 1204 a "History of the Crusaders and Their Voyages to the Holy Land," wherein he described the compass as being in familiar use among the Saracens on the coast of Syria, although a novelty to himself, and he speaks of the magnetic needle as "most necessary for such as sail the sea."

An English monk, Alexander Neckham, who lived from 1157 to 1217 A.D.—the author of several works on varied subjects, wrote in one of them as follows:

"The sailors, moreover, as they sail over the sea, when in cloudy weather they can no longer profit by the light of the sun, or when the world is wrapped in the darkness of the shades of night, and they are ignorant to what part of the horizon the prow is directed, place the needle over

the magnet, which is whirled round in a circle, until, when the motion ceases, the point of it (the needle) looks to north."

In the Paris Library there is an Arabian manuscript written in 1240 A.D. by Bailac Kipdjatie, in which a more definite description of the compass of the period is thus given:

"The captains who navigate the Syrian Sea, when the night is so obscure that they cannot perceive any star to direct them according to the determination of the four cardinal points, take a vessel full of water which they place sheltered from the wind and within the ship. Then they take a needle, which they enclose in a piece of wood or reed formed in the shape of a cross. They throw it in the water contained in the vase, so that it floats. Then they take a magnet stone large enough to fill the palm of the hand, or smaller. They bring it to the surface of the water, and give to the hand a movement of rotation toward the right, so that the needle turns on the surface of the water. Then they withdraw the hand suddenly, and at once the needle, by its two points, faces to the south and to the north. I have seen them, with my own eyes, do that, during my voyages at sea from Tripoli to Alexandria in the (Arabian) year 640 (or 1240 A.D.)."

In the Spanish *Leyes de las Partidas* of date 1263 A.D., occurs this passage:

"And as sailors are guided in an obscure night by means of the magnet needle, which is their mediator between the star and the lodestone, and shows them where to go as well in good weather as in bad; so those who have to aid and to counsel the king should always be guided by justice, which is the mediator between God and the world, always giving safety to the good and punishment to the wicked, each according to his deserts."

These varied accounts—French, Arabian, English, and

Spanish—all treat of an accomplished fact—of a magnetized needle used by all the sea-faring nations of Europe at the time to guide their ships; so that the *date* of the invention of the Compass (in so far as a single needle unassociated with a graduated circle and lubber's point—nothing but a needle lying in the magnetic meridian—can be called a compass) seems to be *anterior to* 1200 A.D.: after this, descriptions multiply, so that it would be burdensome to quote them—they merely corroborate the testimony already given.

219. The prototype of the present compass.—But now appeared a man who may be said to have devised the essential features of the compass of to-day.

Pierre de Maricourt, known in literature as Peter Peregrinus, was born in Picardy, France. In the year 1269 A.D., he wrote a letter (of which the manuscript is in the Paris Library) to a friend named Sigerus of Foucancourt; and in this, he states many things regarding the compass.

In ancient times the prevalent idea was, that the pole-star governed the magnet, then that it influenced mountains which in turn controlled the lodestone; but Peregrinus *indicated* the Earth as the controlling power.

Before Peregrinus' time, the needle was not in constant use aboard ship—only in cloudy weather; during the day ships steered along the land, and at night by the stars when visible.

When required, the needle was temporarily rubbed with the lodestone and floated in a bowl of water by means of a reed or piece of wood: it merely indicated the north and south line—with no fixed mark to reckon from.

Peregrinus thus describes the method of preparing the instrument:

"Take a wooden vessel, round, like a dish or platter, and put the stone in it so that the two points of the stone

may be equidistant from the edge; then put this in a larger vessel containing water, so that the stone may float like a sailor in a boat: the stone so placed will turn in its little vessel until the north pole of the stone will stand in the direction of the north pole of the heavens, and the south pole in that of the south pole of the heavens, by the will of God."

In those days the astrolabe was in general use: it consisted of a circle graduated to degrees, provided with cross-arms, fitted with notches or sight bars: when held vertically, the cross pieces could be pointed to the horizon and to the sun and thus a rough altitude of the latter obtained.

Peregrinus combined the astrolabe with the bowl in which the needle floated, and thus produced the first instrument worthy the name of *compass*.

He set the magnet in a diameter of the bowl, and marked on its rim both this diameter and degree marks; the bowl was then placed in a larger vessel containing water, and a light wooden bar having an upright pin at each end, was laid on the rim of the bowl, whereby bearings could be taken.

FIG. 369.

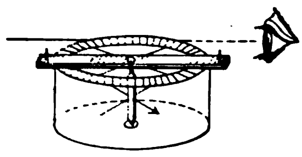


FIG. 370.

He improved upon this by devising the pivoted compass Figs. 369 and 370. [Both figures from *The Intellectual Rise in Electricity*.]

"The floating bowl and the large vessel of water are abolished, and in place of them there is the ordinary cir-

cular compass-box of to-day. Its edges are marked as those of the bowl were—with the degrees of the circle.

"It is covered with a plate of glass. In the center of the instrument, and stepped in the glass cover and in the bottom of the box, is a pivot, through which passes the compass-needle, now no longer a lodestone, but a true needle of steel or iron. Then at right angles to this needle is another needle, which, curiously enough, he says is to be made of silver or copper.

"Pivoted above the glass cover is an azimuth bar, as before, with sight pins at the ends. Now, he says, you are to magnetize the needle by means of the lodestone in the usual way, so that it will point north and south; and then the azimuth bar is to be turned on its center so as to be directed toward the sun or heavenly bodies, and in this way, of course, the azimuth is easily measured." —(*The Intellectual Rise in Electricity.*)

Peregrinus himself says of this instrument:

"You may direct your course toward cities and islands and all other parts of the world, either on land or at sea, provided you are acquainted with the latitudes and longitudes of those places."

And little more can be added to describe the use of the highly finished instrument of to-day: in all its essential features, Peregrinus' compass of the year 1269 is the embryo of our azimuth compass. Subsequent endeavor has been chiefly in the direction of improvement of what he seems to have originated. In Peregrinus' compass, the needle is thrust through a vertical pivot, so that both needle and pivot move together: in the modern compass, the pivot is fixed, and the needle is attached to a card which oscillates on the pivot.

220. Improvement of the Compass.—Interest in the subject now flags: the essential points have been established—that ships were steered by a magnetic needle at

least as early as the year 1200; that the instrument was crude and used only on occasion; and that the prototype of the present form had really its origin about the year 1269. Besides, the improvements have been wrangled over by many nations, and it would be little more than a statement of their pros and cons—a wearisome task—to treat of the additions to the first instrument. That seems to have been the work of Peregrinus; but where or when its ancestor—the simple needle floating in a bowl of water—had its origin, is not positively known; the Chinese, however, seem to have the best title to first using the needle for *guiding* purposes.

The compass gave a bold and confident impulse toward the open sea; and soon all nations—Spanish, English, Portuguese, Dutch, and French—undertook those long, daring, and perilous voyages that have made their seamen famous.

Ere its advent, mariners hugged the coast—ever tied to the leading-strings of Mother Earth:

> "Rude as their ships was navigation then;
> No useful compass or meridian known—
> Coasting, they kept the land within their ken,
> And knew no north but when the pole star shone."
>
> —Dryden.

CHAPTER XIV.

MANUFACTURE OF THE COMPASS.

Section One: The Principles of Magnetism and Mechanics that enter into Compass Construction.

221. **The Compass the Soul of the Ship.**—The principles involved in compass construction will be illustrated as occasion requires by description of the compass used in the U. S. Navy; and, to save repetition, that alone is meant whenever allusion or reference is made to any one.

In unique and striking phrase, Victor Hugo has depicted the old Line-of-battle Ship, and her equipment against the forces of nature and the perils of the deep; he calls the compass the soul of this superb structure, and says of it, that *against the immensity of space she has a needle!*

"Un vaisseau de ligne est une des plus magnifiques rencontres qu'ait le génie de l'homme avec la piussance de la nature.

"Un vaisseau de ligne est composé à la fois de ce qu'il y a de plus lourd et de ce qu'il y a de plus léger, parce qu'il a affaire en même temps aux trois formes de la substance—au solide, au liquide, au fluide—et qu'il doit lutter contre toutes les trois. Il a onze griffes de fer pour saisir le granit au fond de la mer, et plus d'alies et plus d'antennes que la bigaille pour prendre le vent dans les nuées. Son haleine sort par ses cent-vingt canons comme par des clairons énormes, et répond fièrement à la foudre. *L'ocean cherche á l'égarer dans l'effrayante similitude de ses vagues, mais le*

vaisseau a son âme, sa boussole, qui le conseille et lui montre toujours le nord. Dans les nuits noires, ses fanaux suppléent aux étoiles. Ainsi contre le vent, il a la corde et la toile; contre l'eau, le bois; contre le rocher, le fer, le cuivre, et le plomb; contre l'ombre, la lumiere; CONTRE L'IMMENSITÉ, UNE AGUILLE."—*Les Miserables.*

The old Ship-of-the-Line was the symbol of grandiose majesty—high out of water, tier upon tier of guns piercing her ports, lofty spars, a maze of ropes, and spreading sails: the modern Battle-ship is the embodiment of clean-cut power and strength, which would reduce her ancient prototype to a wreck of splinters at the first broadside; and yet the compass is no less the soul—the guiding spirit of the one than of the other—enabling an OREGON to find her way from Atlantic to Pacific, and an OLYMPIA to lead a squadron into action.

222. **Conditions that exist in the compass.**—In the compass three sources of contention arise—inertia, friction, and magnetism. Two of these generally conspire against the third, and the aim of the maker should be—to distribute inertia, reduce friction, and strengthen magnetism; and the extent to which he succeeds in this endeavor, to that extent will the instrument fulfill its mission.

The card upon which the degrees and points are graven, together with the material of the needles themselves, constitute the inert mass that must be moved by the force of magnetism resident in the needles; and the weight of this mass not only contributes nothing to the accuracy of the compass, but is in conflict with it, tending to keep the card at rest when magnetism would move it, and to maintain a swinging motion when magnetism would incline to steadiness.

By reducing the weight of the card, then, to the least that suitable material will permit, we lighten the burden put upon magnetism; and incidentally this lessens the

friction between cap and pivot as the card swings to and fro.

The center of gravity of the mass must be below the point of suspension, and symmetry of material should be kept in view; that is, for each particle on the right of a vertical plane through the magnetic axis of the system, there should be another particle equal in weight and distance on the left of the same plane, otherwise the card would wabble.

If the card happens to stop while the magnetic axis of the system makes an angle with the meridian, friction conspires with inertia to keep it there; and thus both oppose magnetism in its effort to direct the needle aright: on the other hand, when the card is in motion, friction aids magnetism, and both oppose inertia, to bring about a state of rest.

But friction is a vicious helpmate which it were better to curb than give free rein to, for any adventitious assistance it may render.

The weight of the card and needles is considerable, and the pressure on the pivot would be very great—a strong opponent to the moving power, if the bowl were not filled with liquid, which buoys up the card, reduces friction, and incidentally counteracts protracted motion.

Both the pivot and sapphire cap are hard and highly polished in order to reduce friction.

As the card oscillates, its needles excite electric currents in the copper bowl, which oppose its own motion; and thus the needles may be said to put a check upon themselves—one which is the more tightly drawn the greater the motion of the card.

From theoretical considerations as well as experience, it is found that the same weight of steel may be varied in form, quality, and temper, with corresponding variations in its magnetic strength; and as the least weight of

metal for a specific degree of magnetism is the object sought, the efforts that will conduce to that end should be made.

223. Tempering the needles.—By "*needle*," throughout the following articles, is meant the BUNDLE of wires forming one of the magnets attached to the compass-card. If pure wrought iron be highly heated, and then allowed to cool slowly, it acquires a condition said to be *soft;* and this is the most susceptible to magnetism, but it does not abide: a certain percentage of carbon, however, baked into the iron, makes it steel—a hard metal; and this when heated and then suddenly chilled, becomes still harder—*tempered:* this condition is resistent of magnetism, but what little enters, remains. In fact, hardness of metal and retention of magnetism are co-relative, so that the harder the steel, the more tenacious it will be of the magnetism it acquires, and also, the more intense and powerful must be the means employed to give it the highest charge. The condition "glass-hard," or one of extreme brittleness, affords the best and most enduring magnets for compasses.

To temper the needles, they are put into an iron tube, which is placed in a furnace until the wires become bright-red, when they are withdrawn and plunged into cold water; then they will break almost with the facility of glass. The conducting power of the liquid seems to be the essential quality in this procedure: mercury conducts away heat more rapidly than water, and should therefore be better; oil more slowly and hence less suited. Suddenness of chill from high heat affords the best temper for compass needles.

As illustrative of the effect of temper on magnetic moment, an instance will be cited from Coulomb. A steel bar was heated to different temperatures, plunged into cold water at each, magnetized to saturation, and then

the period of ten oscillations noted; these are the results:

Number of trial.........	1st	2d	3d	4th
Heat of tempering.......	875° C.	975° C.	1075° C.	1187° C.
Time of ten oscillations..	93 s.	78 s.	64 s.	63 s.

When tempered *below* 870° C., the bar always made ten oscillations in the same time: from 875° C. upward there was a decided increase in the magnetic moment—first rapidly, then slowly—as shown above by reduction of the time of ten oscillations.

224. Magnetizing the needles.—While the magnetic condition may be induced in a rod of steel by holding it in the Line of Dip and striking it on end, still this will make but a weak magnet; bending, twisting, or pulling the rod will increase the induction; and large bar-magnets drawn over it at the same time will further add to its strength: but all these means fall far short of the powerful effects of electricity.

Bundles of the wires are prepared of the size eventually to be fixed on the card; each bundle is held for a few seconds between the poles of an electromagnet, as at *G*, Fig. 357, Vol. I; the current is a direct one, from a dynamo, and the wires are converted into strong magnets.

Other methods of using the current have been found very efficacious—that, for instance, illustrated by Fig. 371:

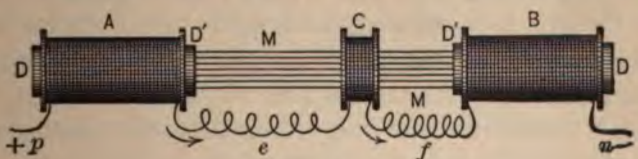



FIG. 371.

A and *B* are two strong electromagnets, between whose poles the bundle of wires is placed; *C* is a small, thick coil of many turns, which is moved back and forth between *A* and *B*, the *same number* of times each way; the current

enters at p, passes from A to C by means of e, thence to B by f, and out at n; both e and f are parts of the wire forming the turns on A and B.

Another method is to place the bundle in the axis of a coil of pure copper wire; the wire should be well insulated and wound on a tube of wood or glass of very thin walls; the length and diameter of the tube to be only a little more than those of the bundle of wires to be magnetized: a tube of metal is likely to become the seat of induced currents which militate against the magnetization.

The efficacy of a helix is reckoned proportional to the number of ampère-turns: we may therefore have one ampère of current and five hundred turns of wire; or ten ampères and fifty turns; or any other number of ampères and turns that will give the same product; and the action of all is said to be the same: but it would seem that to magnetize hard-tempered steel, the preference should be given to small ampèrage and many turns of wire. It is stated that high tension, and not quantity, produces the best results.

While the current flows in the coils of wire round the steel, the *lines of force* pass in at one end of the solenoid and out at the other, and therefore *through* the bundle of wires in its core.

In Articles 191 and 192 a theory of magnetism is set forth: briefly, it is, that the atoms of matter are encircled by currents of electricity—that the magnetic condition becomes manifest when the *axes* of these molecular circuits are turned all one way—and that the neutral state results from their indiscriminate mixture.

Both tempering and magnetizing give a violent wrench to the molecules of a mass of steel—the former may be said to set them as a rigid conglomerate, the latter to turn them into a symmetrical form; and it would seem reasonable that high tension electricity should produce

the best effects—that the sharp, concentrated blow, not the steady, diffused push, were needed.

Both set and blow do not fall equally on all the molecules; irregularity of temper and magnetic condition, not only from wire to wire, but even in parts of the same wire, is frequently observed.

The magnetizing should always be done in the *same direction:* to reverse the poles, heats the steel, which opposes magnetization.

Magnetizing the needles is the most important step toward producing a good compass: if they are strong they readily overcome inertia and friction of the parts; but, if weak, the compass will be unreliable—wabbling with the roll of the ship, and perhaps sticking with some mote that adheres to the pivot.

After final magnetization, the needles should be laid aside, north poles toward the Earth's north for at least six months before placing them in compasses.

225. To determine the relative magnetic strength of the needles.—When needles for a lot of compasses are required, an order is given the manufacturers (Messrs. Ritchie of Boston) to supply them; and many more than the number actually needed are sent to the Superintendent of Compasses for test as to magnetic strength: the best are selected, and those alone used.

The needles are prepared by weight, and sometimes the number of wires in a bundle varies one more or less; before magnetizing, each bundle is tightly encased in a paper tube, in which it remains: this preparation, as well as the tempering and magnetizing, are done by the maker.

Upon receipt, the needles are placed in grooves in wooden blocks, like-poles in the same direction, but separated by about an inch, and all with those poles toward the Earth's magnetic north, that would naturally seek it, if free to move: to save this locution, this pole of the needle will

be called a NORTH pole, though in reality it is a south pole, since it seeks the Earth's magnetic *north:* it is the pole that is *marked* on the paper case.

At first, needles have a higher magnetic charge than they will retain, but it dissipates—rapidly in some, slowly in others: all are tested in the manner to be presently stated—the weaklings are rejected, and only the strong put aside in the same way as before for further trial. This is made again and again during a long period, according to the exigency for compasses, until the necessary number of the best possible ones are obtained: such examination of a single lot has extended over months.

Each needle is weighed and numbered, and a record kept of its varying strength while under trial. When finally passed, the date, weight, and strength are marked on the paper case. The four needles for each card are then selected, placed in a trial-card in the exact positions they will finally occupy, and tested as a magnetic *system.*

The principal tests of magnetic strength are by the methods of torsion, oscillation, and deflection.

The method by Torsion: The torsion balance has been described on page 441; the principle and mode of using it to test needles will now be stated. The instrument, Fig. [illegible], page 440, is cleared of magnets, a copper bar is placed in the stirrup, the suspension wire freed of twist, and the zero diameter of the circle C turned into the magnetic meridian previously found with the magnet *NS* suspended.

The copper bar is now replaced by a needle: it settles along the magnetic meridian, and to deflect it to any angle θ, the circle B must be turned in azimuth to some angle ϕ, [illegible] the angle of torsion is then $(\phi - \theta)$.

It is shown on page 442 that the *force* of torsion is proportional to the angle of torsion; and on page 295, [illegible] moment urging a deflected needle back to the

meridian is $m'.l.H.\sin\theta$; this is proportional to the *sine* of the angle of deflection, since, for the same magnet at the same time and place, $m'.l.H$ is practically constant.

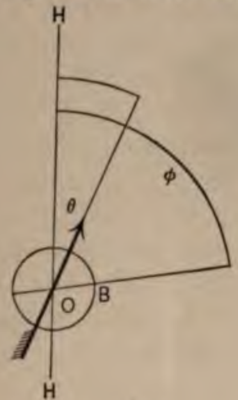


FIG. 372.

When the needle is balanced between the force of torsion and that of the Earth's magnetism, we have

$$m'.l.H\sin\theta = (\phi - \theta), \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or,

$$\frac{(\phi-\theta)}{\sin\theta} = m'.l.H, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and this, according to experiment, is constant (whatever the values of ϕ and θ) with the same needle and suspension wire.

But now if we make θ constant, that is, test all needles at a specific deflection, the angle ϕ will probably vary with each needle, since they are of different lengths (l) and powers (m'); and by (2), H being constant for the same time and place, we thus determine the magnetic moment ($m'.l$) of each needle from the variation of the angle of torsion, $(\phi-\theta)$.

This method is liable to small errors which are set forth on page 446; they are negligible, however, for testing

compass needles, but the method is slow and tedious, though accurate.

The method of oscillation: The principle is this: if a needle be suspended horizontally by a fine wire, and set in motion, it will oscillate slowly or rapidly according to its own magnetic strength; and the square of the number of oscillations in a given time is an index of such strength: if several needles of *identical weight* and *form* be oscillated in the same magnetic field, the square of the number of oscillations of each is therefore an index of its strength. But in any lot of needles there is a slight variation both of weight and form, and as these enter as factors into the moment of inertia to influence the rate of oscillation—*irrespective of the magnetic charge*—this method is not suitable for compass needles; besides, it is slow and tedious.

Deflection—the Tangent method: The method by deflection, on the contrary, is expeditious, easy, and susceptible of great accuracy: its analytical investigation has been given in connection with Figs. 272 and 274, and its practical performance described on pages 274, 275, and 425; but, as it is the method used for testing compass needles, it will be treated specially here.

Figs. 373 and 374 illustrate the principle of the procedure, and Fig. 375 the instrumental means of carrying it out—a *suggested* variation of an instrument already in use. The parts of Fig. 375 will first be named—their use is obvious: G is a long, thick board mounted on foot-screws, K, with levels at D; B is a short cylindrical vessel, preferably glass, set in a recess of the board; it is covered by a glass plate from whose center the tube T rises, containing the suspension wire pendent from the screw S; on an inner projecting ledge of the vessel B, there is a circle C, graduated to 10′ of arc; the magnet M (1½ *inches long*) has aluminum pointers extending over this circle—it is shown apart at M'; L, L are lenses for observing

the pointers and circle—they are attached to a ring H, movable in azimuth on a rim of B; O is a little carriage

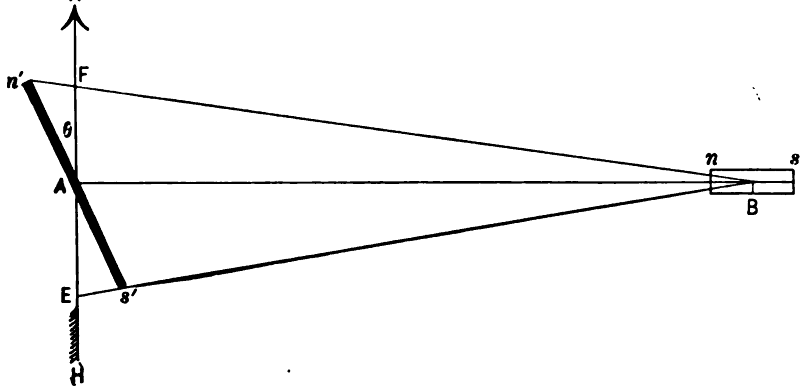


FIG. 373.

(with a groove in the middle) for supporting the needle, which should be on a level with the magnet M; the carriage slides on rails and has a pointer P to indicate the

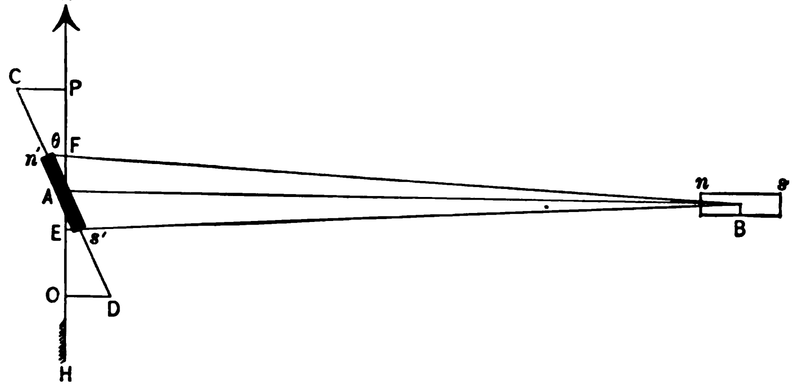


FIG. 374.

distance in inches (E), or centimeters (F), from center to center of needle and magnet.

The procedure of test is this: all iron being removed from the instrument and its vicinity, it is leveled and

adjusted at right angles to the magnetic meridian, so that the magnet M shall hang *in* the meridian when its suspension wire is free of twist and the pointers are at 0° of the circle; the carriage is set at the distance decided upon for all needles of the same length; one of these is placed on the carriage, and the deflection of M noted and recorded with the number of the needle; the latter is turned end for end and the deflection again noted and recorded; the mean of both angles is taken, and the tan-

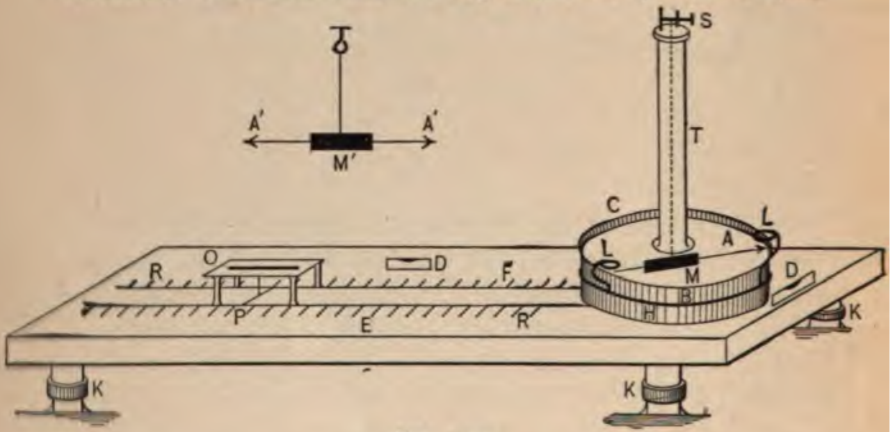


FIG. 375.

gent of this mean deflection is an index of the needle's strength—compared with others tested at the same time and place.

Care must be taken to keep the pointers at 0° of the circle, for there will be fluctuation about it, due to the diurnal change in the Variation: from 9 to 11 A.M. is the best time to test needles.

One advantage of this method is that the needle being across the Earth's field, it is not affected by this in the direction of its axis, and hence the magnetism of the needle alone is determined; but the method is liable to errors whose sources will now be stated.

Consider an extreme case—Fig. 373, and let it repre-

sent actual dimensions of magnet, needle, deflection, and distance; that is, $\widehat{n's'}=30$ mm., $\widehat{ns}=10$ mm., $\theta=25°$, and $\widehat{AB}=90$ mm., or the distance from center to center of needle $\widehat{ns}$ and magnet $\widehat{n's'}$ is three times the length of the latter.

The directive force of the Earth's magnetism may be represented by $\widehat{AF}$ and the resultant force of $\widehat{ns}$—either a push or a pull according to the pole presented—by $\widehat{n'F}$, the magnet $\widehat{n's'}$ resting in equilibrium between both; $\widehat{AF}$ remaining constant throughout the period of test, $\widehat{n'F}$ may alone change with each needle, giving a new value to θ: if the forces were at right angles to each other, we should have $F=E=90°$, and then

$$\tan\theta=\frac{\widehat{n'F}}{\widehat{AF}}=\frac{\widehat{s'E}}{\widehat{AE}}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This is the condition *assumed* in making the *tangents* of deflection represent the relative strength of the needles.

But the angle E is slightly acute and F correspondingly obtuse, hence the actual performance is in error to the extent of this inaccuracy.

Again, supposing the resultant strength of ns located at its middle point, we should have, by the law of inverse squares, for the force it exerts at the point A, $\frac{1}{\widehat{AB}^2}$; at the point n', $\frac{1}{\widehat{n'B}^2}$; and at the point s', $\frac{1}{\widehat{s'B}^2}$; the effect of $\widehat{ns}$ on each pole of $\widehat{n's'}$ should be the same, or the sum of both effects should be double that on one pole, or double the effect at the point A, that is,

$$\widehat{n'B}^2+\widehat{s'B}^2=2\widehat{AB}^2. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

But with numerical values deduced from the dimensions given above, and introduced into equation (4), we should find a difference between its two members, and this indicates the second source of error. A third is, that the push or pull exerted by the needle B on the magnet A is oblique to the latter's length.

ALL THESE ERRORS INCREASE WITH GREATER LENGTHS OF $\widehat{n's'}$, LARGER VALUES OF θ, AND SHORTER DISTANCES OF B FROM A.

But now consider Fig. 374, where the same needle is at B, but deflecting a much shorter magnet at A—only 10 mm. long, and hence $\widehat{AB}$ is nine times the length of $\widehat{n's'}$; θ remains the same. Considering the dimensions of Fig. 374 as actual, measuring $\widehat{n'B}$ and $\widehat{s'B}$, and introducing their numerical values into equation (4), we should find the difference between the two members of the equation less than half the amount of the previous case; and the action of $\widehat{ns}$ upon $\widehat{n's'}$ is more direct—nearly realizing the requirement of (3). The value of θ is still abnormally large: reduced to angles usually employed—say 8° or 10°—it further approximates the method to accuracy—makes it substantially correct.

The conditions, then, that not only conduce to accuracy, but are essential to it, are: a short deflected magnet; small angles of deviation; and long distances between centers of magnet and needle, that is, long compared with the length of the deflected magnet.

The method by Torsion is accurate, and in order that the method by Deflection may be appreciated to its full value, a comparison of both will be made in Table 29—taken from Dr. Scoresby.

The magnets used were all of the *same length*—six inches

TABLE 29.

COMPARISON OF TANGENT-DEFLECTION AND TORSION AS METHODS OF DETERMINING THE STRENGTH OF MAGNETS.

No. of Magnet.	Weight in Grains.	TANGENT METHOD. Distance from deflected needle = 24 in. = 4 lengths of magnet.			TORSION METHOD.	
		Mean Deflection.	Tangent.	Relative Magnetic Strength. No. 1 = 100.	Degrees of Torsion	Relative Magnetic Strength. No. 1 = 100.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	4075	17° 58′	324	100.0	48.0	100.0
2	{ 4075 / −318 }	14 17	255	78.7	38.4	80.0
3	{ 4075 / −640 }	13 15	236	72.8	36.2	75.4
4	2040	9 46	172	53.1	26.0	54.1
5	2050	7 22	130	40.1	19.5	40.6
6	658	4 00	70	21.6	10.8	22.5
7	318	2 50	52	16.1	8.0	16.7
8	145	1 50	32	9.9	5.0	10.4

—but of *diverse mass, form, and strength:* bars, plates, and needles—all hardened steel.

The following is a particular description of them, the numbers corresponding to col. (1) of table 29: 1. combination of 24 bars; 2. the same, with power reduced by placing a thin plate in reversed polar order between the bars as shown in col. (2) by its minus weight (−318); 3. same, power further reduced by inserting two thin plates reversed (−640); 4. combination of twelve plates; 5. a solid bar of one half inch cross-section; 6. a solid bar, one half inch wide, one eighth inch thick; 7. a plate, one half inch wide, one sixteenth inch thick; and 8. a thin needle, three eighths inch wide. Considering the first value, 324, of col. (4) and that, 48, of col. (6), each as 100, the other numbers of each of those columns become, in proportion thereto, the numbers of columns (5) and (7)—that is, the magnetic powers of the respective magnets; and it will be perceived how closely these, cols. (5) and (7), agree for the same magnet, notwithstanding the differ-

ence in weight and composition of the several magnets, the diversity of deflection produced, and the varied degrees of torsion required to deviate each by the same amount.

Deflection—the sine method: The condition expressed by equation (4) may be fulfilled by moving $\overset{\frown}{ns}$ in azimuth, *while deflecting*, until its axis is perpendicular to that of $\overset{\frown}{n's'}$, Fig. 376, when $\overset{\frown}{n'B}$ equals $\overset{\frown}{s'B}$: then by Trig.

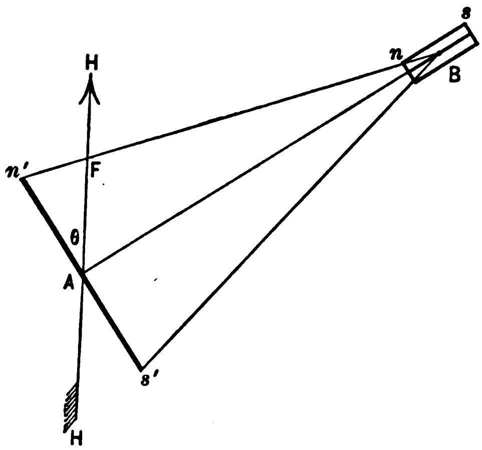


FIG. 376.

$$\overset{\frown}{AF} : \overset{\frown}{n'F} = \sin An'F : \sin\theta, \quad . \quad . \quad . \quad . \quad (5)$$

whence

$$\overset{\frown}{n'F} = \frac{\overset{\frown}{AF} . \sin\theta}{\sin An'F}. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

With needles of varying strength but of the same length, successively placed at B, and at the same distance from A, the angle $An'F$ will be constant; $\overset{\frown}{AF}$, the horizontal intensity, may be considered constant for the period of test, so that θ alone varies, thus giving different values of $\overset{\frown}{n'F}$ for the strength of the needles.

This is an accurate but inconvenient method, for it necessitates turning the board G, Fig. 375, with every needle tested.

For needles of identical form, length, and mass, the method by Oscillation is absolutely correct: an example will therefore be given in Table 30, from Scoresby, of the

TABLE 30.

COMPARISON OF THE *TANGENT* AND *SINE* METHODS OF DEFLECTION WITH THAT BY *OSCILLATION*.

Tangent.				Sine.		
Designation of Magnet.	Mean Deflection.	Tangent.	Relative Magnetic Strength. $A_1 = 100$.	Mean Deflection.	$\widehat{n'F} = \frac{\widehat{AF} . \sin\theta}{\sin An'F}$.	Relative Magnetic Strength $A_1 = 100$.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
A as A_1	12° 30′	221.7	100.0	12° 46′	222.3	100.0
B	8 38	151.8	68.5	8 43	152.4	68.6
C	7 20	123.4	55.5	7 60	124.3	55.9
D	5 16	92.2	41.6	5 18	92.9	41.8
A as A_3	3 36	62.9	28.4	3 33	62.3	28.0

OSCILLATION EXPERIMENT.

A as A_2 performed 21 oscillations in 5 m. 0 s. = rate of 4.2 per minute.
A as A_4 performed 135 oscillations in 17 m. 4 s. = rate of 7.91 per minute.

trial of the same four magnets by Tangent-deflection, Fig. 374, by Sine-deflection, Fig. 376, and by Oscillation.

The magnets were hard steel bars, $\frac{1}{2}$ inch cross-section, 6 inches long, seasoned, and as like as possible, except in power; all were tried at the distance of $19\frac{1}{4}$ inches from the center of the deflected needle; col. (1) contains their designations, and cols. (4) and (7) are derived from (3) and (6) in the same way that cols. (5) and (7) of Table 29 were from cols. (4) and (6) of *that* table.

It will be seen by cols. (4) and (7), Table 30, that the two methods—Tangent and Sine—give almost identical strength for the same magnet.

Bar A was oscillated twice: first, immediately after having produced the deflections opposite A_1; and second, just after those opposite A_3 — its strength having been

artificially reduced after the *first oscillation* experiment. The results of both oscillations are given opposite A_3 and A_4; and the corresponding powers of the magnet are to each other as the squares of the number of oscillations: assuming for the greatest, 100, as in the other cases, we have

$$(7.91)^2 : (4.2)^2 = 100 : x = 100 : 28.19,$$

and this value (28.19)—the magnetic strength at the second trial, is almost the same as its values opposite A_3 by the Tangent and Sine methods.

Thus it has been established by comparison with other methods of undisputed accuracy, and for magnets differing in every way, that the Tangent method is worthy of entire confidence, *provided the essentials to accuracy heretofore stated are observed.*

In connection with Figs. 373 and 374, it was stated that the resultant of both poles of B was supposed located at its center: the real state of the case will now be stated.

Could a small mass of steel be imbued with magnetism of one kind only—a single pole—its effect at any point would vary inversely as the square of the distance therefrom; but in every magnet the two poles are inseparable, and the tendency of the remote one is to nullify the effect of the near one—not even in a constant ratio, but in an increasing one with every remove of the magnet from the point acted upon. Consider the removal to be by multiples of the magnet's own length, and that each pole has the value unity: then when the near pole is one length from a point, the remote one is two lengths from it, and the effect of each pole at the point is $\frac{1}{1^2}$ and $\frac{1}{2^2}$; and the resultant effect is $\frac{1}{1^2} - \frac{1}{2^2} = 1 - \frac{1}{4} = \frac{3}{4}$; when the near pole is two lengths away, the remote one is three lengths, and

the resultant effect is $\frac{1}{2^2}-\frac{1}{3^2}=\frac{1}{4}-\frac{1}{9}=\frac{5}{36}$; similarly, for three lengths' distance of near pole we have $\frac{1}{3^2}-\frac{1}{4^2}=\frac{1}{9}-\frac{1}{16}=\frac{7}{144}$; and for four, $\frac{1}{4^2}-\frac{1}{5^2}=\frac{1}{16}-\frac{1}{25}=\frac{9}{400}$; or, the resultant effect of a real bar with two poles and that of an ideal bar with one pole at the same distance are represented by these two series:

$$\text{real}=\frac{3}{4},\quad \frac{5}{36},\quad \frac{7}{144},\quad \frac{9}{400},\quad \text{etc.} \quad . \quad . \quad . \quad (7)$$

$$\text{ideal}=1,\quad \frac{1}{4},\quad \frac{1}{9},\quad \frac{1}{16},\quad \text{etc.} \quad . \quad . \quad . \quad (8)$$

In the foregoing calculation the poles are supposed to be at the very ends of the bar, but such is not the case; they are a little inside the ends, not enough, however, to vitiate the computations.

In 1882, while the writer was Superintendent of Compasses, wires were introduced to replace the long, thin plates that had been used for compass-needles; and the weights, lengths, and positions (upon the card) of the new wire-needles were decided upon: the *distances* at which the needles were to be tried, were also fixed, but not the angle of deflection. A certain maximum deflection was always kept in view, however, and the effort was not only to attain this for every needle accepted, but to surpass it: this elastic limit, if one may so call it, was an incentive to improve every lot of needles—each to establish its own maximum, but never to fall below the highest average of previous lots.

In order to form an idea of a standard of magnetization, consider (as is practically the case) that one of the wires of which our needles are composed, is *uniformly*

magnetized throughout: if several such wires be united into a bundle, or if a single wire be cut into different lengths, it is evident that the magnetic moment of either bundle or length, divided by its volume, will give a constant; and this constant is called the *intensity of magnetization* of the substance: it may be estimated in C.G.S. units per gram of steel, and has been found, by trial, to be a maximum in steel wire tempered glass-hard, and of which the length exceeds the diameter by sixty times: such wires retain their permanent magnetism almost unimpaired.

By varied experiments with magnets differing in every way—length, width, thickness, and power—and using both the Tangent and the Torsion method for the same magnets, to confirm his deductions, Scoresby established the following law for determining the powers of magnets of *similar form:*

$$P = l^3 \cdot \tan\theta; \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

in which P is the power, l the length of the magnet, and θ the deflection it produces: magnets are to be tried at distances proportional to their own lengths, from the deflected needle.

Incidentally, from these experiments, two facts came prominently out: first, that small magnets were, without exception, stronger, *for their size*, than large ones to which they were proportional in linear dimensions; and second, that the powers of the several magnets were almost identical by both methods of test—Tangent deflection and Torsion—thus again proving the reliability of the deflection method.

Long previous to Scoresby, however, Coulomb expressed the law in general terms—that the magnetic moments of magnets are proportional to the cubes of their homologous dimensions; and this he proved by extensive experiments with the torsion balance in the manner already

described, and with a great number of solid bars of diverse form, cut from the same mass of steel.

As another part of Coulomb's work bears directly on our wire-needles for compasses, an account of it will be given.

He selected a length of 120 feet of very pure wire just as it left the *filière*—a heavy plate pierced with holes of varied size through which the wire is successively *drawn* until it is of the required diameter. The wire was cut in different lengths, and each piece twisted about its axis while it was kept stretched by a weight at the lower end: this twisting was a means of hardening it—a kind of tempering. The wires were then magnetized to saturation, and tied in bundles of different length and different diameter, by fibers of silk. Each bundle was tried in the torsion balance by deflecting it to the same angle from the meridian by twisting its suspension wire. The general result of these experiments was, that for bundles of similar form, the magnetic moments are proportional to the cubes of the homologous dimensions.

In the test of needles, they should never be subjected to undue strain—more than they will experience in service; for it needlessly injures them.

226. Mutually destructive influence of magnets.—If a charge of static electricity be communicated to the inside of a spherical shell, it will speedily find its way to the outer surface; if a current of electricity be sent through a cable composed of many strands, it will flow first in the outer wires, because induction impedes its rapid growth in those near the core; if a steel magnet be immersed in acid, it will be found to have lost most of its virtue when only a moderate layer has been dissolved away; and if a number of bar magnets—cut from the same mass of steel, tempered and magnetized alike—be tested for strength, first individually, and then collectively (up to any number bound

together), it will be found that the power of the combination is not the sum of the individual powers, but much less; and that when the group is broken up and all tested singly again, it will be found that they have lost strength—variously—some falling off to almost nullity, and others having their poles reversed—and generally the weaklings are those that occupied the middle of the group, while the outer layers almost invariably suffer least: these facts, established by experiments of the most eminent investigators, prove the tendency of the electromagnetic condition to seek the surface of bodies; or rather, the tendency of the condition to neutralize itself in the inner recesses of its abode.

The fact is indisputable, so that only a few typical cases will be adduced to illustrate it and pave the way to showing the effect in the combination of wires to form needles, and of these to constitute the directing power of the compass-card.

Consider Table 31 from Scoresby: The magnets of

TABLE 31.

RATIO OF INCREASE OF MAGNETISM WITH MASS IN *SOLID* BARS.

Designation of Magnet.	Weight in Grains.	Highest Power.		Reduced Power.		Tenacity of Hold Upon Magnetism.	
		Mean Deflection.	Tangent.	Mean Deflection.	Tangent.	Difference of Tangents.	Loss per cent.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
E	2050	38° 20′	791	....	....	....	....
F	1404	31 50	621	28° 15′	537	84	13.5
G	786	27 14	514	23 54	443	71	13.8
H	340	14 15	254	1 50	10	235	92.5
K	175	10 00	176	9 50	161	15	8.5

col. (1) were *solid* bars—each six inches long, one half inch wide, and $\frac{1}{32}$, $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$ inch thick, respectively, thus forming a regular series whose volumes bore the constant ratio 2: they were tempered hard, magnetized to saturation, and tested at a uniform distance of twelve

inches; if the power increased as the mass, the quantities in col. (4) should form a series with a constant ratio 2; but this is not the case—far from it; magnet *E*, which has seventeen times the mass of *K*, should cause a deflection whose tangent is $176 \times 17 = 2992$, whereas it is only 791. After producing the deflections of col. (3), each bar was placed with its sides successively in contact with a powerful magnet: this tested the hold it had upon its magnetism; col. (6) shows the result—all lost, and magnet *H* made such little resistance as to part with 92 per cent. (col. 8).

The increase of power with mass in *laminated* magnets is shown by Table 32, from Coulomb.

TABLE 32.

Number of plates together.....	1	2	4	6	8	12	16
Angles of torsion	82°	125°	150°	72°	182°	205°	229°

From a long, thin sheet of steel, 21 mm. wide, sixteen plates, each 162 mm. long, were cut: they were magnetized to saturation, and bundles formed by successively placing 2, 3, 4, . . . , 16 plates side by side, and testing each bundle in the torsion balance.

The first line shows the number of plates in combination; and the second, the torsion required to deflect the bundle 30° from the meridian. If the power were proportional to the mass, the final bundle should require a torsion of $82° \times 16 = 1312°$ to deflect it 30°, instead of 229° only, as it did. On taking the last bundle apart and testing each plate separately, it was found that the outside one was strongest and the middle weakest, with quite a regular gradation between.

In order that it may not be inferred that the destructive effect is due to size (the magnets used for Tables 31 and 32 being small), the following experiments by Scoresby are cited; they relate to heavy 12-inch bars, alike in cross-

section and of the hardest steel. Ten were placed in combination by adding one at a time, and observing the deflection produced by the growing pile at a distance of 24 inches: 1 bar = 19° 5′; 2, = 27° 20′; 4, = 39° 20′; 6, = 46° 20′; 8, = 48° 30′; and 10, = 48° 50′. Again, with eight selected bars of the same kind: 1 bar = 19° 5′; 2, = 30° 20′; 4, = 41° 0′; 6, = 47° 40′; and 8, = 51° 34′. While the destructive effect is prominent, it is also most noticeable how small the increment of magnetism is from two heavy bars in the final groups—in the first case only that represented by 20′ and in the second by 3° 54′, whereas these bars of themselves should each produce a deflection of about 19°.

The tendency of magnetic power toward a limit, even while the mass goes on increasing by equal amounts, is forcibly illustrated by Table 33, from Scoresby. The

TABLE 33.

No. of Plate.	Order of Combination.				Ratio of Increase of Power.			
	Direct.		Converse.					
	Deflection.	Tangent.	Deflection.	Tangent.	Mean of Cols. (3) and (5).	Gain of Power.	Power Individually.	Loss by Combination.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	16°.8	289	16°.3	298	293	293	290	....
2	29 .3	555	27 .5	527	541	248	580	39
3	38 .2	791	36 .2	734	762	221	870	108
4	45 .6	1004	42 .2	911	957	195	1160	203
5	51 .0	1235	46 .4	1058	1146	189	1450	304
6	54 .2	1392	50 .2	1202	1297	151	1740	443
7	57 .2	1563	53 .5	1368	1465	168	2030	565
8	59 .1	1678	56 .8	1490	1584	119	2320	736
9	60 .5	1794	57 .5	1589	1691	107	2610	919
10	61 .6	1875	59 .5	1670	1772	81	2900	1128
11	63 .1	1980	60 .3	1736	1858	86	3190	1332
12	64 .4	2056	61 .3	1808	1932	74	3480	1548

magnets designated by number in col. (1) were plates 24 inches long, 1.5 inches wide, and 0.042 of an inch thick; they were tempered hard and thoroughly magnetized. In order to subject them to uniform violence, a pile was made of the whole number, and this was varied so that those

on the exterior interchanged place with those in the middle, the top with the bottom, and one side with the other. They were then numbered, each tested at one length from the deflected needle, and a record made of the deflections; these varied between 15° and 18°. Next, the plates were magnetized anew and tried as recorded in Table 33—that is, the strongest, No. 1, was placed at the distance of one length from the deflected needle, producing the deviation 16°.8 of col. (2); the next plate in strength, No. 2, was laid upon the first, and the deflection increased to 29°.3; and thus, plate by plate in the decreasing order of power was added until the last and weakest, No. 12, was on the pile, when the whole deflection reached only 64°.4. The pile was now broken up and again formed, but in the converse order, from weakest to strongest, with the result shown in col. (4). The mean of cols. (3) and (5) is col. (6); col. (7) shows the increment of power due to each added plate, and it will be noted how small this is toward the end—the increment of No. 2 being 248, whereas that of No. 12 is only 74; these increments are the difference between the successive tangents of col. (6). Individually, the plates under test exhibited much uniformity of strength: the average of the deflections was 16° 10′, tangent 290; assuming this as a basis and that each plate varied but little from the average, they should, when added one at a time to a pile, produce deflections whose tangents are given in col. (8) *if no destructive influence were at work;* but there was, and the difference between cols. (6) and (8), given in col. (9), shows the enormous destructive influence due to combination.

As between *solid* and *laminated* magnets, the following comparison (data taken from Scoresby) leaves no doubt: a single bar weighing 6.23 lbs. produced a deflection of 54° 12′; a magnet from the same steel, of similar dimensions, tempered and magnetized like the first, and weigh-

ing 6.15 lbs., *but composed of fifteen plates*, produced a deflection of 64° 30′ at the same distance.

To make a similar comparison for solid cylinders and tubes, Daguin states that Nobili tried two cylinders from the same steel, of the same length and exterior diameter, tempered and magnetized alike, but one solid and the other bored out; the latter produced a deflection of 19°, while the former only 9°.5 at the same distance, although the solid cylinder had almost double the weight of the hollow one: also, that De Haldat varied the experiment by magnetizing a tube by means of electricity; 1st, when empty, 2d, when closed by a tightly fitting solid-steel rod, and 3d, when compactly filled with iron filings; the magnetic power was the same in all cases, hence the empty tube, as being lightest for the same effect, had the advantage.

The effect has been tried of separating the plates in a bundle: thirty, as like as possible in every respect, were placed in one pile so that their surfaces touched, producing a deflection of 25°.5; then little blocks of wood, 0.14 of an inch thick, were interposed between every two plates and the pile thus formed, produced a deflection of 35°.5—as was to be expected.

From the foregoing facts, two principles stand clearly forth to guide in the attainment of the greatest power in compass needles: first, they should be built of parts—not a solid mass; and second, these parts should come in contact as little as possible.

Separation by blocks only introduces dead weight—a drag equal to the increased effort, besides forming a clumsy mass for attachment to the card.

Since, then, actual contact is unavoidable, that form of element which will bring the fewest points in touch, is the best; and this is a cylinder: and by reducing its diameter to the smallest practicable, we partly attain the

advantage of the tubular magnet, without boring. Indeed, when the needle is formed of several wires, it is as if we had a tube of many cores; and even in the actual case, contact exists only along mere surface threads of the wires.

Therefore, the wire realizes the best form and size for the primary element; and besides, it can be tempered harder and more uniformly, and magnetized more thoroughly than plates or other masses.

Experiment has shown that the *ratio* of diameter to length in a rod of best steel, temper, and magnetization, is a factor in the effect of a temporary magnetic field upon it: when this ratio is about 10—that is, a magnet comparatively short and thick—the effect is considerable; but with a ratio of 60 and upward—in reality a long, slender magnet—the induced effect is imperceptible: the wires for the two sizes of needle of our compasses have dimensional ratios of 70 and 84 respectively; and as they are of the very best steel, tempered to the utmost hardness, and magnetized by a powerful electric current, they should be free from change in the magnetic field of either Ship or Earth.

The influence upon each other of the wires forming a needle, and of the needles constituting the directive power of a card, will now be illustrated by experiments made by the writer: they relate to wires and needles that are fairly representative—such as would be acceptable for new compasses.

The needles were of two lengths, 5.3 and 4.4 inches, but of the same diameter, 0.3 inch, and were composed of 36 and 32 wires respectively, each 0.06 inch diameter.

Table 34 refers to a 4.4-inch needle: it was received in a lot June 30, 1898, laid aside (*n*-pole toward the Earth's north) until July 28, 1898, and then tested in different ways, as stated below, by the method of Tangent-deflec-

TABLE 34.

Needle R: 4.4 in. long; 0.3 in. diameter; weight, 375 grains; 32 wires, each $\frac{1}{32}$ inch in diameter, and of 11.7 grains. Distance in all cases 14.2 inches from *center* to *center* of deflect*ing* and deflect*ed* magnets.

Designation of Wire.	*N*-pole acting.	*S*-pole acting.	Mean.	Number of Wires acting together.	*N*-pole acting.	*S*-pole acting.	Mean.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
a	0° 20′	0° 20′	0° 20′	1	0° 20′	0° 20′	0° 20′
b	30	25	27.5	2	0 50	0 45	0 47.5
c	25	25	25	3	1 10	1 10	1 10
d	25	25	25	4	1 30	1 40	1 35
e	25	25	25	5	1 55	2 0	1 57.5
f	15	10	12.5	6	2 0	2 10	2 05
g	20	15	17.5	7	2 30	2 30	2 30
h	15	25	20	8	2 50	2 55	2 52.5
i	25	25	25	9	3 10	3 10	3 10
j	20	20	20	10	3 40	3 40	3 40
k	25	25	25	11	3 50	4 0	3 55
l	30	30	30	12	4 10	4 15	4 12.5
m	25	25	25	13	4 30	4 35	4 32.5
n	30	30	30	14	4 50	5 0	4 55
o	15	05	10	15	5 15	5 0	5 07.5
p	30	20	25	16	5 35	5 15	5 25
q	25	15	20	17	5 45	5 55	5 50
r	30	25	27.5	18	6 0	6 10	6 05
s	25	25	25	19	6 30	6 20	6 25
t	30	20	25	20	7 0	6 35	6 47.5
u	20	15	17.5	21	7 05	7 0	7 02.5
v	25	25	25	22	7 30	7 15	7 22.5
w	30	25	27.5	23	7 45	7 35	7 40
x	30	20	25	24	8 0	8 0	8 0
y	25	20	22.5	25	8 20	8 20	8 20
z	15	10	12.5	26	8 35	8 25	8 30
a′	20	20	20	27	8 45	8 50	8 47.5
b′	25	25	25	28	8 55	9 0	8 57.5
c′	10	30	20	29	9 15	9 15	9 15
d′	15	20	17.5	30	9 30	9 45	9 37.5
e′	15	30	22.5	31	9 45	10 0	9 52.5
f′	25	15	20	32	10 0	10 0	10 0

Deflection produced by needle *R* both before beginning and after ending the above series of experiments, 10° 0′ with each pole, distance 14.2 inches from center to center of magnets.

tion; the distance in all cases was measured from center to center of the deflect*ing* and deflect*ed* needle. First, as a whole—in its paper case—each pole produced a deflec-

tion of 10°. Then the wires were pushed *en masse* out of the case, and spread on a table parallel to each other, half an inch apart, *n*-poles toward the Earth's north. Each wire was now tested by itself and produced the deflections of cols. (2) and (3), Table 34: inspection of these will show the individual character of the wires. Next, wire was added to wire as in col. (5), the growing bundle producing the deflections of cols. (6) and (7): the wires were kept close together by tying the bundle at the ends with thread, which was loosened and tied anew every time, and the wire was laid beside the others, not pushed along them through the loops of the thread, *as in a former experiment this practice greatly increased the power of the bundle*—it was a kind of remagnetizing. Finally, when the last wire had been added, the bundle was inserted in its paper tube and tested as at first, producing the same deflection, 10°.

The increment of power with each added wire may be found by taking the difference of the tangents of the angles in col. (8); but the effect of combination will be shown in another way: the sum of col. (4) is 715′, that of col. (8) is 605′; their difference, 110′ = 1° 50′, represents the destructive effect—about 3′.4 per wire. Of the 32 wires, only four—*f*, *o*, *u*, *z*—show individual weakness; while in the combination, the 6th, 15th, 26th, 28th, and 32d added less than the average increment to the total power: this affords a means not possible with *solid* magnets, of weeding out the puny and making needles of only strong elements.

Four needles—two of each length—constitute the directive power of a card, and pairs are selected so that both needles of each size are almost identical in weight and magnet power. The influence of the needles upon each other will now be shown: Table 35 contains data of four, designated as *L*, *M*, *P*, *Q*. When *L* and *M* were inserted in their own metal tubes on the card, as in Fig.

TABLE 35.
DATA RELATIVE TO FOUR COMPASS-NEEDLES.

Designation of Needle.	Length in Inches.	Diameter in Inches.	Weight in Grains.	Deflections.		Mean.	Distance; Center to Center of Needles.
				n-pole.	*s*-pole.		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
L	4.4	0.3	385	10° 0′	9° 0′	9° 30′	14.2 inches
M	4.4	0.3	382	9 45	9 0	9 22	14.2 "
P	5.3	0.3	457	9 45	10 0	9 52	15.7 "
Q	5.3	0.3	463	10 0	10 0	10 0	15.7 "

377, they jointly produced these deflections: *n*-poles acting, 13° 30′; *s*-poles, 14° 0′; mean, 13° 45′; distance 15.7 inches. The sum of their individual efforts (from Table

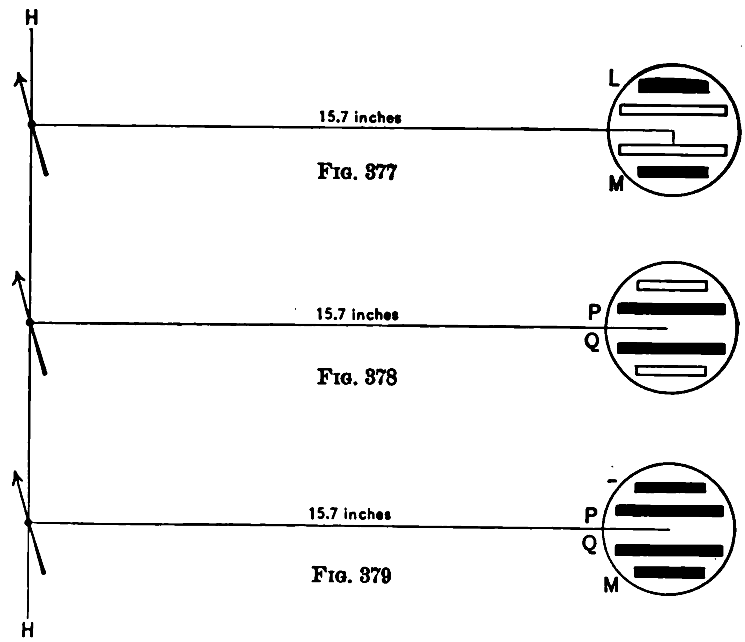


FIG. 377

FIG. 378

FIG. 379

35) is 18° 52′, so that there is an *apparent* loss of 5° 22′ by placing them as in Fig. 377; but this is not really so, for the distance in the *joint* test is 15.7 inches whereas it was only 14.2 in the trial singly.

Removing L and M from the card, and inserting P and Q in *their* own metal tubes, as in Fig. 378, the joint deflections were: *n*-poles acting, 18° 30′; *s*-poles, 19° 0′; mean, 18° 45′; distance, 15.7 inches. The sum of the individual deflections at the same distance (from Table 35) is 19° 52′, so that there is a real destructive influence of 1° 07′ due to their proximity as in Fig. 378.

Now all four needles were inserted in their proper places on the card, as in Fig. 379, and tested at the distance of 15.7 inches, with this result: *n*-end of card acting, 29° 30′; *s*-end, 30° 0′; mean, 29° 45′. The sum of the deflections of the two *pairs* when each pair acted by itself at the same distance, 15.7 inches, is 32° 30′, so that by the combination of all four there is a real loss of 2° 45′ over their effort as pairs. But that the needles hold tenaciously ever after to their magnetism is shown by an instance that will now be cited.

In 1884, after wires had been adopted for needles, a card was prepared: its needles represented the average of a lot then intended for new compasses—not specially selected, but in all respects like those described in Tables 34 and 35. The needles were sealed in their metal tubes on the card as in Fig. 379, and were thus tested as a whole in 1884, at the distance of 15 inches from the center of the card to the center of the deflect*ed* needle; these were the deflections: *n*-end of card acting, 30°; *s*-end, 30°. When tested in 1898, in the same way at the same distance, the deflections were: *n*-end of card acting, 26° 30′; *s*-end, 26° 30′. This card was never used in any compass, but between 1884 and 1898 lay midst pieces of iron and steel and magnets heterogeneously mixed, and in the periodical moving of articles about, was battered and subject to jars and ill-usage such as no compass would ever experience; and yet under this severe treatment its original strength fell off only by 3° 30′ in fourteen years: or, to state the

matter more accurately, the tangent of 30° is 5774 and that of 3° 30′ is 612; the former represents the original strength, the latter the loss—about 10 per cent. of the whole—*not near one per cent. a year*—with the needles still powerful enough to be used in any compass.

227. The liquid used in the Compass.—The primary object of the liquid is to buoy up the card and lessen its pressure on the pivot; incidentally, it steadies its motion and reduces the oscillations to three or four short, quick movements.

The card complete, with the four needles sealed in it, is very heavy, and although cap and pivot come in contact only in a mere point, still the pressure is so great in air, that the powerful needles would never move the mass; but in liquid the buoyancy is so adjusted that this card of 3054 grains exerts a pressure of only 70 grains on the pivot, thus facilitating rapid return of the needles to their natural direction from even the slightest deflection.

The qualities of a liquid are prime factors in its selection; the particles must have the freest movement among themselves, and the liquid must not become viscous like glycerine, nor freeze at ordinary temperatures: alcohol most nearly fulfills these requirements, and it is claimed that by means of liquid air the impurities usually found in the commercial article may be successively frozen out—the water at 32° F. and the fusel oil and other deleterious ingredients at 150° to 190° below zero, leaving the alcohol pure; this freezes at a lower temperature than 200° F. below zero. Alcohol is the ideal liquid: the actual one used, however, is a mixture of 45 per cent. alcohol and 55 per cent. distilled water. Equal volumes of alcohol and water whose sum (when separated) is 100, will be only 94 when mixed—a more dense liquid; but on the other hand, these two components are among the greatest absorbers of heat, which again facilitates mobility of their

particles and motion of the card. The expansibility with heat necessitates a flexible chamber in the bowl, to prevent bursting; and when the liquid cools, the chamber contracts, so that by this means the liquid is made to fill the bowl exactly at all temperatures. Between 32° and 212° F., water increases its volume by $\frac{1}{23}$ and alcohol by $\frac{1}{9}$.

The liquid saves the pivot and cap from injury by concussion and jar such as would otherwise occur—it renders all motions of the card smooth and easy.

228. Metals excluded from compass construction.—There are only three metals in which the magnetic condition may be developed to such degree as to affect the compass injuriously—iron, nickel, and cobalt; and if a bowl be made of any one of these, it can readily be seen from Art. 173, wherein the injury lies—it is in the metal gathering unto itself those lines of terrestrial magnetic force that should give direction to the needles.

The three metals named are injurious in different degrees; but as massive parts, or plating, or fittings of any kind, they should be rigidly excluded from everything surrounding the needles: pure copper is alone suitable for the massive parts, and gold or silver for plating, or small parts.

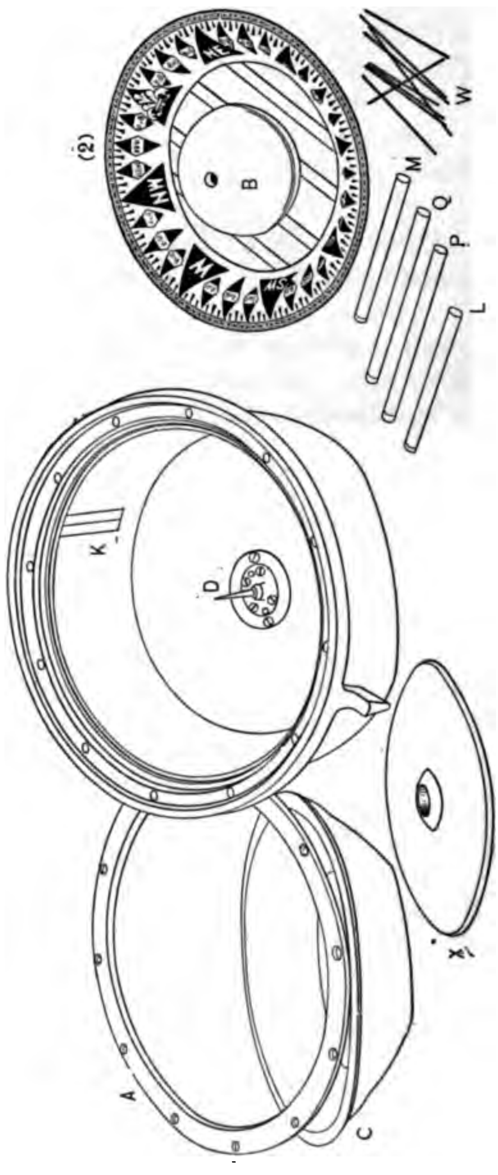
All metals for compass construction and binnacles are examined by the Superintendent of Compasses as to their non-magnetic character.

229. The card, pivot, cap, and bowl.—The compass will be further illustrated by Plates *N* and *O* and Figs. 380 to 385; and in them, the same letters and numbers refer to the same parts in all.

The bowl is seen at (1), Plates *N* and *O*, both in perspective and section; it is of cast bronze, turned to a smooth surface in a lathe, dipped in liquid tin and lead to coat it, and painted white inside and black outside;

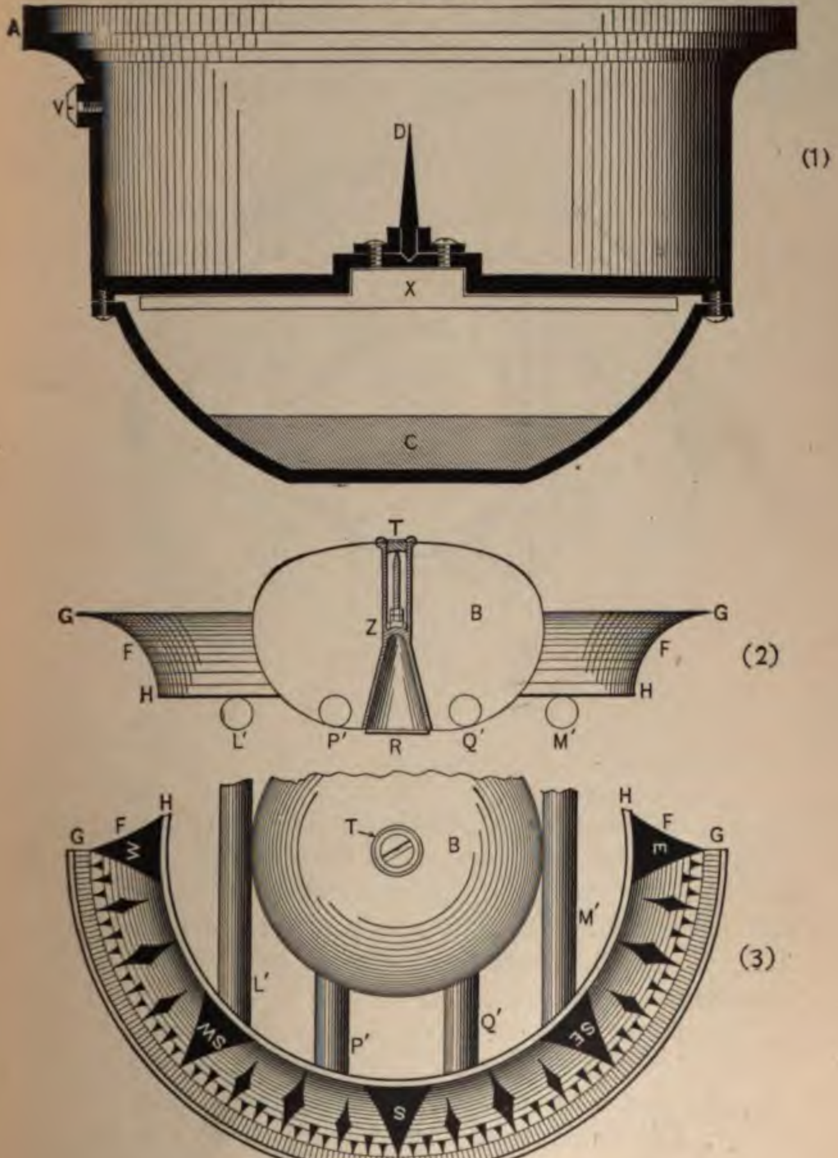
PLATE N.

(1)



Parts of Compass-bowl: Card, Needles, and Wires.

PLATE O.



(1) Vertical section through center of compass-bowl.
(2) Vertical section through center of card.
(3) Top view of card.

K is the "keel-line"—a name I have tried to introduce instead of "lubber's-line"; it is painted on porcelain, which is firmly set in a groove in the bowl; there are two, 180° apart, and the diameter of the bowl thus indicated,

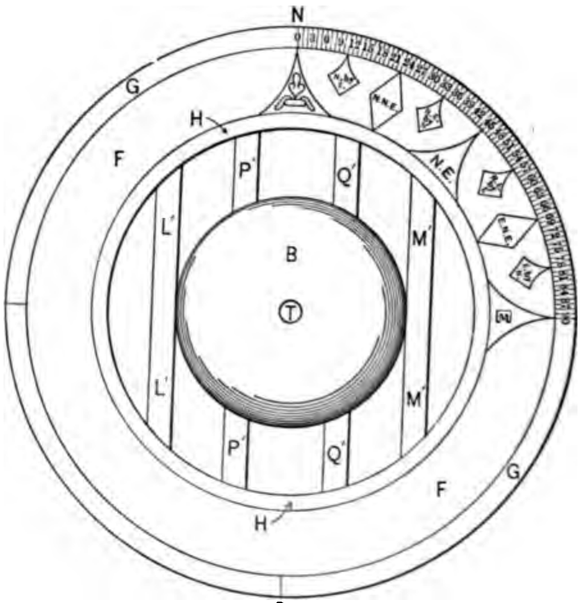


FIG. 380.

is accurately set in the midship vertical plane through the keel; it is the line that points out the compass course, and should divide the ship symmetrically in two equal sections whence the appropriateness of the name, for it indicates the line of the keel, whereas lubber's-line has no such innate signification.

The pivot is shown at *P*; it is made of bronze with hardened tip, and ground down smooth, and polished; Fig. 381 shows the method of setting it in place; *N* is a [illegible] like an inverted saucer, screwed to the bottom of the bowl; upon this, a plate *S* is screwed, and the pivot

D is screwed into S, projecting through it to rest on N; the screws of the plate S can be raised or lowered to center D, and the method of doing this is shown in Fig. 384;

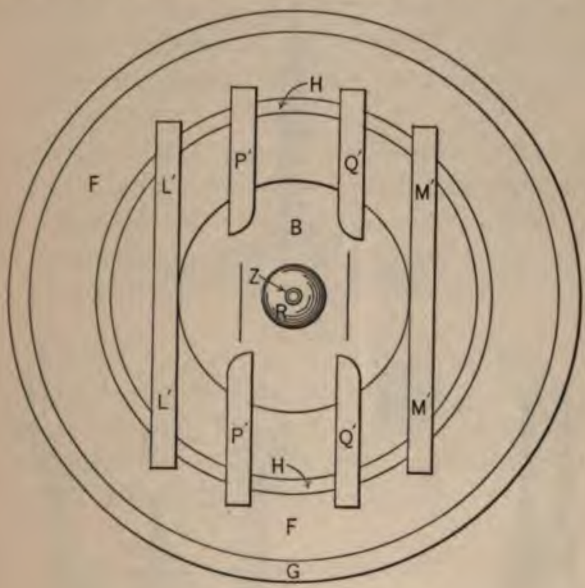


FIG. 381.

a metal ring A', like an azimuth circle, is placed on the rim of the bowl; a cross-bar of this ring supports an inverted pivot D' and a rod with a prism U; this reflects up to the eye the tips of both pivots, and if they appear in the same line while A' is revolved, the lower pivot is then centered exactly.

Below the bridge N, the expansion chamber X, of sheet brass, is soldered to the bottom of the cylindrical part of the bowl, and there is a hole in the bridge to permit free flow of the liquid into the chamber. A hemispherical bottom C, weighted with lead, is screwed to the bowl—to ballast it. The top is closed with glass, rubber packed, which is kept in place by a metal ring A. When the bowl

is filled with liquid, and the card on its pivot, it is placed under the receiver of an air-pump and exhausted, the final addition of liquid, to fill the bowl and chamber com-

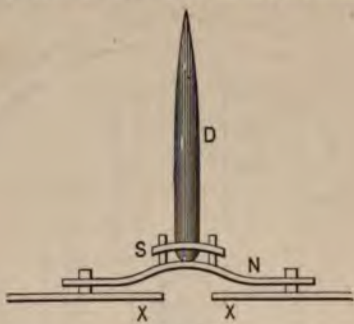


FIG. 382.

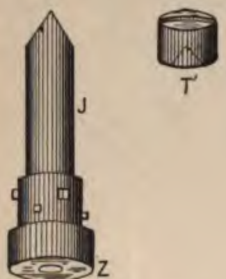


FIG. 383.

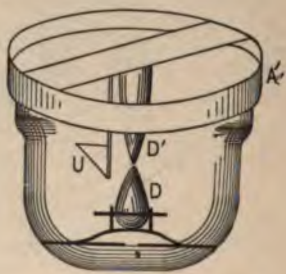


FIG. 384.

pletely, being made through the screw-hole *V*; thus no air bubbles remain in the bowl.

The card is seen in perspective and section at (2), Plates *N* and *O*, with top and bottom views at (3), Plate *O*, and in Figs. 380 and 381; it consists of several parts: *B*, a central ellipsoid which gives buoyancy; *F*, a curved surface for the graduation in degrees and points; *L'*, *M'*, *P'*, *Q'*, metal tubes soldered to both the preceding parts, *P'* and *Q'* running through the ellipsoid as shown in Fig. 381; the needles are sealed in these tubes; they are shown in their paper cases at *L*, *M*, *P*, *Q*, Plate *N*, and beside them at *W* is a group of the wires of which they are com-

posed; the ellipsoid is formed of two parts soldered together; a conical opening is at *R*, (2) Plate *O*, into which the pivot projects; the sapphire cap at *Z* is fixed to the bottom of a spindle *J*, shown in Fig. 383; this spindle is placed in the tubular chamber *T*, (2) Plate *O*, and rests upon a ledge; it is firmly secured in place by a screw seen on top, and at *T′*, Fig. 383.

The cap is a sapphire—a jewel next to the diamond in hardness; a conical depression is ground in it and highly polished; into this the point of the pivot enters: both the sapphire and pivot are examined under a microscope by the Superintendent of Compasses for roughness, flaws, or cracks.

The division into points is painted by hand—that in degrees is printed on linen from an engraved plate and pasted on a flat mica ring which is then attached to the rim of the card; the 0° of the graduation is at North, and the magnetic axis of the *system* of needles is made to coincide with it.

The material of all parts of the card is very thin sheet brass, tinned, and united by thin layers of solder. The separate parts—ellipsoid, cone, curved-surface, rims, and tubes are all stamped out on dies or heavy formers of steel, and are put together on a metal model, working from a center, so that everything on and about the card is symmetrical with respect to this center—the pivot of the compass.

The *intersection* of the diameters through the gimbal bearings coincides with the pivot.

The card is 7½ inches in diameter, and when complete with needles sealed in their tubes, weighs on an average (in air) 3056 grains, of which 1648 grains are magnetic steel and 1408 other matter. The size of the ellipsoid is designed by its buoyancy, to reduce this pressure; and the means of finally determining the weight on the pivot

is shown in Fig. 385: a large glass jar is partly filled with the liquid used in the compass, at a temperature of 60° F.; the card is first placed on the pivot *D* to see if it hangs level; then it is suspended from one arm of a balance and weights are placed in the pan *E* until equilibrium is obtained with the card clear of the pivot; the dimensions of the ellipsoid are so calculated that the weights in *E* range between 60 and 75 grains for different cards, and this is

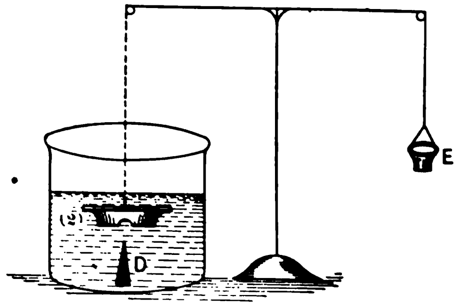


FIG. 385.

all the pressure the card of 3056 grains exerts upon the pivot.

The *directive* force upon the card consists of two factors—the Earth's horizontal component and the magnetism of the needles themselves: each is variable, the former with geographical position, the latter with lapse of time—and both through the ship's magnetism with every change of course. Thus it is important that this variable directive power should have the least resistance possible to contend with, lest at times it be little stronger than the resistance and move the card only languidly, or fail to do so altogether.

Friction between cap and pivot is the principal opponent of the power of the compass: and friction depends upon the pressure of the card, the area of the surfaces in contact and the hardness and smoothness of the materials.

It is conceivable that a viscous liquid like glycerine,

or one that may become such at low temperatures, would not only impede the quick movement of the card, but stop it entirely; and this the more easily, the lower the magnet power and greater the friction: but such conditions do not prevail in our compass—quite the contrary: the liquid is among the most mobile and least prone to viscosity; the magnet power is the greatest attainable for the size of the system; and the friction is an absolute minimum, for cap and pivot come in contact only in a point, and both cap and pivot are of the hardest material, highly polished, and the bearing of one upon the other can scarcely be qualified as pressure, it is so light.

230. **Formulas relative to magnetic moment of the compass.**—The magnetic moment of a system of needles on a card may (in part) be determined by an oscillation experiment; and the formulas requisite for the calculations are those of circular motion in general adapted to magnetic force as the motive power: they are differential expressions, and when integrated with respect to the function t (time) that enters into them, afford the period of oscillation.

This is the usual procedure, but for the purpose of this article, another course will be pursued: it will show more clearly the applicability of the method of oscillations to the determination of magnetic force.

Gravity tends to bring a swinging pendulum into line with its own direction, and the magnetic force of the Earth does the same with a dipping needle in oscillation—indeed this is but a *magnetic* pendulum. The pendulum will have a definite period of swing—so will the needle: this period depends on two quantities—the intensity of the directive force, and the form and weight of the moving body; varying either force or body will change the period. The formulas for calculating each quantity will now be deduced.

Consider Fig. 386: a leaden pellet C hangs by a fine wire from a fixed point O: when drawn to A, and let go,

it will swing to an equal height on the other side, and continue to oscillate until friction with the air brings it to rest in the vertical position.

At every point of the arc ACZ the force of gravity g acts vertically; resolving it in the direction of the wire

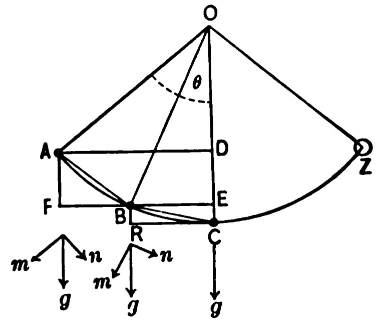


FIG. 386.

and at right-angles thereto, the component m is neutralized by the point O, and n alone tends to give the pellet motion.

The arc in Fig. 386 is greatly enlarged for clearness of illustration, but in reality the demonstration that follows is applicable only to a *small* arc, which ACZ must be considered, so that a portion of it, as AC, may not differ appreciably from its chord.

In its descent, the pellet gradually increases its velocity until the point C is reached; in its ascent toward Z, the velocity gradually diminishes: at any point B, the velocity is the same that it would be at F, if—unattached to any wire—the pellet should fall freely from A to F, the vertical distance between A and B. By Mechanics, this vertical velocity is

$$v=\sqrt{2 . g . \widehat{AF}} . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Let

$$\widehat{OC}=l, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

the semi-diameter of the arc through which the pellet swings;

$$\text{arc } \widehat{BC} = \text{chord } \widehat{BC} = x; \quad . \quad . \quad . \quad . \quad (12)$$

and

$$\widehat{AC} = s, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

the half amplitude of oscillation. By Geom., the chord is a mean proportional between the diameter and its projection on this diameter: whence

$$\widehat{EC} : \text{chord } \widehat{BC} = \text{chord } \widehat{BC} : 2.\widehat{OC}. \quad . \quad . \quad (14)$$

$$\widehat{DC} : \text{chord } \widehat{AC} = \text{chord } \widehat{AC} : 2.\widehat{OC}. \quad . \quad . \quad (15)$$

In Fig. 386,

$$\widehat{AF} = \widehat{DE} = \widehat{DC} - \widehat{EC}. \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Whence, from (14) and (15), by means of (11), (12), and (13),

$$\widehat{EC} = \frac{\widehat{BC}^2}{2.\widehat{OC}} = \frac{x^2}{2.l}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

and

$$\widehat{DC} = \frac{\widehat{AC}^2}{2.\widehat{OC}} = \frac{s^2}{2l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Substituting (17) and (18) in (16), and then this in (10), the latter becomes

$$v = \sqrt{2.g\left(\frac{s^2}{2.l} - \frac{x^2}{2.l}\right)} = \sqrt{\frac{g}{l}(s^2 - x^2)}. \quad . \quad . \quad (19)$$

If $x = 0$, we get the maximum velocity (at C) that is,

$$v' = s\sqrt{\frac{g}{l}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

If $x = s$, we obtain the velocity (v''') at the limits of oscillation (A and Z), that is,

$$v''' = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

To find the *time* of swing, or interval from departure of the pellet at A to its arrival at Z: construct Fig. 387 so

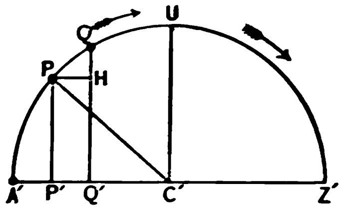


FIG. 387.

that the line $A'C'Z'$ shall be the same length as the arc ACZ of Fig. 386 when straightened out, that is,

$$\widehat{A'C'} = \widehat{AC} = s. \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Upon $\widehat{A'Z'}$ as a diameter, draw the semi-circle $A'UZ'$; and along this $(A'UZ')$, suppose a body to move *uniformly* with the velocity

$$v' = s\sqrt{\frac{g}{l}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

the successive projections of this *uniform* motion upon $A'C'Z'$ will form unequal spaces along this diameter, and the velocity at any point P' will be identical with the velocity for a corresponding length $\widehat{AB}$ of the arc ACB, Fig. 386. To prove this, let P be any position of the moving body, and P' its projection upon $A'Z'$: while the body moves uniformly from P to Q with the velocity v', its projection moves with a variable velocity from P' to Q'—denote it by v''; if the spaces and the time required to traverse them be *very* small, the velocity along $P'Q'$ will be proportional to its space, and we should have

$$v' : v'' = \widehat{PQ} : \widehat{P'Q'} = \widehat{PQ} : \widehat{PH}. \quad . \quad . \quad . \quad (23)$$

The triangles PQH and $PP'C'$ are similar, and therefore by Geom.,

$$\widehat{PH} : \widehat{PQ} = \widehat{PP'} : \widehat{PC'}. \quad . \quad . \quad . \quad . \quad (24)$$

Also, by Geom.,

$$\widehat{PC'}^2 = \widehat{PP'}^2 + \widehat{P'C'}^2;$$

$$\therefore \widehat{PP'} = \sqrt{\widehat{PC'}^2 - \widehat{P'C'}^2}. \quad . \quad . \quad . \quad . \quad (25)$$

But $\widehat{PC'} = \widehat{A'C'} = s$, and let $\widehat{P'C'} = x$, whence $\widehat{PP'} = \sqrt{s^2 - x^2}$, and (24) becomes

$$\widehat{PH} : \widehat{PQ} = \sqrt{s^2 - x^2} : s. \quad . \quad . \quad . \quad . \quad (26)$$

Substituting this in (23), it is

$$v' : v'' = s : \sqrt{s^2 - x^2}, \quad . \quad . \quad . \quad . \quad . \quad (27)$$

and introducing the value of (20) in (27), this last becomes

$$s\sqrt{\frac{g}{l}} : v'' = s : \sqrt{s^2 - x^2}, \quad . \quad . \quad . \quad . \quad . \quad (28)$$

whence

$$v'' = \sqrt{\frac{g}{l}{}^2(s - x^2)}. \quad . \quad . \quad . \quad . \quad . \quad (29)$$

The second members of (19) and (29) are the same, whence $v'' = v$, and thus it is proven that the variable velocity along the arc ACZ of Fig. 386 is identical with the variable velocity along the straight line $A'C'Z'$ of Fig. 387.

The time t, then, that the pellet takes to make the oscillation on the arc ACZ, Fig. 386, is the same as that required for the *projected* motion to traverse the straight line $A'C'Z'$, Fig. 387, *and also the same that the body moves with uniform velocity v' over the semicircle $A'PUZ'$.* This

latter time, by Mechanics, is contained in the general equation for *uniform* motion,

$$d = v'.t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

where d is the distance. Whence

$$t = \frac{d}{v'}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

in this case, d is a semi-circumference ($A'PUZ'$), which, by Geom., is equal to ($\pi.\widehat{A'C'}$), or, {by (22)}, ($\pi.s$), where $\pi = 3.1416$. Substituting this value of d and that of v' from (20) in (31), this becomes

$$t = \frac{\pi.s}{s\sqrt{\frac{g}{l}}} = \frac{\pi}{\sqrt{\frac{g}{l}}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

In this, it will be seen, as stated at the outset, that the period of oscillation depends on two quantities—the weight and form of the moving body represented by l and the force g acting upon it.

Theoretically, t is not the same for all arcs of swing, but increases slowly with the semi-arc.

In June, 1898, I made some experiments with the needle used for the ordinary intensity observations on board ship: it is 3 inches long, mounted on a pivot in a circular brass box covered with glass. Friction between cap and pivot as well as (probably) electric currents excited in the material of the box by the needles' motion put a strong check upon its freedom—it came to rest under the influence of the Earth's field alone after making twenty oscillations, beginning with a semi-arc of 15°; and after making thirty oscillations, with a semi-arc of 25°.

TABLE 36.

No. of Set.	Limits of Semi-arc. Begins.	Limits of Semi-arc. Ends.	No. of Oscillations.	Period of Each Set.
(1)	(2)	(3)	(4)	(5)
1	15°	7°	9	16.37 s.
2	15	7	9	16.37
3	15	6	9	16.37
4	16	8	9	16.12
5	15	7	9	16.00
6	16	8	9	16.00
7	15	6	9	16.00
8	16	6	9	16.00
9	15	6	9	16.12
10	16	7	9	16.12
11	15	6	9	16.00
12	15	6	9	16.00
13	16	7	9	16.00
14	15	7	9	16.37
15	14	5	9	16.12
16	16	7	9	16.12
17	16	7	9	16.00
18	16	7	9	16.00
19	16	7	9	16.12
20	16	7	9	16.12
			Mean	= 16.116

TABLE 37.

No. of Set.	Limits of Semi-arc. Begins.	Limits of Semi-arc. Ends.	No. of Oscillations.	Period of Each Set.
(1)	(2)	(3)	(4)	(5)
1	25°	9°	9	16.37 s
2	25	11	9	16.37
3	25	11	9	16.37
4	25	10	9	16.50
5	25	10	9	16.12
6	26	11	9	16.12
7	25	10	9	16.25
8	26	12	9	16.37
9	26	12	9	16.37
10	23	8	9	16.50
11	26	9	9	16.50
12	25	7	9	16.00
13	24	9	9	16.50
14	26	12	9	16.37
15	25	10	9	16.37
16	26	15	9	16.37
17	25	10	9	16.37
18	25	12	9	16.12
19	27	14	9	16.62
20	26	15	9	16.00
			Mean	= 16.328

Tables 36 and 37 show the results of two series of trials, each consisting of twenty sets of oscillations: one series beginning with a semi-amplitude of about 15°, and ending at about 7°; the other with limits of about 25° and 10°.

A stop-watch marking quarter-seconds was used to time the oscillations. Fig. 388 illustrates the semi-arc, its decrement, and method of counting the oscillations: the watch was started on counting "one" and stopped at "ten," so that the times in col. (5) of each table are the periods of nine swings, both right and left; each of the forty sets in both tables is composed of nine such swings.

It will be seen from the *mean* of the periods of both series that there is considerable difference (0.212 second) between a set of oscillations whose semi-arc begins at 25° and one beginning at 15°: it is therefore essential that all oscillations *for the same series of observations* should begin

with the same amplitude. The varying conditions experienced on different ships will of course necessitate amplitudes suited to the conditions.

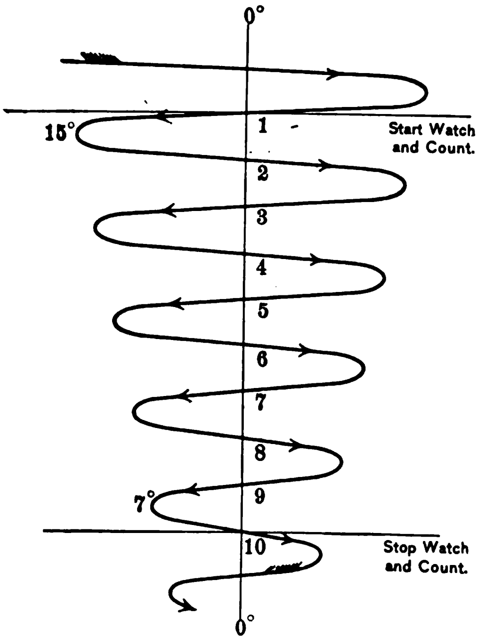


FIG. 388.

To return to the pendulum motion, and pass from the pellet to a body of some weight and volume, consider Fig. 389: *B* is a steel rod to which knife-edges are attached at *O*; they rest on a frame so that the rod may oscillate in a vertical plane under the impulse of a horizontal force *F*. Denote the mass of a particle of the rod by *dm*, its distance from the point of suspension by *r*, and the semi-arc of swing by *θ*. While all particles of the rod swing through the same *angle*, all do *not* describe equal *extents of space;* those nearest the point of suspension pass over the shortest length, and this increases with each remove

from that point, that is, with the radius, so that to get the length traversed in any specific time—the *linear* velocity of a particle *dm*, we must multiply the arc by its

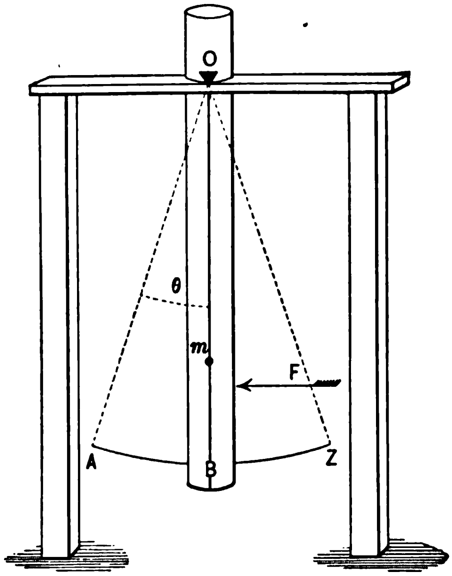


FIG. 389.

radius, or, $r.\theta$; then, momentum (p) being the product of mass and velocity, we have

$$p = dm.r.\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

The *moment* of this momentum—its tendency to produce rotary motion round a point—is measured by the product of the perpendicular from the point upon the direction of the force: hence, from (33),

$$r.p = dm.r^2.\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

The same is true of every other particle of the rod, so that we should have as many equations like (34) as we conceive

the rod to be divided into particles; and, taking their sum, we have for the whole rod, denoting sum by $\int$,

$$\int r.p = \int \theta.dm.r^2 = \theta\int dm.r^2. \quad . \quad . \quad . \quad (35)$$

The quantity $\int dm.r^2$ is called the *moment of inertia:* it represents either the tendency to motion in a swinging body, or the inertness that must be overcome to start one from rest.

The small particles are supposed to be equal—it is only the distance of each that varies; therefore the length of the oscillating body may in a measure be said to represent the moment of inertia: the length here is denoted by r and in Fig. 386 by l, so that l in eq. (32) stands for one of the two quantities that change the period of oscillation, the other being the force represented by g.

By Fig. 389, the nearer the point of suspension is to the end of the rod, the more of its mass the force F will have to move—hence, the greater the inertia, and also its moment: on the other hand, if the point of suspension approaches the middle of the rod, the portion above acts as a counterpoise to that below; when the suspension is at the center of gravity, the rod will hang indifferently and the moment of inertia is least.

Consider this case in particular, and in order to lead up to the real object of this investigation—the oscillation of a magnetic needle round a vertical axis—let Fig. 390 represent a short rectangular bar suspended by a stout wire, $\widehat{ZA}$; O is the origin of coordinates X, Y, Z, which are fixed in the bar and rotate with it.

A rigid rod $\widehat{Z'B}$ may be soldered to the bar—another vertical axis round which it may swing in a horizontal plane if the suspension $\widehat{ZA}$ be removed: thus $\widehat{ZA}$ and $\widehat{Z'B}$

are parallel axes in the vertical plane ZOX; the latter, $\widehat{Z'B}$, is analogous to the *suspension* at O in Fig. 389, only that the body in Fig. 389 oscillates in a vertical plane, and in Fig. 390 in a horizontal plane; $\widehat{ZA}$ constitutes a

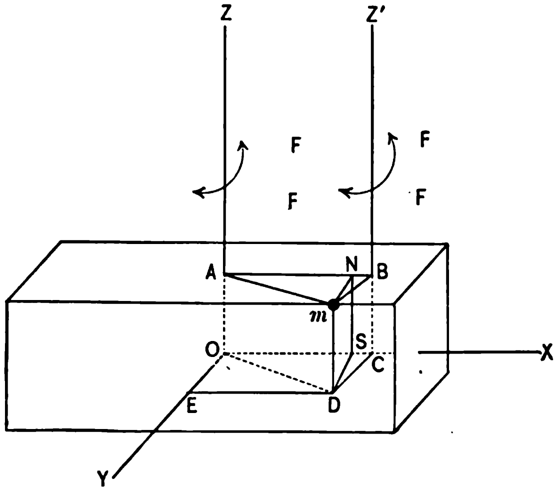


FIG. 390.

new condition—an axis through the center of gravity, round which it is sought to find the moment of inertia.

In Fig. 390, let m be any particle of the bar denoted by dm, and $\widehat{mN}$ a perpendicular on $\widehat{AB}$, which is itself at right angles to both axes; let

$$\widehat{mA} = \widehat{OD} = r, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

$$\widehat{mB} = p, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

and

$$\widehat{BA} = h; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

the r... the axis
sum, w...

. . . . (39)

. . . . (40)

The q...
repres...
body, ...
from r... . . . . (41)
Th...
the di...
the os...
the m... . (42)
and in...
the tw... $y^2 - x^2 = r^2$,
the ot...
By...
the en... . . . (43)
have ...
its m... mass of the par-
appro...
as a ...
at th... . . (44)
and t...
C... other par-
up t... by f,
of a ...
repr...
wire. . (45)
are ...
... gravity,
A...
vert...
plan... . . . . (46)

and (45) becomes, by making K represent, *in general*, the moment of inertia,

$$K = \int dm \cdot p^2 = \int dm \cdot r^2 + \int dm \cdot h^2. \quad . \quad . \quad (47)$$

$\int dm \cdot p^2$ is the moment of inertia about the axis $\widehat{Z'B}$, and $\int dm \cdot r^2$ that about $\widehat{ZA}$: hence from (47) the moment of inertia about *any* axis $(\widehat{Z'P})$ is equal to that about a parallel axis $(\widehat{ZA})$ through the center of gravity, increased by a quantity $\left(\int dm \cdot h^2\right)$ composed of the product of the mass $\left(\int dm\right)$ and square of the distance (h) between both axes; thus, the moment of inertia round the center of gravity is least, as was otherwise stated with regard to the rod in Fig. 389.

Now let $M' = \int dm$, the mass of the bar; then (47) becomes

$$\int dm \cdot p^2 = \int dm \cdot r^2 + M' \cdot h^2. \quad . \quad . \quad . \quad (48)$$

Assume k such that

$$\int dm \cdot r^2 = M' \cdot k^2 = \frac{W}{g} \cdot k^2 \quad . \quad . \quad . \quad . \quad (49)$$

and (48) becomes

$$\int dm \cdot p^2 = M' \cdot k^2 + M' \cdot h^2 = M'(k^2 + h^2). \quad . \quad (50)$$

$M' \cdot k^2$ is the moment of inertia round an axis through the center of gravity, and $M'(k^2+h^2)$ that round any other axis parallel to it at a distance $h (= \widehat{AB})$ as in Fig. 390.

F indicates the to-and-fro motion, according
used; the coordinates of dm are

$$x = \widehat{OS} = \widehat{AN} = \widehat{ED}, \quad . \quad .$$

$$y = \widehat{OE} = \widehat{Nm} = \widehat{SD}, \quad . \quad .$$

and

$$Z = \widehat{OA} = \widehat{SN} = \widehat{Dm}. \quad .$$

By Geom., from Fig. 390,

$$\widehat{mB}^2 = \widehat{mN}^2 + \widehat{NB}^2 = \widehat{mN}^2 + (\widehat{AB}$$

Expanding $(\widehat{AB} - \widehat{AN})^2$, and observi
eq. (42) becomes, by means of (36) to

$$p^2 = r^2 - 2h . x + h$$

Multiplying this throughout by dm
ticle, it becomes

$$dm . p^2 = dm . r^2 - 2dm . h$$

And there are analogous equa
ticle of the bar; whence taking
we have for the whole bar,

$$\int dm . p^2 = \int dm . r^2 - \int$$

The origin of coordinates be
$x = 0$, whence

$$\int 2dm$$

...e in a horizontal plane, they ...ut soon get out of step: those ...sion would oscillate rapidly— ...nd the *difference* in rate would ... approached a point intermediate ...at this point there is an *average* ...the pellets, and the point is called ...

...ulum it is interchangeable with the ... that is, the rod will swing in the ...er center.

...called the *radius of gyration* of a body ... gravity, and in (50), $\sqrt{k^2+h^2}$ is the ...axis parallel to it. The radius of gyra- ... h^2) may—like the center of oscillation ...—be defined as the distance at which ...ge "*moment of inertia*" of all the particles

...nt of inertia may be calculated for many ...lar geometrical form; those usually employed ...l observations are elongated and of varied ... rectangular, round, tubular, solid, and cable; ...ing the appearance of a cut across our com- ...s.

...gth of the magnet, the form of its cross-section, ...e of a thin slice of this from the axis of rotation, ...lement of the mass, are the data which, intro- ...nto eq. (47), afford the means, by integration, ...mining the moment of inertia of any particular ...

...sider, for instance, a long, narrow blade, like that ...able-knife—Fig. 391: let a be the length and dr a ...-section at the distance r from the axis $\widehat{ZC}$; this ...s-section is rectangular, but on account of the thinness

of the blade, it is a mere vertical line, as may be seen at B, and of such vertical lines the whole length of the blade is composed; thus dr may really be taken for an element of mass, dm.

As the axis of oscillation $\widehat{ZC}$ is perpendicular to the length of the magnet through its center of gravity, the

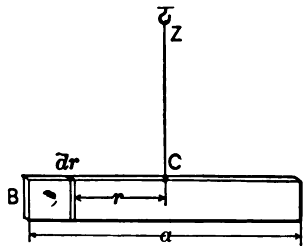


FIG. 391.

last term of eq. (47) must be omitted, as that term applies only to an axis outside the center of gravity.

Introducing, then, the above particulars into eq. (47), it becomes

$$K = 2\int_C^{\frac{a}{2}} r^2.dr. \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

In this, the integration (as indicated) is from the central vertical line or element at C to $\frac{a}{2}$, or for one half the blade, and the result is doubled for the whole blade. Performing the integration of (51), we have

$$K = 2\int_C^{\frac{a}{2}} \frac{r^3.dr}{3.dr} = \frac{2\left(\frac{a}{2}\right)^3}{3} = \frac{a^3}{12} = a\left(\frac{a^2}{12}\right); \quad . \quad (52)$$

whence

$$K = M'\left(\frac{a^2}{12}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

That is, since a is the length of the magnet, it is also for this particular blade its mass M', whence eq. (53) from the last member of eq. (52).

Now suppose that two blades, identical with that in Fig. 391, be successively attached parallel to it, one on each side, so as to build up a symmetrical *bar* of any width; then the cross-section will no longer be a line, but have width as well as depth; denote this width by b: as, in the case of the single blade, the integration was effected by summing up the number of vertical lines composing its length, so, now, in the case of the rectangular *bar* which has been formed by adding *rows* of similar lines on each side of it, we must sum up their number *laterally*—that is, integrate for the width b as well as for the length a; and introducing this directly into eq. (53), we have

$$K = M'\left(\frac{a^2+b^2}{12}\right), \quad . \quad . \quad . \quad . \quad . \quad (54)$$

which gives the moment of inertia of a rectangular bar—a parallelopipedon whose length is a, width b, and mass M', and which oscillates round an axis through its center of gravity, perpendicular to the horizontal plane containing its length and width.

If a pivoted magnet A, Fig. 392, be placed under the

FIG. 392.

influence of another magnet B, the axes of both being in the magnetic meridian (H), and A be set in motion, it will have a definite period of oscillation; if either a stronger

or a weaker magnet replace B, the period will change with each; and if B be removed altogether, the period will still differ from either of the preceding, the magnet A oscillating under the influence of the Earth's field alone. A part of the period in each case is due to the form and weight of the magnet—to its moment of inertia, and is therefore constant for the same magnet A; the variable part is due to the changed magnetic condition in each case.

Recurring to the formula for the period of oscillation,

$$t=\frac{\pi}{\sqrt{\frac{g}{l}}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

we have

$$\sqrt{\frac{g}{l}}=\frac{\pi}{t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

and

$$\frac{g}{l}=\frac{\pi^2}{t^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

It has already been shown that g represents the force and l the moment of inertia: the analogy between a pendulum and a magnet in oscillation is therefore too obvious to require more than directing attention to it; the expression for moment of inertia applies equally to pendulum and magnet, but the force of gravity acting on the pendulum is replaced by the force of magnetism acting on the magnet: this latter force is contained in the expression $M.H$, as explained in Arts. 122, 171, and 176, Vol. I. Representing *in general* the moment of inertia of the magnet by K, {eq. (47)}, and substituting K for l and $M.H$ for g in eq. (56), it becomes

$$\frac{M.H}{K}=\frac{\pi^2}{t^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

Placing this under another form, we have

$$t=\pi\sqrt{\frac{K}{M.H}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

With M and K constant, that is, using the same magnet, let it be oscillated in different fields, H_1 and H_2, for the same period T: the number of oscillations in both fields will differ—denote them by n_1 and n_2 respectively; then from (58), by analogy, we have

$$T=n_1.\pi\sqrt{\frac{K}{M.H_1}} \quad . \quad . \quad . \quad . \quad . \quad (59)$$

and

$$T=n_2.\pi\sqrt{\frac{K}{M.H_2}}, \quad . \quad . \quad . \quad . \quad . \quad (60)$$

whence

$$n_1.\pi\sqrt{\frac{K}{M.H_1}}=n_2.\pi\sqrt{\frac{K}{M.H_2}}. \quad . \quad . \quad . \quad (61)$$

Therefore

$$\frac{n_1^2}{H_1}=\frac{n_2^2}{H_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

or

$$\frac{H_1}{H_2}=\frac{n_1^2}{n_2^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

or

$$H_1 : H_2 = n_1^2 : n_2^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

That is, the intensities of the two fields are proportional to the square of the NUMBER *of oscillations that the same needle makes in them during the same period of time.*

If, on the other hand, we note the *times* T_1 and T_2 required to make the same number of oscillations in both fields, we have by analogy with (59) and (60)

$$T_1 = n.\pi\sqrt{\frac{K}{M.H_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

and

$$T_2 = n.\pi\sqrt{\frac{K}{M.H_2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

Whence

$$\frac{T_1^2}{T_2^2} = \frac{H_2}{H_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

or

$$\frac{H_1}{H_2} = \frac{T_2^2}{T_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

or

$$H_1 : H_2 = T_2^2 : T_1^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

That is, the intensity of the fields is inversely proportional to the squares of the TIMES *in which the magnet makes the same number of oscillations.*

For bodies of other than regular geometrical form, and especially when composed of different materials, such as compass-cards are, the moment of inertia is determined by oscillating the card first alone, and then burdened with some regular body—for instance, a rectangular brass bar: loaded and unloaded, the magnetic moment ($M.H$) will be the same, but the moment of inertia (K) will vary, and with it the period of oscillation (T). Denote the periods by T' and T'', the moment of inertia of the card alone by K', and that of the regular body added to it by K''; then by analogy with (58) we have

$$T' = \pi\sqrt{\frac{K'}{M.H}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

and

$$T'' = \pi\sqrt{\frac{K'+K''}{M.H}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

whence, dividing (70) by (71),

$$\frac{T'^2}{T''^2} = \frac{K'}{K'+K''}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

and

$$T'^2.K' + T'^2.K'' = T''^2.K'; \quad . \quad . \quad . \quad (73)$$

also

$$K'(T''^2 - T'^2) = K''.T'^2. \quad . \quad . \quad . \quad . \quad . \quad (74)$$

Therefore

$$K' = \frac{K''.T'^2}{T''^2 - T'^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

Of the second member, K'' (relating to the added body of regular form) may be calculated; T' and T'' are observed; and hence K' becomes known: then from (70) we have $M.H = \frac{\pi^2.K'}{T'^2}$; hence

$$M = \frac{\pi^2.K'}{T'^2.H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

All the quantities in the second member are now known, so that M, the magnetic moment of the card, or rather of the system of four needles on it, is also known.

It must be understood that the foregoing procedure is possible only with the card suspended in air by a fiber of silk; for, once sealed in the bowl, it is inaccessible for placing a weight on it to determine K', and thus there would be this unknown quantity in equation (76).

231. Arrangement of the needles on the card.—This is not a matter of mere symmetry, but of symmetry based on the greatest advantage obtainable from the moment of inertia of the needles.

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that just made—that increased moment of inertia facilitates action of the magnetic moment. It will now be shown how K' is made a maximum.

By reference to Plate N (2), it will be seen that the needles do not oscillate round axes through their own centers of gravity; but that the whole system swings about its center of figure B as represented in Fig. 393: it is the case of an axis of gyration ($\sqrt{k^2+h^2}$) parallel to one (k) through the center of gravity, which has already been investigated for Figs. 389 and 390.

Consider a diagram of the compass-card—Fig. 393: L, M, P, Q are the four needles forming chords of a circle; for simplicity of this investigation, they are supposed to be thin rectangular plates—knife-blades, as in Fig. 391.

$$\overset{\frown}{BD}=r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

the radius of this circle; $\overset{\frown}{AC}$ is the magnetic axis of the system, parallel to the needles; by Geom.,

$$ABD=BDH=\phi; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (78)$$

let

$$\overset{\frown}{DM}=l, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

the length of the needle M; and

$$\overset{\frown}{BH}=h, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

a perpendicular from the pivot upon $\overset{\frown}{DM}$, the pivot is the center of moments, both magnetic and of inertia. Denote the moment of inertia of the needle M about its center of gravity H, by K_m: as it is a uniform body, the mass is proportional to the length; hence, by (52),

$$K_m=\frac{l^3}{12}=l\left(\frac{l^2}{12}\right). \quad . \quad . \quad . \quad . \quad . \quad (81)$$

It is not, however, the moment of inertia about H that is required, but that about a parallel axis through B, so

that the case reverts to that expressed by (47), or by the last member of (50), in which h^2 there corresponds to $\widehat{BH}^2$ in Fig. 393; M' in (50) to l in (81); and k^2 in (50) to $\left(\frac{l^2}{12}\right)$ in (81).

Let K'_m be the moment of inertia of the needle M about the axis through B; then from (50), (80), and (81),

$$K'_m = l(k^2 + h^2) = l\left(\frac{l^2}{12} + h^2\right). \quad . \quad . \quad . \quad (82)$$

Determining l and h in terms of the radius, keeping (77) to (80) in view, we have, by Trig., from Fig. 393,

$$\sin BDH = \frac{\widehat{BH}}{\widehat{BD}}, \quad \text{or} \quad h = r.\sin\phi; \quad . \quad . \quad (83)$$

and

$$h^2 = r^2.\sin^2\phi; \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

also,

$$\cos BDH = \frac{\widehat{DH}}{\widehat{BD}}, \quad \text{or} \quad \tfrac{1}{2}l = r.\cos\phi, \quad . \quad . \quad (85)$$

and

$$l = 2r.\cos\phi;$$

also,

$$l^2 = 4r^2.\cos^2\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad (86)$$

Substituting values from (84) and (86) in the last member of (82), this becomes

$$K'_m = (2r.\cos\phi)\left\{\frac{4r^2.\cos^2\phi}{12} + r^2.\sin^2\phi\right\}. \quad (87)$$

By Trig.,

$$\sin^2\phi = 1 - \cos^2\phi; \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

substituting (88) in (87), we have,

$$K'_m = (2r.\cos\phi)\left\{\frac{4r^2.\cos^2\phi}{12} + r^2 - r^2\cos^2\phi\right\}. \quad (89)$$

$$K'_m = r^3(\tfrac{2}{3}\cos^3\phi + 2\cos\phi - 2\cos^3\phi). \quad . \quad . \quad . \quad . \quad (90)$$

By Trig., Art. 76, $\cos 3\phi = 4\cos^3\phi - 3\cos\phi$; whence

$$\cos^3\phi = \tfrac{1}{4}\cos 3\phi + \tfrac{3}{4}\cos\phi. \quad . \quad . \quad . \quad . \quad (91)$$

Substituting this in (90), it becomes

$$K'_m = r^3[\tfrac{2}{3}(\tfrac{1}{4}\cos 3\phi + \tfrac{3}{4}\cos\phi) + 2\cos\phi$$
$$- 2(\tfrac{1}{4}\cos 3\phi + \tfrac{3}{4}\cos\phi)], \quad . \quad (92)$$

$$K'_m = r^3(\cos\phi - \tfrac{1}{3}\cos 3\phi) = \tfrac{1}{3}r^3(3\cos\phi - \cos 3\phi). \quad . \quad . \quad (93)$$

In this, the quantity

$$(3\cos\phi - \cos 3\phi) \quad . \quad . \quad . \quad . \quad . \quad . \quad (94)$$

is alone variable, and it is sought to find what value of ϕ will make it a maximum, in order that K'_m may be such.

By Calculus, to determine the maximum of a quantity, is to differentiate it twice—place the first differential equal to zero—find the roots of the resulting equation—substitute these in the second differential—and see if the result is negative: if so, the root is a maximum for the original quantity. Performing this operation on (94), and remembering that all constants may be omitted from the process, we have, by making

$$y = 3\cos\phi - \cos 3\phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (95)$$

$$dy = -3\sin\phi.d\phi + 3\sin 3\phi.d\phi.$$

$$\therefore \frac{dy}{d\phi} = -3\sin\phi + 3\sin 3\phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad (96)$$

and

$$\frac{d^2y}{d\phi^2} = -3\cos\phi + 9\cos 3\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

Placing (96) equal to zero, we have

$$-\sin\phi+\sin 3\phi=0. \quad . \quad . \quad . \quad . \quad (98)$$

By Trig., Art. 75,

$$\sin 3\phi=4\sin\phi \,.\, \cos^2\phi-\sin\phi. \quad . \quad . \quad (99)$$

Substituting this in (98), it becomes

$$-\sin\phi+4\sin\phi \,.\, \cos^2\phi-\sin\phi=0; \quad . \quad . \quad (100)$$

dividing by $\sin\phi$, it is

$$-1+4\cos^2\phi-1=0; \quad \text{or} \quad \cos^2\phi=\tfrac{1}{2}. \quad . \quad (101)$$

By Trig., Art. 30,

$$\tfrac{1}{2}=\cos^2 45; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (102)$$

whence

$$\phi=45^\circ. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (103)$$

From (97), the second differential is

$$-\cos\phi+3\cos 3\phi. \quad . \quad . \quad . \quad . \quad (104)$$

By Trig.,

$$\cos 3\phi=4\cos^2\phi-3\cos\phi. \quad . \quad . \quad . \quad (105)$$

Substituting this in (104), it is

$$-\cos\phi+12\cos^2\phi-9\cos\phi. \quad . \quad . \quad . \quad (106)$$

Dividing by $\cos\phi$, it becomes

$$-10+12\cos\phi. \quad . \quad . \quad . \quad . \quad . \quad (107)$$

By Trig., since $\phi=45^\circ$, $\cos 45^\circ=\frac{1}{2}\sqrt{2}$, whence (107) becomes

$$-10+12(\tfrac{1}{2}\sqrt{2})=-7.6. \quad . \quad . \quad . \quad . \quad (108)$$

This being negative, shows that 45° is the maximum angle of which the needle $\widehat{DM}$ should be the chord, in order that its moment of inertia K'_m may be a maximum.

Were no other consideration, then, than the maximum moment of inertia to govern in the number and arrangement of the needles on a compass-card, we should have only two—placed symmetrically on the chords of 45°, as at L and M, Fig. 393; but other reasons *do* exist to decide upon four needles, rather than two: *first*, the destructive influence upon each other's magnetism is less with the same number of wires divided into four bundles, and thus real power is gained; *second*, the needles must be so disposed that their moments of inertia shall be the same around all diameters of the card, otherwise there would be a wabbling motion instead of uniform oscillation.

To investigate this matter, let Fig. 394 represent two

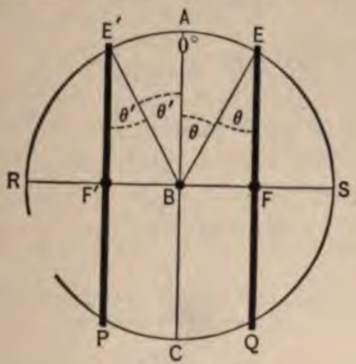


FIG. 394.

parallel needles P and Q placed at angular distances θ and θ' from the central diameter $\widehat{AC}$, parallel to them. If the moments of inertia around $\widehat{AC}$ are the same as those around the diameter $\widehat{RS}$, perpendicular thereto, the identity will hold for all other pairs of diameters at right

angles to each other, that is, all round the circle. The question, then, is: What values must be given θ and θ', that is, what arcs must P and Q be the chords of, in order that their moments of inertia shall be the same round all diameters?

The needles being regular cylindrical bodies, only one half their lengths need be considered; hence, by Mechanics, M being the mass, the moment of inertia K' of $\widehat{EF}$ with respect to the point F of the diameter $\widehat{RS}$ is

$$K' = \tfrac{1}{3}.M.\widehat{EF}^2; \quad . \quad . \quad . \quad . \quad . \quad (109)$$

and with reference to the point B of the diameter $\widehat{AC}$, the moment of inertia of $\widehat{EF}$ is proportional to

$$M.\widehat{BF}^2 = K''. \quad . \quad . \quad . \quad . \quad . \quad . \quad (110)$$

Similarly, for $\widehat{E'F'}$ we find

$$K_1 = \tfrac{1}{3}.M.\widehat{E'F'}^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (111)$$

and

$$K_2 = M.\widehat{BF'}^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (112)$$

Let

$$\widehat{BA} = \widehat{BE} = \widehat{BE'} = r, \quad . \quad . \quad . \quad . \quad (113)$$

the radius of the circle of which the needles are the chords: then from Fig. 394, $\sin\theta = \dfrac{\widehat{BF}}{r}$;

$$\therefore \quad \widehat{BF} = r.\sin\theta,$$

and

$$\widehat{BF}^2 = r^2.\sin^2\theta; \quad . \quad . \quad . \quad . \quad . \quad (114)$$

also,

$$\widehat{BF'} = r.\sin\theta',$$

and

$$\widehat{BF'}^2 = r^2.\sin^2\theta'. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (115)$$

$$\cos\theta = \frac{\widehat{EF}}{r}.$$

$$\therefore \quad \widehat{EF} = r.\cos\theta = M,$$

and

$$\widehat{EF}^2 = r^2.\cos^2\theta; \quad . \quad . \quad . \quad . \quad . \quad . \qquad (116)$$

also,

$$\widehat{E'F'} = r.\cos\theta' = M,$$

and

$$\widehat{E'F'}^2 = r^2.\cos^2\theta'. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (117)$$

The masses M {in the equations (115) to (117)} of the half lengths of the needles are proportional to $\widehat{EF}$ and $\widehat{E'F'}$.

Then, in order that equality may exist, we must have the moments of inertia of both half needles round the diameter $\widehat{RS}$ the same that they are round the diameter $\widehat{AC}$, or, stated analytically, from equations (109) to (112),

$$K' + K_1 = K'' + K_2, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (118)$$

which, by means of (109) to (117), becomes

$$\begin{aligned}\tfrac{1}{3}(r.\cos\theta)(r^2.\cos^2\theta) + \tfrac{1}{3}(r.\cos\theta')(r^2.\cos^2\theta')\\ = (r.\cos\theta)(r^2.\sin^2\theta) + (r.\cos\theta')(r^2\sin^2\theta'). \qquad (119)\end{aligned}$$

That is,

$$\tfrac{1}{3}(\cos^3\theta + \cos^3\theta') = \cos\theta.\sin^2\theta + \cos\theta'.\sin^2\theta'. \qquad (120)$$

By Trig., $\sin^2\theta = 1 - \cos^2\theta$; substituted in (120), this becomes

$$\tfrac{1}{3}\cos^3\theta + \tfrac{1}{3}\cos^3\theta' = \cos\theta - \cos^3\theta + \cos\theta' - \cos^3\theta') \quad (121)$$

whence

$$\cos^3\theta + \cos^3\theta' = \tfrac{3}{4}(\cos\theta + \cos\theta'). \quad . \quad . \quad (122)$$

If we *make*

$$\cos^2\theta - \cos\theta\cos\theta' + \cos^2\theta' = \tfrac{3}{4}, \quad . \quad . \quad (123)$$

this *assumption*, if substituted in (122), will make it

$$\cos^3\theta + \cos^3\theta' = \cos^3\theta - \cos^2\theta\cos\theta' + \cos\theta\cos^2\theta'$$
$$+ \cos^2\theta\cos\theta' - \cos\theta\cos^2\theta' + \cos^3\theta'. \quad (124)$$

Both members of (124) reduce to an identity, so that the assumption made in (123) is correct. By subtracting $\tfrac{3}{4}\cos^2\theta'$ from both members of (123) it becomes a perfect square; thus,

$$\cos^2\theta - \cos\theta\cos\theta' + \cos^2\theta' - \tfrac{3}{4}\cos^2\theta' = \tfrac{3}{4} - \tfrac{3}{4}\cos^2\theta', \quad (125)$$

or

$$\cos^2\theta - \cos\theta\cos\theta' + (1 - \tfrac{3}{4})\cos^2\theta'$$
$$= \tfrac{3}{4}(1 - \cos^2\theta') = \tfrac{3}{4}\sin^2\theta'. \quad (126)$$

Extracting the square root, this becomes

$$\cos\theta - \tfrac{1}{2}\cos\theta' = \pm\sin\theta'\sqrt{\tfrac{3}{4}}$$
$$= \pm\sin\theta'\frac{\sqrt{3}}{2} = \pm\tfrac{1}{2}.\sqrt{3}.\sin\theta'. \quad (127)$$

That is,

$$\cos\theta = \tfrac{1}{2}\cos\theta' \pm \tfrac{1}{2}\sqrt{3}.\sin\theta'. \quad . \quad . \quad . \quad (128)$$

By Trig., pages 19 and 20, $\cos 60° = \tfrac{1}{2}$ and $\sin 60° = \tfrac{1}{2}\sqrt{3}$, whence (128) becomes

$$\cos\theta = \cos 60°\cos\theta' \pm \sin 60°\sin\theta'. \quad . \quad . \quad (129)$$

This, by Trig., page 21, becomes

$$\cos \theta = \cos (\theta' \pm 60^\circ). \quad . \quad . \quad . \quad . \quad . \quad (130)$$

Hence $\theta = \theta' \pm 60^\circ$, or

$$\theta - \theta' = \theta' - \theta = 60^\circ. \quad . \quad . \quad . \quad . \quad . \quad (131)$$

That is to say, an arc of 60° must intervene between the ends of the needles, or they should be placed 30° on each side of the central diameter $\widehat{AC}$, as at $\widehat{E'P}$ and $\widehat{EQ}$, Fig. 394.

Now, *four* chords—at 15° and 45°—one of each on the same side of the central diameter, are equivalent, as regards symmetry of position and equality of moment of inertia, to two chords, one at 30° on each side—that is to say, the *same weight* distributed among four needles or divided between two needles: therefore, by placing four needles on chords of 15° and 45°, two on each side of the pivot, we combine two most important points—maximum moment of inertia with its equality of distribution.

An investigation into the effect of the *length* of needle upon the deviations shows that, when not abnormally long, some of the less familiar deviations, such as the octantal, sextantal, and decantal, are entirely avoided by placing *four* needles on chords of 15° and 45°.

Hence, to sum up the advantages of this arrangement: 1st, increase of magnet power; 2d, maximum moment of inertia; 3d, equalization of this on the system, thereby insuring steady oscillation of the card; and 4th, avoidance of deviations having more frequent maxima and minima than the semicircular and quadrantal have.

Accordingly, the four needles of the U. S. Navy Compass are placed practically on the chords of 15° and 45° of the circle through their magnetic foci, as L, M, P, Q, Fig. 393, reckoning the *angular* distance from the magnetic axis $\widehat{AC}$ of the system.

232. Dip, and steadiness, of the card.—If a knitting-needle be balanced on an axis through its center of gravity, it will hang level at all points of the Earth to which it may be carried; but when magnetized, it will no longer do so: it dips, and variously, according to the latitude.

Dry compasses have a small sliding weight to restore the card to level. This is not necessary on our card: the center of buoyancy is slightly below the center of suspension, and the center of gravity is far below both—about three-quarters of an inch; this means that there is a great tendency—a mechanical moment—of the card itself to keep level.

To illustrate, consider Fig. 395: a magnet is poised

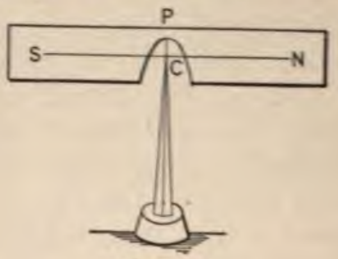


FIG. 395.

with its center of gravity C well below the point of suspension; when carried toward polar regions, the tendency of the magnetic couple $M.H$ is to tilt the magnet as in Fig. 396: if effectual, the center of gravity is displaced to O, and the mechanical couple thus called into action counteracts the magnetic couple.

The mechanical couple is illustrated by Fig. 397: O is the displacement of the center of gravity from C, which will increase with $\widehat{PC}$; that is, with the distance between the centers of gravity and suspension; and the mechanical couple will be the product of the arm $\widehat{OC}$ and weight of the bar.

Now, "It has been shown that the effect of magnetizing a bar, under the most advantageous circumstances of form, and at the part of the globe where the vertical component of the magnetic force is greatest, is the same (as to its position of equilibrium) as if its center of gravity had been transferred about the one-fortieth ($\frac{1}{40}$) of an inch toward the north end; so that the moment of the force, exerted by the vertical component of the Earth's

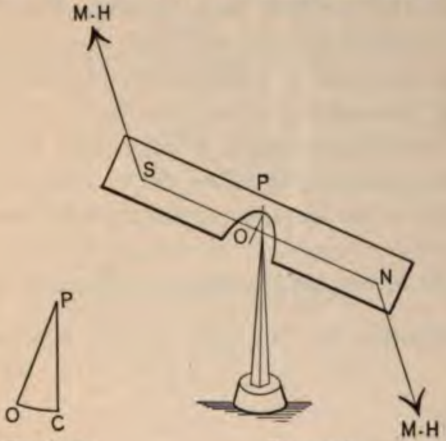


FIG. 397. FIG. 396.

magnetism, can never exceed this small quantity multiplied by the weight of the bar (Lloyd)." That is to say, if in Fig. 396 the pivot were moved $\frac{1}{40}$ inch from the center toward the *N*-end of the magnet, this would make it hang level.

As this is the maximum effort of the vertical magnetic couple, it can exert no appreciable influence on our card with its large mechanical couple: practically, the card remains level in all latitudes to which our ships go.

Regarding steadiness, it was shown in Art. 231 how the arrangement of the needles contributes to that end when motion at all is set up by the rolling and pitching of the ship: the great distance between pivot-point and

center of gravity of the card also conduces to it in a high degree.

Upon the SCORESBY—Fig. 428, Part Third—a compass was mounted as on a ship, and the little vessel was then rolled more quickly and deeply than ever a 90-day gunboat could approach, and yet the card remained steadfast to its course.

The fact that the center of gravity of the card is always practically in the vertical axis through the pivot, leaves no eccentric weight for the roll and scend of the ship to act upon: in Fig. 396, if a weight had to be placed on the upper end of the magnet to restore level, we should have a *horizontal* pendulum (on a small scale) to which every roll of the ship, especially when on courses parallel to the needles, would be an impulse to set up troublesome oscillations.

233. Effect of the length and arrangement of the needles on the deviations.—In Fig. 398, let $\widehat{NS}$ represent a needle

FIG. 398.

deflected to any angle from the meridian by a magnet whose near-pole is at *D*—the remote pole not being considered, as its influence only lessens the effect of the near-pole. This point and others relating to the present investigation are treated in vol. 1, pages 252, 272, 288, 294, 385, 392, and 437.

Let m be the magnetic intensity of the N-end of the needle, and m' that of the S-end: they often *appear* unequal, as is found in testing compass needles; but although not really so, yet it is the apparent action that must control in what follows, as all deduced results depend upon that: let q be the magnetic intensity of the deflecting influence at D; then the attractive force exerted on the N-end of the needle is $\frac{m \cdot q}{\widehat{ND}^2}$, and the moment F of this force—the effort to turn the needle—is

$$F = \widehat{CP} \cdot \frac{m \cdot q}{\widehat{ND}^2}, \quad . \quad . \quad . \quad . \quad . \quad (132)$$

$\widehat{CP}$ being a perpendicular from C upon the direction PND of the force. In Fig. 398, let $\widehat{NS} = 2l$, or

$$\widehat{CN} = \widehat{CS} = l; \quad . \quad . \quad . \quad . \quad . \quad (133)$$

$$\widehat{CD} = e; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (134)$$

$$\widehat{ND} = h; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (135)$$

$$HCN = HCS = \phi; \quad . \quad . \quad . \quad . \quad (136)$$

$$CND = \alpha; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (137)$$

and

$$NCD = \beta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (138)$$

Then, by Trig.,

$$\sin CNP = \frac{\widehat{CP}}{l},$$

$$\therefore \ \widehat{CP} = l \, . \sin CNP; \quad . \quad . \quad . \quad . \quad (139)$$

Substituting the last member of (154) for $\frac{1}{h^3}$ in (142), this becomes

$$F=\frac{1}{e^3}\left(1+\frac{9}{4}\cdot\frac{l^2}{e^2}+3\cdot\frac{l}{e}\cdot\cos\beta+\frac{15}{4}\cdot\frac{l^2}{e^2}\cdot\cos 2\beta\right)\left(e.l.m.q.\sin\beta\right), \quad (155)$$

or

$$F=m.q.\frac{l}{e^2}\left[\left(1+\frac{9}{4}\cdot\frac{l^2}{e^2}\right)\sin\beta+3.\frac{l}{e}.\sin\beta.\cos\beta + \frac{15}{4}\cdot\frac{l^2}{e^2}.\sin\beta.\cos 2\beta\right]. \quad (156)$$

By Trig.,

$$\sin\beta\cos\beta=\tfrac{1}{2}\sin 2\beta, \quad . \quad . \quad . \quad . \quad (157)$$

and

$$\sin\beta\cos 2\beta=\tfrac{1}{2}\sin 3\beta-\tfrac{1}{2}\sin\beta. \quad . \quad . \quad (158)$$

Substituting (157) and (158) in (156), it becomes

$$F=m.q.\frac{l}{e^2}\left[\left(1+\frac{9}{4}\cdot\frac{l^2}{e^2}\right)\sin\beta+\frac{3}{2}\cdot\frac{l}{e}.\sin 2\beta + \frac{15}{8}\cdot\frac{l^2}{e^2}.\sin 3\beta-\frac{15}{8}\cdot\frac{l^2}{e^2}.\sin\beta\right], \quad (159)$$

or

$$F=m.q.\frac{l}{e^2}\left[\left(1+\frac{3}{8}\cdot\frac{l^2}{e^2}\right)\sin\beta+\left(\frac{3}{2}\cdot\frac{l}{e}\right)\sin 2\beta + \left(\frac{15}{8}\cdot\frac{l^2}{e^2}\right)\sin 3\beta\right]. \quad (160)$$

This is the attractive force of D upon the N-end of the needle; but there is also a repulsive force F' on the S-end; to find this, we must determine the distance $\widehat{SD}=h'$, which, equally with h, is a function of the length of the needle: the procedure to get F' is obviously identical with that just gone through for F, with a similar resulting expression;

we may therefore write at once an equation like (160) for F', with such change as the value of h' naturally introduces.

In Fig. 398, by Geom.,

$$SCD = 180° - NCD = 180° - \beta; \quad . \quad . \quad . \quad (161)$$

by Trig.,

$$\widehat{SD}^2 = \widehat{SC}^2 + \widehat{CD}^2 - 2\widehat{SC}.\widehat{CD}.\cos(180° - \beta), \quad (162)$$

or

$$h'^2 = l^2 + e^2 - 2l.e.\cos(180° - \beta), \quad . \quad . \quad (163)$$

and

$$h' = [l^2 + e^2 - 2l.e.\cos(180° - \beta)]^{\frac{1}{2}}. \quad . \quad . \quad (164)$$

But by Trig.,

$$\cos(180° - \beta) = -\cos\beta; \quad . \quad . \quad . \quad . \quad (165)$$

therefore, where $\cos\beta$ occurs in (156) we must substitute $-\cos\beta$ for it; this is in the third term of the second member, so that the corresponding term in eq. (160), that is, $\frac{3}{2}.\frac{l}{e}.\sin 2\beta$, must be negative. Hence

$$F' = m'.q.\frac{l}{e^2}\left[\left(1 + \frac{3}{8}.\frac{l^2}{e^2}\right)\sin\beta - \left(\frac{3}{2}.\frac{l}{e}\right)\sin 2\beta + \left(\frac{15}{8}.\frac{l^2}{e^2}\right)\sin 3\beta\right]. \quad (166)$$

A quantity of which the *sine* of an angle, as $\sin\beta$, is a factor, will pass through all its numerical values—having one maximum and one minimum,—while the angle changes through a semicircle; and when such a quantity is a deviation, it is hence called semicircular: similarly, a deviation affected by the *sine* of double the angle, as $\sin 2\beta$, will have two maxima and two minima in a semicircle, that is, one of each in a quadrant, and is therefore called quadrantal; likewise, one affected by the *sine* of treble the angle, as $\sin 3\beta$, will have three maxima and three minima in a semicircle, or six in a circle, and is called

sextantal; by analogy, the octantal deviations are connected with sin 4β, the decantal with sin 5β, and so on.

Now it will be observed in (160) and (166) that there are terms affected by sin β, sin 2β, and sin 3β—that is, semicircular, quadrantal, and sextantal deviations: therefore, when there is only a *single* needle, placed on a diameter of the card, as in Fig. 398, and the needle is ***not*** of the hypothetical infinitely small size, but of such practical length as may be used in a compass, the semicircular deviation is increased by the quantity $\frac{3l^2}{8e^2}$; a quadrantal term, $4.\frac{3}{2}.\frac{l}{e}$, is introduced; and a sextantal amount equal to $\frac{15}{8}.\frac{l^2}{e^2}$ is added; and it will be further noticed that in these additions (semicircular and sextantal) the increase is in proportion to the square (l^2) of the half-length of the needle.

By comparing (160) and (166), it will be seen that m and m' are the only factors that differ: if equal, that is, if the magnetic intensity is symmetrically distributed in both ends of the needle, the quadrantal term will disappear by adding the two equations, or

$$[illegible] = 2F - 2m.q.\frac{l}{e^2}\left[\left(1 + \frac{3}{8}.\frac{l^2}{e^2}\right)\sin\beta + \frac{15}{8}.\frac{l^2}{e^2}\sin 3\beta\right]. \quad (167)$$

This is the total turning force of D, Fig. 398, deflecting the needle from the meridian, while the combined magnetic moment of Earth and needle, $M.H.\sin\phi$ (see Part [illegible] pages above cited), pulls it back: when equilibrium obtains we have

$$[illegible] = 2F - 2m.q.\frac{l}{e^2}\left[\left(1 + \frac{3}{8}.\frac{l^2}{e^2}\right)\sin\beta + \frac{15}{8}.\frac{l^2}{e^2}\sin 3\beta\right]. \quad (168)$$

We now pass to the consideration of the force acting on *two* needles placed as chords of the card—Fig. 399: a comparison of this with Fig. 398 shows that the conditions are similar—the needle of Fig. 398 is simply moved out of its central position to form a chord, as in Fig. 399, and shortened to the length of that chord, while another, its

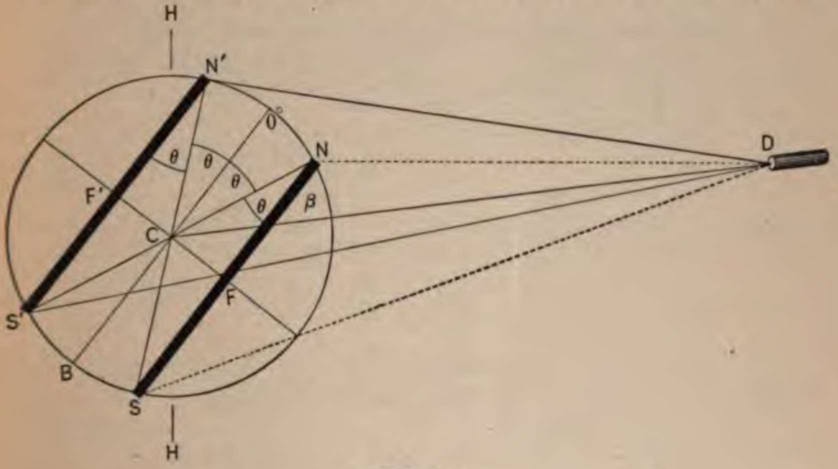


FIG. 399.

exact equal, is symmetrically placed on the other side; the only change then entailed in the previous investigation is to obtain new values for the corresponding angles.

From Fig. 399, by Geom., we have

$$NCD = \beta - \theta; \quad . \quad . \quad . \quad . \quad . \quad (169)$$

$$SCD = 180° - \beta - \theta = 180° - (\beta + \theta). \quad . \quad . \quad (170)$$

$$N'CD = 2\theta + NCD = 2\theta + \beta - \theta = \beta + \theta; \quad . \quad (171)$$

and

$$S'CD = 180° - NCD = 180° - \beta + \theta = 180° - (\beta - \theta). \quad (172)$$

In the formulas for *one* needle, for *two*, and for *four*, l stands for the same thing, viz., the radius of the circle through

the ends of the needles: it *happens* to be the half-length of the needle in Fig. 398, where it forms the diameter of the circle.

Designating by N, S, N', and S' the force acting on each end of the respective needles, we have, by analogy with (160) and (166), for the needle $\widehat{NS}$, by substituting the values from (169) and (170) in (160) and (166):

$$N = m.q.\frac{l}{e^2}\left[\left(1+\frac{3}{8}\frac{l^2}{e^2}\right)\sin(\beta-\theta)+\frac{3}{2}\frac{l}{e}\sin 2(\beta-\theta) + \frac{15}{8}.\frac{l^2}{e^2}\sin 3(\beta-\theta)\right]. \quad (173)$$

$$S = m'.q.\frac{l}{e^2}\left[\left(1+\frac{3}{8}\frac{l^2}{e^2}\right)\sin(\beta+\theta)-\frac{3}{2}\frac{l}{e}\sin 2(\beta+\theta) + \frac{15}{8}.\frac{l^2}{e^2}\sin 3(\beta+\theta)\right]. \quad (174)$$

And similarly for the needle $\widehat{N'S'}$, by substituting the values from (171) and (172) in (160) and (166):

$$N' = m.q.\frac{l}{e^2}\left[\left(1+\frac{3}{8}\frac{l^2}{e^2}\right)\sin(\beta+\theta)+\frac{3}{2}\frac{l}{e}\sin 2(\beta+\theta) + \frac{15}{8}\frac{l^2}{e^2}\sin 3(\beta+\theta)\right]. \quad (175)$$

$$S' = m'.q.\frac{l}{e^2}\left[\left(1+\frac{3}{8}\frac{l^2}{e^2}\right)\sin(\beta-\theta)-\frac{3}{2}\frac{l}{e}\sin 2(\beta-\theta) + \frac{15}{8}\frac{l^2}{e^2}\sin 3(\beta-\theta)\right]. \quad (176)$$

Considering the magnetic intensity symmetrically distributed in the two needles, so that $m=m'$, and adding the four equations (173), (174), (175), and (176), the quadrantal terms cancel, and the following is the result:

$$N+S+N'+S' = 4N = 2m.q.\frac{l}{e^2}\left[\left(1+\frac{3}{8}\frac{l^2}{e^2}\right)\left\{\sin(\beta+\theta) + \sin(\beta-\theta)\right\} + \frac{15}{8}.\frac{l^2}{e^2}\left\{\sin 3(\beta+\theta)+\sin 3(\beta-\theta)\right\}\right]. \quad (177)$$

But, by Trig.,

$$\sin(\beta+\theta)+\sin(\beta-\theta)=2\sin\beta\cos\theta, \quad \ldots \quad (178)$$

and

$$\sin 3(\beta+\theta)+\sin 3(\beta-\theta)=2\sin 3\beta\cos 3\theta; \quad (179)$$

substituting these in (177), it becomes

$$4N=M.H\sin\phi=2.m.q.\frac{l}{e^2}\left[\left(1+\frac{3}{8}\frac{l^2}{e^2}\right)2\sin\beta\cos\theta + \frac{15}{8}\cdot\frac{l^2}{e^2}(2\sin 3\beta\cos 3\theta)\right], \quad (180)$$

or

$$M.H\sin\phi=4m.q\frac{l}{e^2}.\cos\theta\left[\left(1+\frac{3}{8}\frac{l^2}{e^2}\right)\sin\beta + \left(\frac{15}{8}\frac{l^2}{e^2}\cdot\frac{\cos 3\theta}{\cos\theta}\right)\sin 3\beta\right]. \quad (181)$$

This is the total pull on the two needles by D, and is balanced by $M.H.\sin\phi$: if the intensity of the needles be not symmetrically distributed, the quadrantal term must be inserted in (181).

It will be seen in this equation that $\cos 3\theta$ enters the coefficient of the sextantal term, $\sin 3\beta$; therefore, if $3\theta=90^\circ$, [$\cos 90^\circ=0$], and hence the sextantal term disappears: but when $3.\theta=90$, $\theta=30^\circ$, or, in Fig. 399, the needles should be placed 60° apart, 30° on each side of the central diameter.

This result is identical with that for the equal distribution of the moment of inertia, so that these two important principles coincide.

Finally, to ascertain the pull upon four needles is obviously but an extension of that for two: there are simply more needles for which the force must be determined and more angles whose equivalents must be found and substituted in (160) and (166); the procedure is so

similar to that just gone through, that only the result need be written; it is:

$$M.H.\sin\phi = 8m.q.\frac{l}{e^2}.\cos\left(\frac{\theta+\theta'}{2}\right)\cos\left(\frac{\theta-\theta'}{2}\right)\left[\left\{1+\frac{3}{8}.\frac{l^2}{e^2}\right\}\sin\beta + \left\{\frac{15}{8}.\frac{l^2}{e^2}.\frac{\cos 3\left(\frac{\theta+\theta'}{2}\right)\cos 3\left(\frac{\theta-\theta'}{2}\right)}{\cos\left(\frac{\theta+\theta'}{2}\right)\cos\left(\frac{\theta-\theta'}{2}\right)}\right\}\sin 3\beta\right]. \quad (182)$$

This is the total pull by D on the four needles of Fig. 400

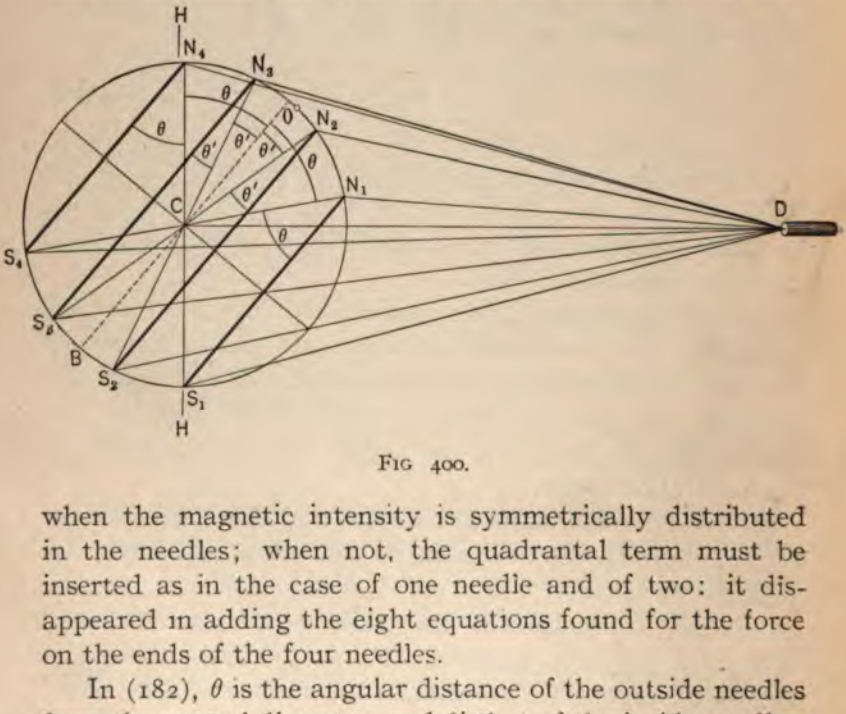


FIG 400.

when the magnetic intensity is symmetrically distributed in the needles; when not, the quadrantal term must be inserted as in the case of one needle and of two: it disappeared in adding the eight equations found for the force on the ends of the four needles.

In (182), θ is the angular distance of the outside needles from the central diameter, and θ' that of the inside needles. These angles determine the lengths of the needles.

In the coefficient of the sextantal term, $\sin 3\beta$, there is the factor $\cos 3\left(\frac{\theta+\theta'}{2}\right)$; if we make $\left(\frac{\theta+\theta'}{2}\right)=30°$, then $3(30°)=90°$, and $\cos 90°=0$, whence, as with the two-needle system, the sextantal term disappears: that is, if the needles be placed on chords of $15°$ and $45°$, we shall have $\left(\frac{15°+45°}{2}\right)=30°$. Thus, for the four-needle card, the avoidance of sextantal deviations is attained by placing the needles on the same chords ($15°$ and $45°$) that are requisite for the equable distribution of the moment of inertia—another fortunate coincidence of essential principles.

While the foregoing investigation is based on a single pole at D, still it covers any disturbing field, however large, remote, or near; but it is evident that in concentrated fields close-to, the effect upon the system in Fig. 400 will not be equal on all four needles: the one nearest D, as $\widehat{N_1S_1}$, will feel the influence more than any of the other three, and will exert a more controlling effort on the movement of the card. This means that the advantage of placing four needles as chords of $15°$ and $45°$ is much lessened, if not almost lost, by the vitiating effect belonging to a *single* needle, and *that* not even centrally located. We should, therefore, expect large sextantal deviations from strong, concentrated fields, close-to; and indeed not only this, but also octantal deviations which are realized in the case of the U. S. S. ATLANTA's steering compass, whose deviations are fully investigated in Art. 327.

The condition otherwise is not at all imaginary: there are already magnets, soft iron, and hard iron, in close proximity to the compass on our ships of war, and the probability is that such are acting more on the nearest needles, than on all equally, thereby detracting from the

gain their proper position on the card would naturally afford.

This remark bears directly on Art. 234, relative to the size of the compass.

By referring to equations (150) and (151), it will be seen that the differentiation of (145) was carried only to second differentials, thus affording {with (145) itself when $l=0$} coefficients for only three terms of the series (146); therefore all subsequent deductions are accurate only to the degree that those three terms afford: series (146) is infinite, however, and the more terms that are employed, the more accurate will be the results dependent on them.

Differentiating (148) we obtain $f'''(l)$, and then by differentiation of this we get $f^{\text{IV}}(l)$; that is, the third and fourth differentials; making $l=0$ in both, we obtain the coefficients $f'''(0)$ and $f^{\text{IV}}(0)$ of series (146), which, when substituted in that series and the various mathematical processes are performed that led up to (182), will give the following as the value of (182) based on differentiation carried to fourth differentials instead of second, as in (182):

$$M.H.\sin\phi = 8m.q.\frac{l}{e^2}\cos\left(\frac{\theta+\theta'}{2}\right)\cos\left(\frac{\theta-\theta'}{2}\right)$$

$$\left[\left\{1+\frac{3}{8}\cdot\frac{l^2}{e^2}+\frac{15}{64}\cdot\frac{l^4}{e^4}\right\}\sin\beta\right.$$

$$+\left\{\frac{15}{8}\cdot\frac{l^2}{e^2}+\frac{105}{128}\cdot\frac{l^4}{e^4}\cdot\frac{\cos 3\left(\frac{\theta+\theta'}{2}\right)\cos 3\left(\frac{\theta-\theta'}{2}\right)}{\cos\left(\frac{\theta+\theta'}{2}\right)\cos\left(\frac{\theta-\theta'}{2}\right)}\right\}\sin 3\beta$$

$$\left.+\left\{\frac{315}{128}\cdot\frac{l^4}{e^4}\cdot\frac{\cos 5\left(\frac{\theta+\theta'}{2}\right)\cos 5\left(\frac{\theta-\theta'}{2}\right)}{\cos\left(\frac{\theta+\theta'}{2}\right)\cos\left(\frac{\theta-\theta'}{2}\right)}\right\}\sin 5\beta\right]. \quad (183)$$

This is the total pull by D on the four needles of Fig. 400, provided their magnetic intensity be symmetrically

distributed: if not, a quadrantal term with sin 2β must be added.

Comparing (182) and (183), it is seen that the use of more terms of series (146) introduced the following: $\frac{15}{64}\cdot\frac{l^4}{e^4}$, an increase in the semicircular term; $\frac{105}{128}\cdot\frac{l^4}{e^4}$, an increase in the sextantal; and an entirely new term—the decantal, denoted by sin 5β.

If, however, we make $\frac{\theta-\theta'}{2}=18°$, then $5\left(\frac{\theta-\theta'}{2}\right)=5(18°)=90°$, and $\cos 90°=0$, whence the decantal term disappears. When $\frac{\theta-\theta'}{2}=18°$, then $\theta-\theta'=36°$, and this should be the distance apart of the needles of each pair on the same side. If the needles of a pair be moved a little apart—the outer ones to chords of 48° and the inner to chords of 12°, this will satisfy the above condition, and the decantal term disappears.

Now chords of 12° and 48° differ but little from those of 15° and 45°, so that even these, on which the needles are practically placed, reduce the decantal term, if not wholly destroy it.

The angle ϕ, to which the compass-card is deflected, whether it carries one, two, or four needles, is therefore represented by a series whose terms, as in (183), have a certain angle β and its multiples for one factor, and constant coefficients for other factors: this series has reference exclusively to the condition of the compass itself—to the length of the needles and their location on the card; and by suitably placing them, some of the terms are reduced in value, or caused to vanish.

The Scoresby is admirably fitted for testing experimentally all the results of the foregoing investigation.

234. The Size of the Compass.—The *size* of the compass is a favorite point of attack with those prone to innovation in this instrument: a large card graduated to degrees is the object of their periodic strife, and none would dispute the contention, if it were merely a question of more material wrought into seemly shape; but the size of the compass—meaning thereby, essentially, the length of the needles and their arrangement on a card—rests, as shown in the preceding article, upon a mathematical investigation that will ever resist the assaults of those unacquainted with the subject. And it is often only such that charge headlong into it: those who have some knowledge of the matter, know well that it is an entanglement of opposing requirements through which a careful conservative course is alone practicable.

The mathematical theory of the deviations conceives the needle to be a material particle—a needle so short that the poles are scarcely separated—a needle, in fact, that is ideal, whereas the real one has considerable length, is not very distant from iron, and must be quite strong to control the direction of the card. Here, then, is the first unavoidable departure from exact conditions, with a corresponding difference between theoretical and actual deviations.

Connected with the theory of the deviations is their compensation, for their analysis affords the only intelligent means of correcting them; and if defect enters the foundation, the superstructure will be faulty.

Now, every inch added to the needles beyond the unavoidable one they *must* have, introduces more and new deviations—the primary curves, semicircular and quadrantal, are increased, while octantal, sextantal, and others less familiar, appear, as in the case of the U. S. S. ATLANTA already cited.

The *theory* takes cognizance of these and all others—

they are, as it were, but the varied magnetic harmonics superposed upon the fundamental note, which may be readily disentangled and spread to view: but, to compensate them—that is another question.

Restraints enough are already put upon the compass, and if we unduly lengthen the needles, we but make every mass of iron in the vicinity so many new irritants; and all must concede that masses of iron and steel—fittings of the ship, to say nothing of soft iron and magnets used as correctors—are now dangerously enough near the needles to deter any one from prolonging them into more remote fields, or to be affected more strongly by proximate ones.

Better not contract a disease than have to dose it constantly.

The *arrangement* of the needles on the card will no longer avail to avoid certain new deviations if the needles be abnormally long.

Without lengthening the needles, the card cannot well be enlarged, for the same magnet power will not suffice to easily move the increased weight of card; and indeed even with longer needles we get a compass less sensitive and strong—proportionately; for it has been abundantly shown in Art. 226 that the magnet power does *not* increase in the same ratio as the mass, but far below it; therefore, while we make the card clearer to the helmsman, we at the same time have one that moves less quickly and surely.

Better, far, have a sensitive card that will invariably return to the meridian from even the slightest deflection.

But by enlarging the card we trench upon another phase of the matter—its period of oscillation; this is increased also: now, it has been shown in Vol. I, that as the Ship and Compass are both magnets, their every motion sets up magnetic waves in the ether—definite in size and period—corresponding to the roll of the one and the oscilla-

[illegible] that these wave
[illegible] if those of th
[illegible] Compass—through
[illegible] we have the recurren
[illegible] a swing, and
[illegible] their joint move
[illegible], if possible.

Table [illegible]

[illegible] OSCILLATION OF COMPASS-CAR[illegible]
[illegible] INCREASED.

[illegible]	[illegible]		[illegible]		[illegible]		16 sec.	
[illegible]	[illegible]		[illegible]		[illegible]		24 sec.	
[illegible]	[illegible]		[illegible]		[illegible]		[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	No. of roll of ship.	No. of oscill. of comp.
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3d with 6th " 9th " 12th " 15th " 18th "	2d 4th 6th 8th 10th 12th
[illegible]	[illegible]		[illegible]		[illegible]		16 sec.	
[illegible]	[illegible]		[illegible]		[illegible]		32 sec.	
[illegible]	[illegible]		[illegible]		[illegible]		[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	No. of roll of ship.	No. of oscill. of comp.
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		2d with 4th " 6th " 8th " 10th " 12th " 14th " 16th " 18th " 20th "	1st 2d 3d 4th 5th 6th 7th 8th 9th 10th

[illegible] let the time of roll
[illegible] sixteen seconds—
[illegible] of the compass

be the same, and then be successively increased by two seconds. When the periods of roll and oscillation are the same, the magnetic waves excited by the ship will speedily set the card oscillating; and as the movements are synchronous, there is no reason why the motion of the ship should not soon become that of the card, and continue so.

When the period of the card is increased 2 s., or when it is 18 s., as in col. (1), it may *start* with the ship, but will soon get out of step, and in twenty rolls will have only two coincidences *nearly*, at the 8th and 17th; in col. (2), with the period of the card 20 s., there are four coincidences, at the 5th, 10th, 15th, and 20th; in col. (3), with the period 22 s., there is none exactly but three *nearly*, at the 7th, 11th, and 18th roll; and similarly with the other columns: it would, therefore, seem that the relative periods of roll and oscillation should be those having few coincidences of the magnetic waves excited by both ship and compass.

With regard to the graduation to degrees, that should be continuous, from 0° to 360°, with only the four cardinal points indicated: all courses and bearings then become specific by merely naming the number of degrees, without even the addition, North, South, East, or West: this is the extreme of simplicity, and an immense gain over present locutions.

Visibility of the degree-marks by the helmsman is of the first importance: it may possibly be effected by an instrument-maker; a magnifying-glass fixed to a ring like an azimuth circle, that could be revolved round the compass as new arcs of the graduation were required in view, might attain the object.

As bearing on the subject of this article and the preceding, an account will be given of two experiments made with the SCORESBY—Fig. 428,—to show the effect of soft iron and steel magnets in close proximity to *different types* of compass: those used are described on Plate *P*.

PLATE P.

FIG. 401.—LIQUID COMPASS No. 6216.
7½-inch card.

A A—two bundles of needles placed close together, so as (practically) to constitute but one, attached to the card in a diameter. Length of each bundle, 3¾ inches; weight, 370 grains; total weight of magnetized steel, 740 grains. Each bundle is composed of steel wires of about the thickness of a fine knitting-needle. This is the compass described in Part Third as having been specially designed for experimental work on the Scoresby.

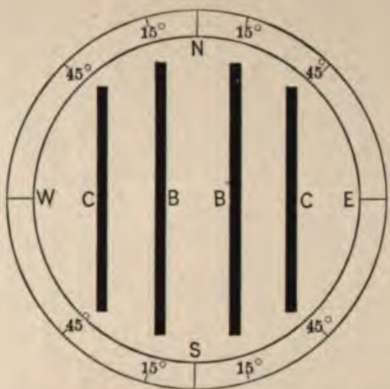


FIG. 402.—LIQUID COMPASS No. 7860.
7½-inch card.

Four bundles of steel-wire needles, each needle about the thickness of a fine knitting-needle. B B—two bundles, each 5½ inches long; weight, 412 grains; placed on the chords of 15° on each side of the diameter. C C—two bundles, each 4½ inches long; weight, 365 grains; placed on the chords of 45° on each side of the diameter. Total weight of magnetized steel, 1,554 grains. This is, in all respects, the type of 7½-inch standard and steering compass adopted in the year 1883 for the U. S. Navy.

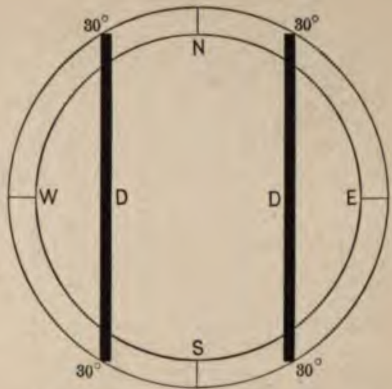


FIG. 403.—LIQUID COMPASS No. 3314.
7½-inch card.

D D—two bundles of very thin steel plates, seven in each bundle; length, 6½ inches; weight, 770 grains; total weight of magnetized steel, 1,540 grains; needles placed on chords of arcs of 30° on each side of diameter. This is the type of 7½-inch standard and steering compass used for many years in the Navy, until the introduction, in 1883, of the four-needle card represented in Fig. 402.

EXPERIMENT 1. SOFT-IRON TUBE: This was twenty-eight inches long, two inches external diameter, walls one-eighth inch thick, and weighed eight pounds. It was placed on the SCORESBY as in Fig. 404—axis horizontal,

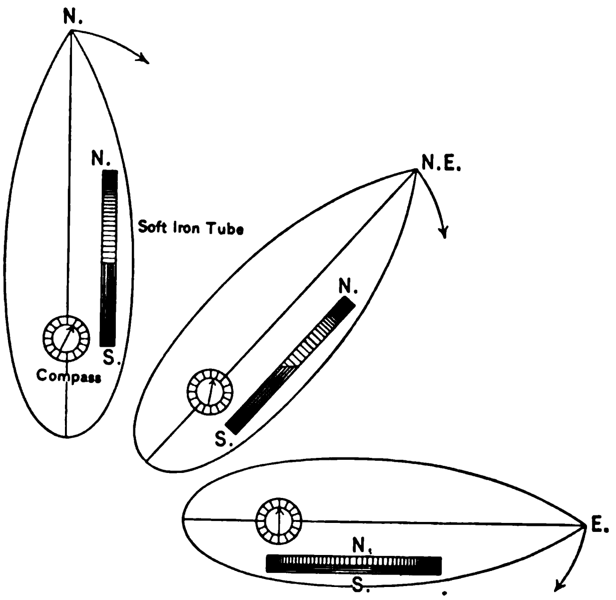


FIG. 404.

parallel to keel, in plane of needle, and with the nearest end FOURTEEN INCHES from the compass pivot: it was not moved during the experiment. The vessel was upright and swung through the *N.E.* quadrant, resting two minutes on each point.

Each compass was successively placed in the Y's and the observations made with it as indicated by the headings of the columns of Table 39; the resulting deviations of the respective compasses, are given in cols. (4), (7), and (10), and illustrated by the curves of Fig. 405.

EXPERIMENT 2. STEEL MAGNET: This was a powerful bar, twenty inches long, of square cross-section, one inch

TABLE 39.

SOFT-IRON TUBE.

Heading of SCORESBY by Compass.	Bearing of True Meridian Line on Wall by Compass No. 6216, *without* Tube on SCORESBY.	Bearing of True Meridian Line on Wall by Compass No. 6216, *with* Tube Placed as in Fig. 404.	Deviations of Compass No. 6216 Produced by Tube Placed as in Fig. 404.	Bearing of True Meridian Line on Wall by Compass No. 7860, *without* Tube on SCORESBY.	Bearing of True Meridian Line on Wall by Compass No. 7860, *with* Tube Placed as in Fig 404.
(1)	(2)	(3)	(4)	(5)	(6)
N.	N. 7° 20′ E.	N. 4° 50′ E.	2° 30′ E.	N. 7° 10′ E.	N. 5° 10′ E.
N. by E.	N. 6 20 E.	N. 4 20 E.	2 0 E.	N. 6 30 E.	N. 4 45 E.
N. NE.	N. 5 40 E.	N. 3 50 E.	1 50 E.	N. 5 55 E.	N. 4 30 E.
NE. by N.	N. 4 55 E.	N. 3 30 E	1 25 E.	N. 5 0 E.	N. 4 10 E.
NE.	N. 4 05 E.	N. 3 30 E.	0 35 E.	N. 4 30 E.	N. 4 0 E.
NE. by E.	N. 3 30 E.	N. 3 20 E.	0 10 E.	N. 4 0 E.	N. 3 50 E.
E. NE.	N. 3 0 E.	N. 3 20 E.	0 20 W.	N. 3 30 E.	N. 3 40 E.
E. by N.	N. 3 0 E.	N. 3 20 E.	0 20 W.	N. 3 10 E.	N. 3 30 E.
E.	N. 3 0 E.	N. 3 0 E.	0 0	N. 3 0 E.	N. 3 15 E.

Heading of SCORESBY by Compass.	Deviations of Compass No. 7860, Produced by Tube Placed as in Fig. 404.	Bearings of True Meridian Line on Wall by Compass No. 3314, *without* Tube on SCORESBY.	Bearings of True Meridian Line on Wall by Compass No. 3314, *with* Tube Placed as in Fig. 404.	Deviations of Compass No. 3314 Produced by Tube Placed as in Fig. 404.
(1)	(7)	(8)	(9)	(10)
N.	2° 0′ E.	N. 7° 35′ E.	N. 4° 45′ E.	2° 50′ E.
N. by E.	1 45 E.	N. 6 45 E.	N. 4 20 E.	2 25 E.
N. NE.	1 25 E.	N. 5 55 E.	N. 4 0 E.	1 55 E.
NE. by N.	0 50 E.	N. 5 10 E.	N. 4 0 E.	1 10 E.
NE.	0 30 E.	N. 4 30 E.	N. 3 55 E.	0 35 E.
NE. by E.	0 10 E.	N. 4 0 E.	N. 4 0 E.	0 0
E. NE.	0 10 W.	N. 3 30 E.	N. 3 40 E.	0 10 W
E. by N.	0 20 W.	N. 3 20 E.	N. 3 35 E.	0 15 W.
E.	0 15 W.	N. 3 0 E.	N. 3 15 E.	0 15 W.

a side. It was placed as in Fig. 406—axis horizontal, in vertical section through keel, and in plane of needles, with its nearest (south) pole distant THIRTY-NINE INCHES from pivot of compass: it was not moved during the experiment. The vessel was upright and swung through the

N.E. quadrant, resting two minutes on each point. Each compass was successively placed in the Y's and the observations made as indicated by the headings of the columns

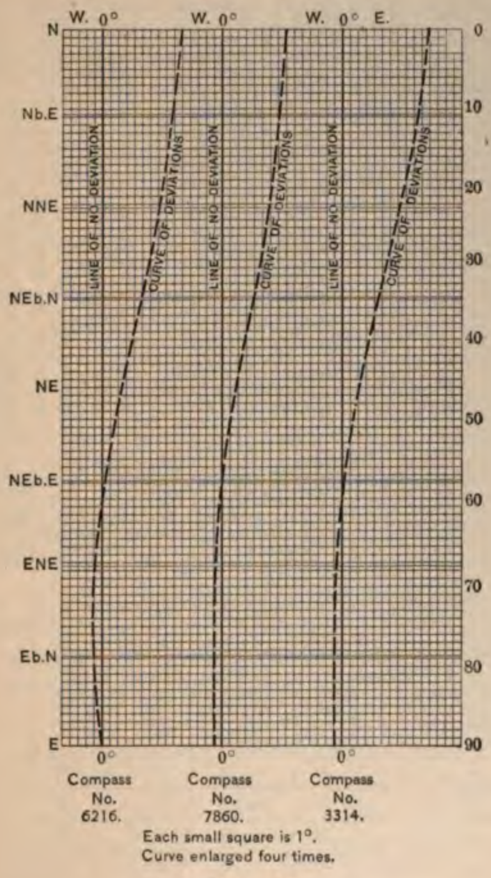


FIG. 405.

of Table 40; the resulting deviations given in cols. (4), (7), and (10) are illustrated by the curves of Fig. 407.

By observing Plate *P*, it will be seen that the three types of compass were as dissimilar as well could be: one had a single short needle of extremely low magnet power,

TABLE 39.

SOFT-IRON TUBE.

Heading of SCORESBY by Compass.	Bearing of True Meridian Line on Wall by Compass No. 6216, *without* Tube on SCORESBY.	Bearing of True Meridian Line on Wall by Compass No. 6216, *with* Tube Placed as in Fig. 404.	Deviations of Compass No. 6216 Produced by Tube Placed as in Fig. 404.	B[illegible] True Lin[illegible] by No. [illegible]	Bearing of True Meridian Line on Wall by Compass No. 7860, *with* Magnet Placed as in Fig. 406.
(1)	(2)	(3)	(4)	[illegible]	(6)
N.	N. 7° 20′ E.	N. 4° 50′ E.	2° 30′ E.	[illegible]	N. 7° 25′ E.
N. by E.	N. 6 20 E.	N. 4 20 E.	2 0 E.	[illegible]	N. 4 35 E.
N. NE.	N. 5 40 E.	N. 3 50 E.	1 50 E.	[illegible]	N. 2 0 E.
NE. by N.	N. 4 55 E.	N. 3 30 E.	1 25 E.	[illegible]	N. 0 35 W.
NE.	N. 4 05 E.	N. 3 30 E.	0 35 E.	[illegible]	N. 2 30 W.
NE. by E.	N. 3 30 E.	N. 3 20 E.	0 10 E.	[illegible]	N. 4 10 W.
E. NE.	N. 3 0 E.	N. 3 20 E.	0 20 W.	[illegible]	N. 5 15 W.
E. by N.	N. 3 0 E.	N. 3 20 E.	0 20 W.	[illegible]	N. 5 55 W.
E.	N. 3 0 E.	N. 3 0 E.	0 0	[illegible]	N. 6 0 W.

Heading of SCORESBY by Compass.	Deviations of Compass No. 7860, Produced by Tube Placed as in Fig. 404.	Bearings of True Meridian Line on Wall by Comp No. 3314, *with* Tube on SCORESBY.	[illegible]	Deviations of Compass No. 3314, Produced by Magnet Placed as in Fig. 406.
(1)	(7)	(8)	[illegible]	(10)
N.	2° 0′ E.	N. 7° 35′ [illegible]	[illegible]	0° 25′ W.
N. by E.	1 45 E.	N. 6 45	[illegible]	1 40 E.
N. NE.	1 25 E.	N. 5 55	[illegible]	3 30 E.
NE. by N.	0 50 E.	N. 5 10	[illegible]	5 20 E.
NE.	0 30 E.	N. 4 3[illegible]	[illegible]	6 55 E.
NE. by E.	0 10 E.	N. 4 [illegible]	[illegible]	8 20 E.
E. NE.	0 10 W.	N. 3 [illegible]	[illegible]	8 55 E.
E. by N.	0 20 W.	N. 3 [illegible]	[illegible]	[illegible] 20 E.
E.	0 15 W.	N. 3 [illegible]	[illegible]	[illegible] 0 E.

a side. It was placed as [illegible] very strong needles vertical section through [illegible] of fine wires, its nearest (south) pole [illegible] third had two pivot of compass: it [illegible] made up of ment. The vessel wa[illegible] considering the

instrumental and observational errors, and the variable effect of the distant pole of the tube and magnet on the compass on different headings of the vessel, the deviations

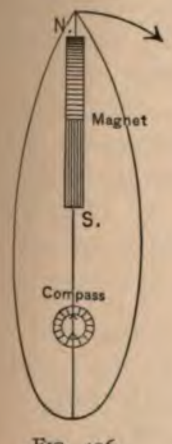


FIG. 406.

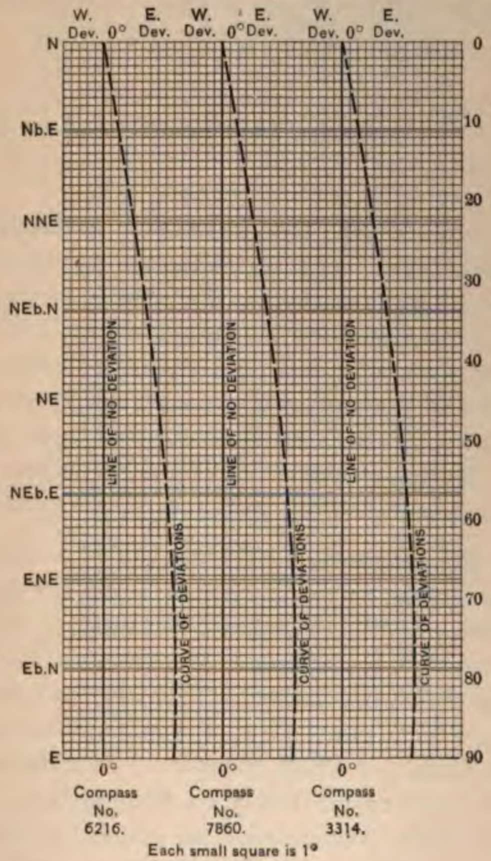


FIG. 407.

of the three compasses, as shown in Tables 39 and 40, are nearly enough alike to warrant the conclusion that *at the distances employed for the disturbing forces*, the *type of compass* does not affect the resulting deviations; but if these distances had been much less—or the single needle had

the length of the two-needle system—or the needles of this and the four-needle card had been arranged on chords, other than those proven to be the best, it is safe to say that the deviations would not be so accordant.

Section Two: Inspection of the Compass.

235. What the inspection determines.—The needles, card, pivot, and sapphire-cap having been examined in Washington in the manner stated in the foregoing chapter, they are returned to the maker in Boston, where they are put in the compass, and the instrument completed. When a lot of compasses are ready, the Superintendent of Compasses proceeds to Boston, and there, in the observatory of Messrs. Ritchie, the final inspection takes place. By this, the following points are determined: 1st, coincidence of the magnetic axis with the $\widehat{N.S.}$-line of the card; 2d, sensibility, or exact return of the needles to the meridian, when slightly deflected; 3d, relative strength of the magnetic system; and 4th, centering of the card on the pivot. The following minor points are also matters of examination: that the face of the bowl is horizontal—determined by placing a small spirit-level in different diameters; that the bowl moves freely on the gimbals; that the keel-lines and their continuation as scores on the rim of the bowl are well defined; that the card and compass-box have the same number; that the marking of the points is correct; and that the general appearance of the instrument is acceptable.

236. Instruments used for the inspection.—Certain apparatus, which will now be described, are required for making the inspection.

PLATE Q.

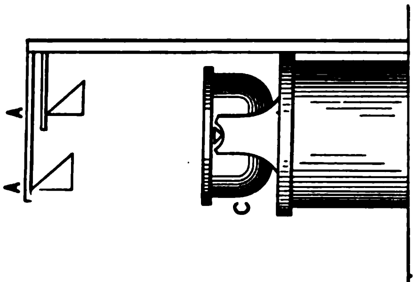


FIG. 410

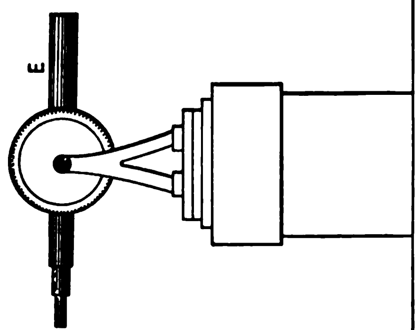


FIG. 409.

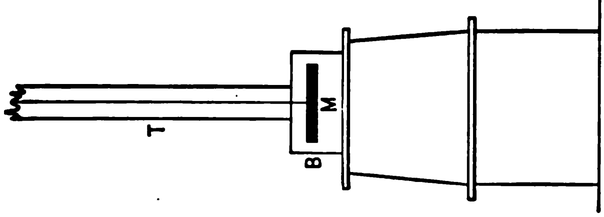


FIG. 408.

prisms which reflect the magnetic axis, or line of the magnetic meridian, into the telescope of the theodolite. Fig. 415 is from a photograph of the compass-stand complete: by means of a screw at the right, the heavy brass block forming the immediate support for the compass is movable east and west on solid rails of brass; the prisms *AA* are contained in the rectangular box above the compass, and several screws are at the side for adjusting the prisms to correct reflection of the magnetic axis. Fig. 413 is a brass ring with four strong magnets attached: when placed on a compass and turned round like an azimuth circle it controls the direction of the card completely; the needles may be made to point with the North toward the South, or the East, or the West, or in any other direction. Finally, two magnets—a compass-needle and a single wire—are required for deflecting the card, several degrees or a few minutes, as desired; and a chronometer or stop-watch (as is usually employed) is needed to note the periods of oscillation.

237. Coincidence of the magnetic axis with the $\widehat{N.S.}$-line of the card.—The points are so painted on the card that the line through North and South forms the 0°-diameter of the degree-graduation; and it is essential that the *axis* of the magnetic system should coincide with this diameter. The adjustment is made before sealing the compass, and the inspection by the Superintendent is to verify its accuracy.

To effect coincidence, the maker places the card on its pivot, the bowl being filled with liquid; he then views the zero-diameter (0°) through the theodolite, and if he finds it separated from the middle vertical hair (which indicates the magnetic meridian, as will be presently seen), he moves one end of a needle until coincidence is attained. The needles had previously been firmly sealed in their tubes—all except one end of a long one; small lateral screws press on this, so that it has a slight side-way

motion; but so accurately are the needles placed at first that it is only in a few instances that the maker has to resort to a turn of the screws to ensure perfect coincidence of the magnetic axis of the system with the zero-diameter (0°) of the card.

The Earth's magnetic field is most nearly quiet during the forenoon: if, then, on a still day (magnetically speaking) the theodolite *E* of Plate *Q* be pointed to the Variation-magnet *M*, Fig. 408, the latter will be observed making very regular oscillations of small amplitude; a mean position is readily determined—it is the magnetic meridian, upon which the central vertical wire of the telescope is set, and the horizontal circle read.

If, now, the telescope be turned 180° in azimuth, it will joint to the compass-stand, and the central wire indicates the magnetic meridian in that direction; a compass being placed on the stand, is to be moved east or west by the large screw on the right, until its North-and-South line is exactly upon the wire, or until it cannot be made to take that direction: in the former case, the magnetic-axis and zero-diameter coincide—in the latter, they do not, and the movable needle must be suitably moved to effect it.

Instead of determining the meridian in this way, it can be done by placing the Variation-compass *C* (Figs. 411 and 412) on the stand, Fig. 410; the axis *HH* of its needle lies in the meridian, and is reflected by the prisms *AA* into the telescope which is moved until exact coincidence with the vertical wire is effected—this wire is then the magnetic meridian: removing the variation-compass and placing any other in its stead, its magnetic axis will be indicated by the vertical wire cutting the central diameter of the card; if the wire coincides with the $\widehat{N.S.}$-line throughout, the requirement is fulfilled, otherwise not. This is the

most expeditious method, and is the one usually practiced. After each compass is tested, the variation-compass is put on the stand, and the position of the vertical wire aligned anew with the magnetic meridian.

The compasses are handled with great care to avoid motion of the liquid and card, and even then some minutes must elapse to ensure quiet, before making the observation.

238. Sensibility of the compass.—While the card is at perfect rest, a single wire is brought toward it from one side and an easterly deflection of 15′ to 30′ is produced; the wire is quickly withdrawn; the movement of the card is watched through the telescope; the magnetic axis returns to the vertical wire—passes beyond—returns—and rests on the wire: a deflection of 15′ to 30′ toward the west is next produced, with a similar result, and thus the compass is found to have fine sensibility. It is a slight force that will cause a deflection of 15′ to 30′, and this small arc affords ample opportunity for friction to do its utmost in preventing return of the card to its original position. The test is a severe one, but I have never known a compass of the many inspected to fail in it.

239. The period of oscillation of the card an index of the strength of its needles.—The period of oscillation is the time elapsed between two successive transits of the 0°-diameter of the card in opposite directions across the keel-line; it depends on many things: 1st, friction between cap and pivot; 2d, mobility of the liquid—whether it be more or less viscous; 3d, total weight of the oscillating mass; 4th, the distribution of this with respect to a center—its moment of inertia; 5th, currents induced in the copper bowl by the swinging needles, and which react upon them—curbing their motion; 6th, temperature of the liquid and needles; 7th, magnetism of the needles; and 8th, the Earth's horizontal intensity at the place of observation.

These may all vary; they may conspire toward an increase or a decrease of the period, or the effect of some may neutralize that of others. In any case, it is a matter of easy inference which way a change will affect the period.

By care and skill, the fluctuation of some of these conditions can be reduced to such narrow limits that usually, and with reason, they are lost sight of as variables: this is the case with all but the 6th and 7th; and thus the period of oscillation at a standard temperature becomes an index of the strength of the needles.

TABLE 41.

Number of Compasses Inspected at a Time in One Lot.	Mean Weight of Short Needles Grains	Mean of Deflections Produced by Short Needles	Mean Weight of Long Needles Grains.	Mean of Deflections Produced by Long Needles.	Mean Total Weight of Magnetic Steel on Card = 2 (Col. 2 + Col. 4). Grains.	Mean Weight of Card Alone, Without Needles.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
19	347	8° 15′	419	9° 15′	1532	1436
21	349	9 15	420	9 15	1538	1440
31	347	9 50	420	9 30	1534	1392
24	347	9 50	420	9 30	1534	1392
28	357	8 45	430	8 40	1574	1368
10	354	9 05	431	9 10	1579	1368
133						

Number of Compasses Inspected at a Time, in One Lot.	Mean Weight of Total Matter in Oscillation: Grains Col 6 + Col. 7.	Mean of Deflections Produced by the Four Needles of Each Card, *on the Card*	Mean Amplitude of Oscillation on Each Side of Keel-line for Period	Mean of Times Between First and Second Passage of 0° Across Keel-line = Period of Oscillation.	Mean Temperature During Oscillation Experiment. F Deg
(1)	(8)	(9)	(10)	(11)	(12)
19	2968	29° 30′	11° 43′	17.9 sec.	70°
21	2978	31 30	11 18	17.8 "	60
31	2026	31 15	11 15	15.8 "	65
24	2926	31 15	11 15	16.8 "	60
28	2942	29 0	11 15	17.5 "	70
10	2938	31 30	11 15	15.0 "	85
133					

Table 41 is the record of 133 compasses inspected at various times during a period of four years: a different number was examined each time, as shown by col. (1), and only mean values of each lot are given on the horizontal

lines. The needles in these compasses were the same as those used for all our compasses, namely, 5.28 and 4.4 inches long respectively; diameter of each, 0.25 inch; diameter of the individual wires, 0.06 inch.

The deflections of col. (3) were produced at a distance of 14.2 inches between centers of deflecting and deflected needles; those of col. (5) at a distance of 15.7 inches; and those of col. (9) at a distance of 14.9 inches from the center of the card to the center of the deflected needle, the needles being in their tubes on the card.

It will be perceived by cols. (2) and (4) how closely the weights of the needles run, and hence by col. (6) how nearly alike are the quantities of magnetic steel in compasses of different lots; also how little the card alone differs from one group to another, and—as a final result—how nearly identical is the oscillating mass throughout: since it is distributed with the same symmetry in all cases, this means that both the weight and its moment of inertia are within such restricted limits of variability as to warrant calling them the same in all compasses. The cap and pivot are identical in all, and hence friction may be said to be constant. The liquid is the same and so is its mobility at a given heat. Induced currents are explained on page 471 *et seq.*; the kind of metal surrounding a swinging magnet, its thickness, proximity to the magnet, and the strength and rapidity of motion of this—are all factors of the intensity of induced currents: except strength of needles, these factors are the same for all compasses, and as the current is proportional to needle strength and motion, the whole factor may thus be said to be constant.

Regarding the effect of heat, the following results of some crude experiments with the same compass will afford an idea:

Temperature, F.	60°	70°	85°
Oscillations, seconds.	16.5	15.5	15.0

The ordinary changes of temperature that a compass experiences, affect the magnetism of its needles but little: it is quite probable that it is the liquid that is sensitive to heat and cold—becoming more or less mobile, and thereby influencing the period of oscillation.

Of all the quantities upon which the period of oscillation depends, we have thus practically eliminated as constants, all except the magnetism of the needles themselves, that of the Earth, and Temperature.

The mean of the periods of the 133 compasses of Table 39 is 16.9 seconds, and this may be taken as a fair average: if, in any compass, it should be found many seconds greater than this, it is an indication of the magnetism of the needles having become weak and the compass unreliable. Of course this supposes the test made where the Earth's horizontal magnetic intensity has the same value as at Washington: if made where less, as in high magnetic latitudes, the period will increase without any necessary diminution of the needles' strength; while, if made where greater, as in low magnetic latitudes, the period will decrease without any consequent augmentation of the needles' strength.

TABLE 42.

THE MAGNETIC ELEMENTS AT WASHINGTON, D. C.

	Force in C. G. S. Units: Yearly Means.	
	1889.	1890.
(1)	(2)	(3)
V = variation.	4° 01′ 31″ W.	4° 05′ 45″ W.
D = dip.	71° 05′ 59″	71° 04′ 31″
H = horizontal force.	0.19869	0.19860
Z = vertical force.	0.58033	0.57927
T = total force.	0.61340	0.61238

Table 42 gives the magnetic elements at Washington: they are from the continuous records of the magnetographs

described in Vol. I, when in the observatory on the old site in the city of Washington.

After the sensibility has been determined, and while the 0°-mark of the card and vertical wire of the telescope are in one, a magnet is brought toward the compass from the side and the card deflected about 11° to the East; the magnet is quickly removed, and, as the card swings back, the transit of its 0°-mark across the vertical wire and keel-line (still in one) is noted by stop-watch; the card continues on, completing its amplitude to the west and then turns back; the second transit of the 0°-mark across the wire to the east is noted, and the elapsed time between both transits is the period of oscillation.

This procedure is repeated, beginning with a deflection to the west, and as many such observations are made as will give a good mean.

The temperature of the air in the room is observed, and is assumed to be that of the liquid and needles, as the compasses should have been in the room some hours.

440. Centering the card on the pivot.—The compass being on the stand, Fig. 410, the magnet-ring, Fig. 413, is placed on it so that the *N*-point of the card is held in the general direction of the Earth's magnetic north, though not necessarily coincident with the meridian; opposite parts of the graduation are reflected into the telescope by the prisms *A A*; the degree marks coincident with the middle wire are noted, and if the centering is accurate with respect to this diameter, these readings will differ by 180°. Then the magnet-ring is turned until the *S*-point of the card is held toward the magnetic north, and the marks on the vertical wire are again noted and recorded; the observation is repeated with the east, and then the west, point, toward the north, and it may be done with the intercardinal points turned in that direction, i. e., in any four of [illegible] diameters at right angles; the two marks of [illegible]

the graduation seen on the vertical wire of the telescope differ by 180°, the card is accurately centered.

This adjustment is made before sealing the compass, and while the card is still accessible to rectify any defects. There are four small screws set at right angles to each other in the side of the spindle, Fig. 386, that carries the sapphire cap, and by a suitable tentative motion of these—observing the effect through the telescope—the card may be, and is, exactly centered.

This completes the inspection of the compass. A record of the procedure both as regards the individual parts and the finished instrument is kept in the Office of Superintendent of Compasses.

241. The azimuth circle.—A few words will suffice for this inseparable adjunct of the compass: every officer who uses it, knows the principles of its construction; it is made by Messrs. Ritchie—the material is brass, painted black—and it is inspected by the Superintendent of Compasses. The chief requisites are: solidity; firm attachment of the parts; clearness of the silvered surfaces; vanes perpendicular to the plane of the ring; central hair vertical; to set level on the bowl, fit closely and concentrically, and turn smoothly and easily.

Section Three: Magnetic Moment of the Compass.

242. Data relative to the needles.—For the purpose of determining the magnetic moment of needles singly, and combined on a card, the following experiments were made in Washington in 1898. The needles were selected from a large number then under trial for new compasses, and would be representative of those in general use: each was encased in a paper wrapper weighing 0.65 gram, and, for convenience of reference, they were marked A, B, C, D, respectively.

TABLE 44.
WASHINGTON, JUNE 14, 1898.

Designation of Needle.	Passage of Needle Across Meridian Line.	Needle Alone. Times Recorded.	Needle Alone. Differences.	Needle Alone. T' Period Reduced to Small Arc.	Needle with Brass Bar Attached. Times Recorded.	Needle with Brass Bar Attached. Differences.	Needle with Brass Bar Attached. T'' Period Reduced to Small Arc.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		h. m. s.	s.		h. m. s.	m. s.	
A	1st	4 5 39.5	48.5		4 26 16.5	1 4.5	
	10th	6 28.0	48.0	sec.	27 21.0	1 4.5	sec.
	20th	7 16.0	48.5	4.836	28 25.5	1 4.5	6.455
	30th	8 04.5	48.5		29 30.0	1 4.5	
	40th	8 53.0	48.5		30 34.5	1 5.0	
	50th	9 41.5			31 39.5		
B	1st	5 33 08.0	47.5		5 44 50.0	1 3.0	
	10th	33 55.5	47.0	sec.	45 53.0	1 3.0	sec.
	20th	34 42.5	47.0	4.726	46 56.0	1 3.0	6.293
	30th	35 30.5	47.5		47 59.0	1 3.0	
	40th	36 18.0	47.5		49 02.0	1 3.0	
	50th	37 05.5			50 05.0		
C	1st	5 9 02.0	42.5		5 21 48.5	1 6.0	
	10th	9 44.5	43.0	sec.	22 54.5	1 6.0	sec
	20th	10 27.5	42.5	4.266	24 0.5	1 5.5	6.584
	30th	11 10.0	43.0		25 6.0	1 6.0	
	40th	11 53.0	42.5		26 12.0	1 6.0	
	50th	12 35.5			27 18.0		
D	1st	4 39 10.0	42.0		4 55 27.5	1 04.0	
	10th	39 52.0	42.0	sec.	56 31.5	1 04.5	sec.
	20th	40 34.0	42.0	4.207	57 36.0	1 04.0	6.405
	30th	41 16.0	42.0		58 40.0	1 04.0	
	40th	41 58.0	42.5		59 44.0	1 04.0	
	50th	4 42 40.5			5 00 48.0		

the card and tested singly in the same way as on June 13th (Table 43), with the results given in Table 45.

TABLE 45.
WASHINGTON, SEPT. 29, 1898.

Designation of Needle.	Deflections. N-end acting.	Deflections. S-end acting.	Deflections. N-end acting.	Deflections. S-end acting.	Distance from Center to Center of Deflecting and Deflected Needle.
A (long)	5° 0′	5° 0′			47.29 cms.
A "			9° 0′	9° 0′	39.70 "
B "	5 0	5 30			47.29 "
B "			9 30	9 30	39.70 "
C (short).............	5 0	5 0			43.68 "
C "			9 0	9 0	36.00 "
D "	5 15	5 15			43.68 "
D "			9 15	9 15	36.00 "

243. Moment of inertia of the brass bar.—The bar being a regular parallelopipedon, of *square* cross-section, with the axis of rotation through its center of gravity perpendicular to the length, its moment of inertia was calculated by formula (54), Art. 230, that is,

$$K'' = M'\left(\frac{a^2+b^2}{12}\right); \quad . \quad . \quad . \quad . \quad (184)$$

in which K'' is the moment of inertia of the bar; M' its mass, 34.55 grams; a its length, 10.2 centimeters; and b its width, 0.65 cm.: substituting these values in (184), it becomes

$$K'' = (34.55)\left[\frac{(10.2)^2+(0.65)^2}{12}\right] = 300.764. \quad (185)$$

244. Moment of inertia of the needles, singly.—The needles being neither tubular nor solid—yet cylindrical in form with cross-section full of small holes—that of a cable—the formula suitable to tube or rod was not deemed strictly applicable to calculating their moments of inertia; although the formula for a solid cylinder would give an approximate result: in this doubt, recourse was had to the method of oscillating the needle, first alone, and then with the brass bar attached; the latter being a regular body, its moment of inertia was calculated as in Art. 243, and then that of the needle determined by formula (75), Art. 230, that is,

$$K' = K''\left(\frac{T'^2}{T''^2 - T'^2}\right); \quad . \quad . \quad . \quad . \quad (186)$$

in which K' is the moment of inertia of the needle; K'' that of the brass bar; T'^2 the square of the period of oscillation of the needle alone; and T''^2 that of the needle with the brass bar attached.

Substituting successively in the above formula the

value of K'' from (185) and those of T'^2 and T''^2 from Table 44, we have for each needle the following:

For needle A,

$$K'_A = (300.764)\left[\frac{(4.836)^2}{(6.455)^2 - (4.836)^2}\right] = 384.8, \quad (187)$$

and similarly,

$$\text{For needle } B, \ K'_B = 388.4. \quad . \quad . \quad . \quad (188)$$

$$\text{For needle } C, \ K'_C = 217.7. \quad . \quad . \quad . \quad (189)$$

$$\text{For needle } D, \ K'_D = 224.7. \quad . \quad . \quad . \quad (190)$$

245. Magnetic moment of the needles.—The magnetic moment was calculated by formula (76), Art. 230, that is,

$$M.H = \frac{\pi^2 . K'}{T'^2}; \quad . \quad . \quad . \quad . \quad . \quad (191)$$

in which M is the magnetic moment of the needle; K' its moment of inertia; T' its period of oscillation alone; H the horizontal component of the Earth's magnetic force at the place of observation; and $\pi = 3.1416$. Substituting in (191) the proper data from Table 44 and eq. (187), we have: For needle A:

$$M.H = \frac{(3.1416)^2 (384.8)}{(4.836)^2}. \quad . \quad . \quad . \quad (192)$$

Hence

$$M.H = 162.4. \quad . \quad . \quad . \quad . \quad . \quad (193)$$

For calculating the *ratio*, we have formula (46), page 424, that is:

$$\frac{M}{H} = \frac{1}{2} . x^3 \tan \theta. \quad . \quad . \quad . \quad . \quad . \quad (194)$$

In this, M and H have the same meaning as in eq. (191); x is the distance from the center of the deflecting needle

to the center of the deflect*ed* one; and θ the angle of deflection. As given in Table 43, each needle was used to produce deflections at two different distances: substituting these distances and the corresponding deflections successively in (194), we have: For needle A:

$$\frac{M}{H}=\left\{\tfrac{1}{2}(47.29)^3(\tan 5^\circ 45')\right\} \quad . \quad . \quad . \quad (195)$$

Hence, for the first distance

$$\frac{M}{H}=5326. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (196)$$

And for the second distance

$$\frac{M}{H}=\left\{\tfrac{1}{2}(39.70)^3(\tan 9^\circ 30')\right\}. \quad . \quad . \quad . \quad (197)$$

Hence, for the second distance

$$\frac{M}{H}=5236. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (198)$$

To obtain the value of M, eq. (193) must be multiplied by eqs. (196) and (198) in succession, that is

$$(M.H)\left(\frac{M}{H}\right)=M^2=(162.4)(5326). \quad . \quad . \quad (199)$$

Hence, for needle A, for first distance,

$$M=929.8. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (200)$$

And also,

$$(M.H)\left(\frac{M}{H}\right)=M^2=(162.4)\ (5236). \quad . \quad . \quad (201)$$

Hence, for second distance

$$M=922.0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (202)$$

As bearing upon what follows, pages 294, 295, 382, 384, 422, and 425 should be consulted. From equation (2), page 416, we have

$$m = \frac{M}{2l}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (203)$$

in which M has the same meaning throughout; that is, the magnetic moment of the needle; $2l$ its *exact* length; and m its pole-strength, or resultant magnetic force for one half the needle: the point of application of this resultant is not at the very extremity of the needle, but a little inside it; therefore, to divide the magnetic moment by the *exact* length, as indicated by (203), will give a pole-strength m a trifle less than is actually the case.

The length ($2l$) of needle A is 13.3 cms. and the two values of M are given by equations (200) and (202); substituting these successively in eq. (203), we have:
For needle A:

$$m = \frac{929.8}{13.3} = 69.9; \quad \text{and} \quad m = \frac{922.0}{13.3} = 69.3. \qquad (204)$$

To get the value of H, we have from eqs. (193), (196), and (198), by dividing the first by the second and the third successively,

$$\frac{(M \cdot H)}{\left(\frac{M}{H}\right)} = H^2 = \frac{162.4}{5320}; \text{ hence } H = 0.1746. \qquad (205)$$

And

$$\frac{(M \cdot H)}{\left(\frac{M}{H}\right)} = H^2 = \frac{162.4}{5230}; \text{ hence } H = 0.1761. \qquad (206)$$

Thus from experiments of oscillation and deflection with needle A, the preceding values of M, m, and H were obtained; and by similar experiments at the same time

and place with needles B, C, and D, and like calculations thereon, other values of M, m, and H for these needles also were obtained: they are all given in Table 46.

TABLE 46.

ALL VALUES ARE IN C.G.S. UNITS.

Designation of Needle.	M	m	H	K'
(1)	(2)	(3)	(4)	(5)
A (long)	929.8 922.0	69.9 69.3	0.1746 0.1761	384.8
B (long)	951.0 955.4	71.5 71.8	0.1786 0.1777	388.4
Means.	939.6	70.6	....	386.6
C (short)	703.9 674.0	63.4 60.8	0.1678 0.1751	217.7
D (short)	708.9 708.6	63.9 63.8	0.1767 0.1768	224.7
Means.	698.9	63.0	0.1754	221.2

246. Meaning of the quantities in Table 46.—Taking the averages in the several columns of Table 46, we find for needles A and B together, that $M=939.6$; $m=70.6$; and $K'=386.6$—as representatives of the magnetic moment, pole-strength, and moment of inertia, respectively, of the *long* needles: and similarly, for C and D together, that $M=698.9$; $m=63$; and $K'=221.2$ for the *short* needles.

The mean of col. (4) is 0.1754; and this was the value of H on the day of observation at the place where the experiments were conducted, that is, on the SCORESBY, in Room No. 80 of the Navy Department.

The only correction applied to the observed quantities was for reduction to small arc. The watch was an excellent one, so that the rate-correction (if any) would be small; the suspension fiber was of fine silk, and thus the torsion couple was insignificant; the hard temper of the

needles probably made the induction factor negligible: and from these considerations, the omitted corrections could affect the values of Table 46 by only small amounts.

To interpret physically the results of Table 46: if one gram of steel could be imbued with south magnetism alone—that is, a pole created separate and apart from its congener—and were free to move along the magnetic meridian, it would experience in the Compass Office at Washington an *increase* of its velocity—an acceleration—of 0.1754 centimeter per second in its attraction toward the north magnetic pole of the Earth; just as the same mass, if let fall from the top of the Washington Monument, would experience an acceleration of 980 centimeters per second in its descent to Earth under the force of gravity.

Now, a center of magnetic force is of the same nature whether it be that of the Earth or of a compass-needle—it is merely large and of widespread influence in the one, and small and concentrated in the other: in both cases, a magnetic particle imbued with one kind of polarity (could such exist) would experience a pull—an acceleration, if in motion, proportional to the intensity of the force and to the inverse square of the distance of the particle from its focus: therefore, by analogy, if a single pole of the mass of one gram should experience at the distance of the city of Washington from the Earth's magnetic north-pole an acceleration of 0.1754 centimeter per second; so this same magnetic gram—not at any distance from the north-pole of the needle A, but in actual contact with it—would feel a force of attraction such as would impart to it, if in motion, an acceleration of 69.9 centimeters per second.

The mean value of the pole-strength for both *long* needles is 70.6, and for both *short* needles, 63; and the meaning of these figures is, that such are the accelerations respectively, in centimeters per second, that the poles of

these needles would cause in a gram of steel imbued with one kind of magnetism, if in motion in the immediate vicinity of their foci.

Both magnetism and gravity are central forces, that is, their powers are exerted toward a point—the center of a field of influence: comparing the force of magnetism in each size of needle with the force of gravity at Washington, it is $\frac{70.6}{980}=0.072$ for the long needles, and $\frac{63}{980}=0.064$ for the short needles; or, to state it differently, the force of gravity is about fourteen times greater than the magnetic force of each pole of the long needles, and sixteen times greater than that of the short needles.

247. Data relative to the needles as a magnetic system on the card.—The following experiments were made August 30 and 31, 1898, in Washington, to determine the magnetic moment of the needles *as a system*. The same four needles were used throughout; they were selected from a lot of two hundred—were among the best—and would fairly represent such as are accepted for compasses: each was in a paper case, and, for convenience of reference, they were marked *L*, *M*, *P*, *Q*, respectively. They were received from the maker June 30, 1898; were laid away parallel to each other, about one inch apart, like-poles in same direction, and north-poles towards the Earth's magnetic north, until July 28th; then they were measured and weighed, and the deflections each produced, observed: these data are given in Table 47.

The needles were then placed in the metal tubes of the card, just as in the finished compass, and this combination known as system I, remained so from July 28th until August 30th, when it was oscillated.

For this purpose, the system was suspended horizontally by a silken fiber attached to a little hook fastened with sealing-wax to the central screw in the ellipsoid of the card.

TABLE 47.

WASHINGTON, JULY 28, 1898.

Designation of Needle.	Length. Cms.	Diameter. Cms.	Weight (Steel Alone, Exclusive of Paper Case). Grams.	Deflections.		Distance from Center of Deflecting to Center of Deflected Needle. Cms.
				N-end acting.	*S*-end acting.	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
L	11.1	0.65	24.30	10° 0′	10° 0′	36.0
M	11.1	0.65	24.16	10 0	10 0	36.0
P	13.3	0.65	29.16	10 0	9 30	39.7
Q	13.3	0.65	29.16	10 15	10 15	39.7

The magnetic meridian was traced on paper fastened to a table close below the swinging card; it had been determined by means of a single magnetic wire (when at rest) suspended by a fiber of silk.

The card was allowed to oscillate until it acquired uniform motion, and then successive passages of its *N*-point across the meridian were noted by a stop-watch marking quarter-seconds: Table 48 contains data relative to system I as it oscillated; Fig. 417 illustrates the method of noting the time and oscillations; semi-amplitude of latter at beginning 12°, at ending 5°; temperature during period 29° C.

TABLE 48.

	Grams.
Total mass (card and needles) oscillating in system I...	197.9
Total mass of magnetic steel (wires only) oscillating in system I..	106.8
Total mass of inert matter (card, and paper cases of needles) oscillating in system I	91.1

When this series was finished, two brass inertia bars were added to system I and it became known as system II—differing from system I only by the addition of the two bars.

System II was now oscillated precisely as system I had been.

The inertia bars were fastened on top of the card parallel to the two long needles, symmetrically, one on each

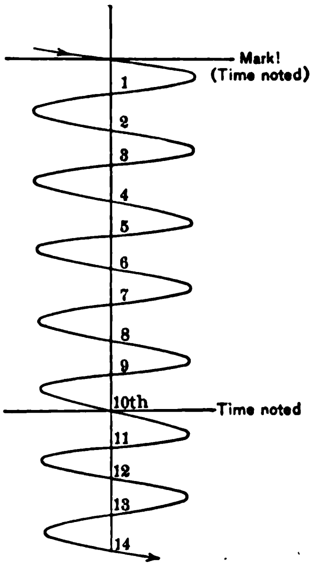


FIG. 417.

side of the axis of rotation, which was through the center of the card: semi-amplitude of oscillation of system II at beginning 14°; at ending 7°; temperature during period 29° C. Table 49 contains particulars of the oscillation of systems I and II.

The oscillations were made August 30th, and the deflections the next day; for the latter, the card, with the needles still in the metal tubes (as in the oscillation experiment) was placed successively at three distances from the deflected needle and the deviations observed: they are given in Table 50.

The needles were now removed from the card and the deflections each produced by itself, observed: the results

TABLE 49. (Washington, Aug. 30, 1898.)

Passage of N-point of Card Across Meridian.	Card and Needles in it. System I.		Card with Needles in it and Inertia Bars on Top. System II.	
	Times.	Differences.	Times.	Differences.
(1)	(2)	(3)	(4)	(5)
Mark!	$10^h\ 04^m\ 25^s.0$		$10^h\ 43^m\ 32^s.0$	
		$1^m\ 28^s.5$		$2^m\ 5^s.5$
10th transit	05 53 .5		45 37 .5	
		1 28 .0		2 4 .5
20th "	07 21 .5		47 42 .0	
		1 28 .5		2 4 .5
30th "	08 50 .0		49 46 .5	
		1 28 .0		2 4 .5
40th "	10 18 .0		51 51 .0	
		1 28 .0		2 5 .0
50th "	11 46 .0		53 56 .0	
		1 28 .0		2 4 .0
60th "	13 14 .0		56 00 .0	
		1 28 .0		2 4 .0
70th "	14 42 .5		58 04 .0	
		1 27 .5		2 4 .5
80th "	16 10 .0		11 00 08 .5	
		1 29 .0		2 4 .5
90th "	17 39 .0		02 13 .0	
		1 27 .0		2 4 .0
100th "	10 19 06 .0		11 04 17 .0	
Period of oscillation, reduced to small arc: $T' = 8^s.8015$.			Period of oscillation reduced, $T'' = 12^s.4451$.	

TABLE 50. (Washington, Aug. 31, 1898.)

Needles on Card as Systems I and II.		Distance from Center of Deflected Needle to Center of Card. Centimeters.
N-end acting.	S-end acting.	
5° 0′	5° 0′	72.39
9 30	9 30	58.42
31 45	31 45	38.10

are given in Table 51; comparing it with Table 47, obtained under precisely similar circumstances on July 28th, it will be seen that the needles lost but little of their magnetism.

TABLE 51.

WASHINGTON, AUG. 31, 1898.

Designation of Needles.	Deflections.		Distance from Center to Center of Deflecting and Deflected Needles. Centimeters.
	N-end acting.	*S*-end acting.	
L	9° 0′	9° 0′	36.0
M	8 30	8 0	36.0
P	9 20	9 40	39.7
Q	9 40	9 40	39.7

248. Moment of inertia of the two brass bars.—The bars were placed on top of the card parallel to the long needles, as *EE*, Fig. 418: they were identical in weight and dimensions—each 17.78 centimeters long; of *square*

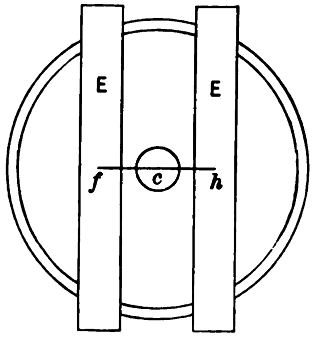


FIG. 418.

cross-section, 0.65 cm. a side; and weighed 60.46 grams; c is the axis of rotation—vertical through the center of gravity of the oscillating system; f and h are the centers of the bars, each distant 1.8 cms. from c, so that $\widehat{fh}$ is 3.6 cms.

As each bar oscillated round an axis c parallel to one through its own center of gravity, formulas (47) and (50), Art. 230, are applicable to the case: the moment of inertia

(K^{IV}) of each bar round an axis through its *own* center of gravity and perpendicular to the length of the bar, is given by formula (54), Art. 230; that is,

$$K^{IV} = M'\left(\frac{a^2+b^2}{12}\right); \quad . \quad . \quad . \quad . \quad (207)$$

in which M' is the mass of the bar, a the length, and b its width. To find the moment of inertia (K''') for the axis through c parallel to that through h or f, we must add to the above, $M' . \widehat{ch}^2$, the formula thus becoming

$$K''' = M'\left(\frac{a^2+b^2}{12} + \widehat{ch}^2\right). \quad . \quad . \quad . \quad (208)$$

Substituting in this the given numerical values of the bar, we have

$$K''' = (60.46)\left[\frac{(17.78)^2+(0.65)^2}{12} + (1.8)^2\right]. \quad . \quad (209)$$

Hence

$$K''' = 1790. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (210)$$

As there are two identical bars, symmetrically placed, this result must be doubled to get the total moment of inertia (K'') of the two bars in oscillation in system II; hence

$$K'' = 3580; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (211)$$

in which K'' is the total moment of inertia of the two bars round the vertical axis through the center of figure of the system.

249. **Moment of inertia of the card and needles combined.**—This system, while symmetrical, is not of the regular form that admits direct calculation of its moment of inertia: it was therefore obtained by formula (75) of Art. 230, that is,

$$K' = (K'')\left(\frac{T'^2}{T''^2 - T'^2}\right); \quad . \quad . \quad . \quad . \quad (212)$$

in which K'' is that determined by eq. (211); T' the period of oscillation of the card with the four needles in the tubes; and T'' the period of the card and needles with the brass bars added. Substituting in (212) the numerical values from (211) and Table 49, we have

$$K' = (3580)\left[\frac{(8.8015)^2}{(12.4451)^2 - (8.8015)^2}\right] \quad . \quad . \quad (213)$$

Hence

$$K' = 3588. \quad . \quad . \quad . \quad . \quad . \quad . \quad (214)$$

That is, K' is the moment of inertia of the *card and needles combined as in the finished compass in Service.*

250. Magnetic moment of the needles—as a system. —This was calculated by formula (76), Art. 230, that is,

$$M.H = \frac{\pi^2.K'}{T'^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (215)$$

Substituting in this the value of K' from (214) and that of T' from Table 49, it becomes

$$M.H = \frac{(3.1416)^2(3588)}{(8.8015)^2}. \quad . \quad . \quad . \quad . \quad (216)$$

Hence

$$M.H = 457.2. \quad . \quad . \quad . \quad . \quad . \quad (217)$$

To obtain the ratio of M to H, formula (194), Art. 245, is employed:

$$\frac{M}{H} = \tfrac{1}{2}.x^3.\tan\theta. \quad . \quad . \quad . \quad . \quad . \quad (218)$$

Here, x is the distance from the center of the deflect*ed* needle to the center of the deflect*ing* system, and θ the angle of deflection: there are three values of each, given in Table 50. Substituting corresponding quantities from

that Table successively in (218), we have for the first distance and angle

$$\frac{M}{H} = (\tfrac{1}{2})(72.39)^3(\tan 5^\circ\ 0') = 16600.0. \quad . \quad (219)$$

For the second,

$$\frac{M}{H} = (\tfrac{1}{2})(58.42)^3(\tan 9^\circ\ 30') = 16680.0. \quad . \quad (220)$$

And for the third,

$$\frac{M}{H} = (\tfrac{1}{2})(38.10)^3(\tan 31^\circ\ 45') = 17110.0. \quad . \quad (221)$$

To obtain M, eq. (217) must be multiplied successively by eqs. (219), (220), and (221); hence for the first value.

$$(M.H)\left(\frac{M}{H}\right) = M^2 = (457.2)(16600.0). \quad . \quad (222)$$

Hence

$$M = 2755.0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (223)$$

For the second value,

$$(M.H)\left(\frac{M}{H}\right) = M^2 = (457.2)(16680.0). \quad . \quad (224)$$

Hence

$$M = 2762.0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (225)$$

For the third value,

$$(M.H)\left(\frac{M}{H}\right) = M^2 = (457.2)(17110.0). \quad . \quad (226)$$

Hence

$$M = 2765.0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (227)$$

To get H, eq. (217) must be divided successively by (219), (220), and (221):

For first value

$$\frac{M.H}{\frac{M}{H}} = H^2 = \frac{457.2}{16600.0}; \text{ hence } H = 0.1660. \quad . \quad (228)$$

For second value

$$H^2 = \frac{457.2}{16680.0}; \text{ hence } H = 0.1656. \quad . \quad (229)$$

For third value

$$H^2 = \frac{457.2}{17110.0}; \text{ hence } H = 0.1635. \quad . \quad (230)$$

The mean value of H is 0.16503 C.G.S.-units, and this represents the magnetic field in that part of the Compass Office in which the experiments were made August 30 and 31, 1898. The mean of eqs. (223), (225), and (227) is 2760.7 C.G.S.-units—that is, the value of M, the magnetic moment of the four needles *as a system* in the metal tubes of a card: it is representative of the magnetic moment of every $7\frac{1}{2}$-inch compass in the Service to the extent that the four needles from which it was derived are fair samples of those finally selected for compasses.

On account of the omission of small corrections such as those enumerated in connection with the single needles, the results are approximate.

251. Magnetic force, and couple, acting on the $7\frac{1}{2}$-inch compass-card.—By the magnetic moment M—determined in the preceding article—is meant the effect of two equal forces acting at the ends of an arm of definite length: it is now proposed to determine both the force and the arm that constitute the magnetic moment of the $7\frac{1}{2}$-inch compass.

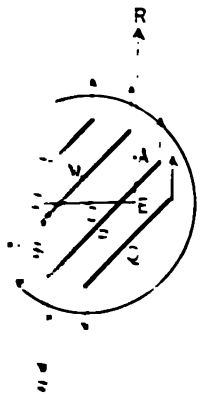
that Table ... the card: the pole-
tance and ... in direction
... they constitute
... resultants R and R'

... the length $\widehat{AB}$ is
For the ... that is,

... (231)

... equal forces that
And for ... compass.
... and the bowl filled

To obta...
by eqs.



Hence

For th...

Hence

For ... field now acts on
... of the horizontal
... action between
... turn the card as
Hen... ... The lines repre-
... parallel, but

their resultants, $\widehat{AR}$ and $\widehat{BR'}$, when prolonged through the points of application, A and B, are at the ends of an arm $\widehat{DE}$; this couple, $R\text{-}\widehat{DE}\text{-}R'$, tends to turn the card into alignment with the Earth's field, as in Fig. 419. If the card had been turned until the needles were across the magnetic meridian, as in Fig. 421, we should have the

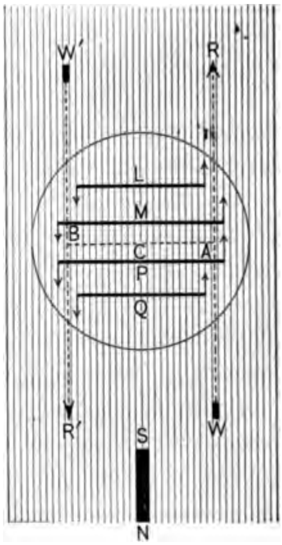


FIG. 421.

longest arm possible, and therefore the maximum effort of the couple; for the forces and their resultants remain the same in all cases. It must be understood, however, that in the actual condition of affairs, there is in Figs. 419, 420, and 421, reciprocal action between field and needles: still the factor *field* is not included in R; this stands only for the united pole-strength (m) of the four needles on the card.

Referring to eq. 203, Art. 245, it is

$$m=\frac{M}{2l}. \quad \ldots\ldots \quad (232)$$

Con... strength ... and an... two gr... have po... the mea... magnetic moment represented ... whose mean value ... C.G.S.-units; $\widehat{AB}$ corre... (231); and m to R ... forces acting at either ... these numerical values in

... C.G.S.-units. . (233)

This is t... represent... is the *combined pole-strength* ...

Let th... pole-strength of a long needle ... needle, 63.0: the sum of ... each length of needle, is ... in (233) by 40.9 C.G.S.-... as indicating the loss of ... combination on a card— ... P, Q would, singly, be ... from which the values ... and which latter were dealt ... 220.29 by 4, we have ... each needle L, M, P, Q in ... but this makes the long and ... whereas they bear a ... the basis of the mean strength ... adjusting the strength of the ... other according to this ratio, ... long ones a pole-strength of ... each of the two short ones 53.18 ... *positions they occupy on the card.*

with liqui... that gives the compass direc-
R-$\widehat{AB}$-R', ... representing a finished com-
lines of for... field and ... in Fig. 42... senting th... and let a magnet $\widehat{NS}$

be presented to it: at a certain point of approach, the card will be turned across the meridian as shown; if, now, access be had to the arm $\widehat{AB}$, and very fine silk threads be tied to it, one at A and the other at B, and both be passed over little wheels without frictional movement, certain weights W, W', might be attached to the end of each thread—the magnet $\widehat{NS}$ removed—and the card would remain across the meridian; the value of this weight is now to be determined—it is the force that gives the compass direction.

Evidently—since it is in equilibrium with the directive magnetic force existing between the needles and the Earth—it is equal to the product $R.H$: the mean value of H at Washington for 1890 was 0.1986 C.G.S.-unit; the value of R from (233) is 226.29 C.G.S.-units; hence

$$R.H = (226.29)(0.1986) = 44.94 \text{ dynes.} \quad . \quad (234)$$

The value of the force of gravity at Washingon is about 980 dynes; hence to get the value of $R.H$ in terms of gravity, we have

$$W = \frac{44.94}{980} = 0.046 \text{ gram.} \quad . \quad . \quad . \quad (235)$$

And this is the weight—a truly small one—that must be suspended as in Fig. 421 from each thread fastened to the end of an arm 12.2 centimeters long, to keep the card across the meridian: it wiil serve to impress one with the delicate forces called into play in the compass.

CHAPTER XV.

THE COMPASS IN USE.

253. The natural magnetic field.—Such facts as tend to explain the Magnetism of the Earth and its extension into Air as Electricity were set forth in Vol. I: the abode of the phenomena was conceived to be the ether of space; and the periodic movements of this ether as well as the fluctuations of these movements were fully described—they give rise to both the regular and erratic motions of every freely suspended needle of small size, and even to some degree affect the compass. Formulas for the *general* investigation of this surging medium were deduced in Arts. 193 to 196, and we have now to determine the component that directly controls the compass: this matter has already been treated from different standpoints, and it will conduce to its better understanding if articles 88 to 91, 122 and 178; and pages 295, 382, 392, 393; and Chapter XI, be consulted.

If a large magnet be laid on a table, and a small needle be suspended at different points above it, as in Fig. 422, the needle will dip—the end of unlike polarity approaching the magnet—except at the middle point, where the needle is horizontal: it is a universal law of the relative action of two magnets—when one is the Earth, as well as when one is only a steel bar.

Magnetic attraction and repulsion produce this result; and the effects might be carried to the extent of the bodily transfer of a magnet if it were floated on a cork slab in

water—it would approach or recede according to the pole of another magnet presented to it. That magnets do not so move under the influence of the Earth alone, is due to the great distance of even its nearest magnetic pole compared to the small length of steel bars; the Earth acts

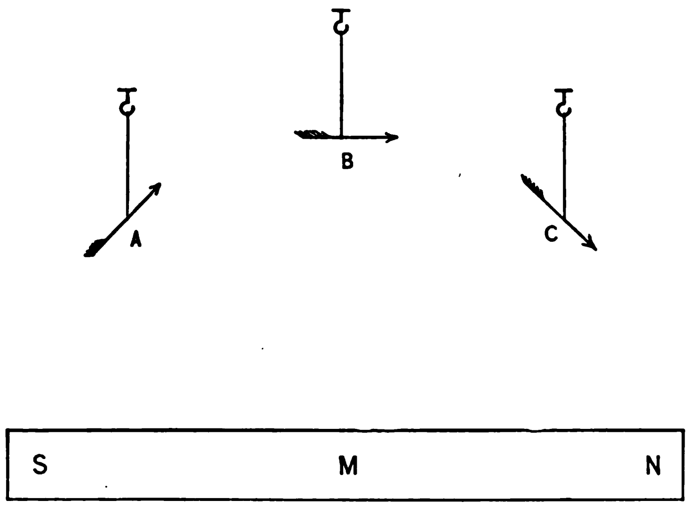


FIG. 422.

equally on each end of the magnet—repels the one as much as it attracts the other, and thus exercises only a directive force.

Consider Fig. 423—it represents a needle under the influence of the Earth's north-pole: this exerts an attractive force $\widehat{FB}$ on the near end, and a repulsive force $\widehat{EB'}$ on the remote end; both are equal in amount and parallel in direction because of the great distance of the Earth's pole compared to the length $\widehat{EF}$ of the needle: similarly, the terrestrial south-pole gives rise to two equal, though lesser, forces—$\widehat{EA'}$, attractive, and $\widehat{FA}$, repulsive. Completing the parallelogram of forces at each end, we have

the resultant effect of the Earth, $\widehat{FR}$ and $\widehat{ER'}$, which gives the needle direction—the dip that takes place at C, Fig. 422: when the needle is at A, Fig. 422, the south-pole of the Earth (simulated by the magnet M) is dominant, and the other end of the needle dips: when it is at B,

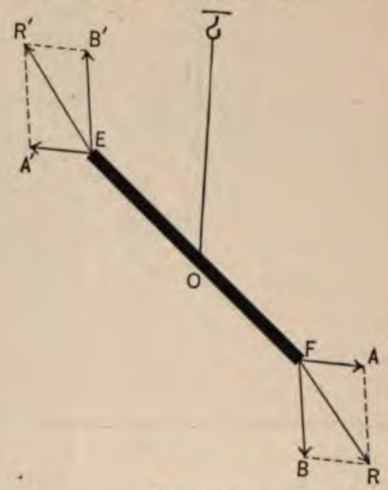


FIG. 423.

both poles act equally, and the needle becomes horizontal—a condition that is experienced at the terrestrial magnetic equator. In Fig. 423 the lines $\widehat{FR}$ and $\widehat{ER'}$ should have been drawn in the prolongation of $\widehat{EF}$.

Now, to deal separately with $\widehat{FR}$, Fig. 423, transfer it to Fig. 424: whatever result is obtained, will be equally applicable to $\widehat{ER'}$; both these forces and the axis of the needle (Fig. 423) are in the vertical plane through the magnetic meridian, which is thus but the trace of this plane on the surface of the Earth: for a small extent the meridian may be represented by a horizontal straight line $\widehat{FC}$ (Fig. 424), to which CR is perpendicular.

In the triangle FCR, then, we have: $\widehat{FR}=T$, the total magnetic force; $\widehat{FC}=H$, the horizontal component; $\widehat{CR}=Z$, the vertical component; and $CFR=D$, the dip. The formulas of a right-angled triangle afford the means

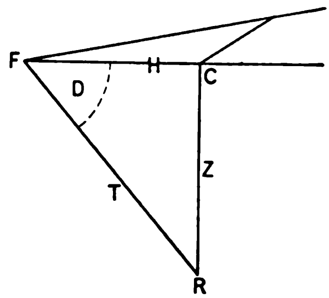


FIG. 424.

of determining any one of these quantities when two others are given, thus:

$$\sin D=\frac{Z}{T};\ \text{hence } Z=T.\sin.D. \quad . \quad . \quad (236)$$

$$\cos D=\frac{H}{T};\ \text{hence } H=T.\cos D. \quad . \quad . \quad (237)$$

$$\tan D=\frac{Z}{H};\ \text{hence } Z=H.\tan D. \quad . \quad . \quad (238)$$

$$\cot D=\frac{H}{Z};\ \text{hence } H=Z.\cot D. \quad . \quad . \quad (239)$$

$$\sec D=\frac{T}{H};\ \text{hence } T=H.\sec D. \quad . \quad . \quad (240)$$

$$\operatorname{cosec} D=\frac{T}{Z};\ \text{hence } T=Z.\ \operatorname{cosec} D. \quad . \quad (241)$$

$$T^2=H^2+Z^2;\ \text{hence } T=\sqrt{H^2+Z^2}. \quad . \quad (242)$$

$$H^2=T^2-Z^2;\ \text{hence } H=\sqrt{T^2-Z^2}. \quad . \quad . \quad (243)$$

$$Z^2=T^2-H^2;\ \text{hence } Z=\sqrt{T^2-H^2}. \quad . \quad . \quad (244)$$

The quantity H, variously determined by some of these formulas, is that component of the Earth's total force which gives the compass direction in every part of the globe; it represents the field (modified by surrounding iron) in which the experiments of the preceding chapter were conducted: usually, H denotes the natural magnetic field of the Earth as pictured by parallel lines—called lines of force—Fig. 425. When the compass-needles are par-

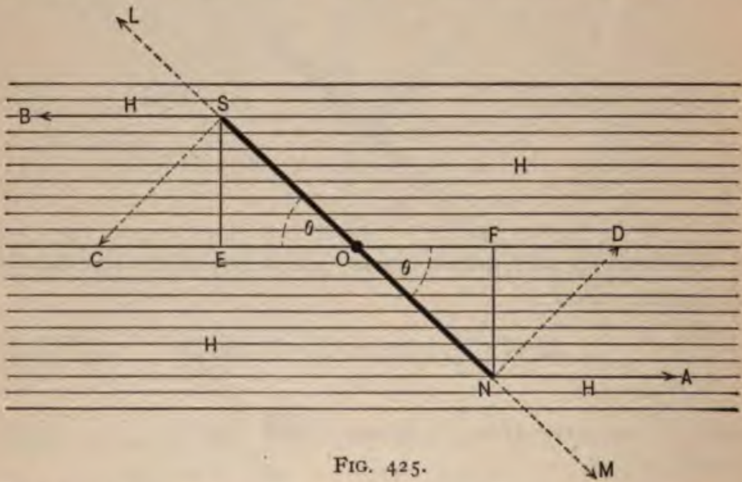


FIG. 425.

allel to them, they are steadiest—when across, most unstable, and the turning moment greatest.

Consider a midway position $\widehat{NS}$—Fig. 425: there are two ways of determining the couple acting on the compass—by resolving either the force or the arm.

TO RESOLVE THE FORCE: in Fig. 425, in place of $\widehat{NA} = \widehat{SB} = H$, substitute components parallel and perpendicular to $\widehat{NS}$; $\widehat{NM}$ and $\widehat{SL}$ being equal and opposite, neutralize each other, leaving the couple $\widehat{ND}$, $\widehat{NS}$, $\widehat{SC} = Q$; hence

$$Q = \widehat{ND}.\widehat{NO} + \widehat{SC}.\widehat{SO}. \quad . \quad . \quad . \quad . \quad (245)$$

By construction

$$\widehat{OD}=\widehat{NA}=\widehat{OC}=\widehat{SB}=H. \quad . \quad . \quad . \quad . \quad (246)$$

Whence

$$\sin\theta=\frac{\widehat{ND}}{\widehat{OD}}=\frac{\widehat{SC}}{\widehat{OC}};\ \text{hence}\ \widehat{ND}=\widehat{SC}=H.\sin\theta. \qquad (247)$$

Also

$$\widehat{NO}=\widehat{SO}=l=\text{one half the length of the needle.} \quad (248)$$

Whence from (245) to (248),

$$Q=2l.H.\sin\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (249)$$

This refers to the field H only in connection with the arm; but it is not at a single point N or S alone that H acts, but throughout the length of the needle, which is magnetic in its finest particles; and the field is not accurately represented by parallel lines, however closely drawn—rather, it is like a solid stream—so that the interaction is between the most minute subdivision of field and needle: the resultant effect is therefore expressed by the product $m.H$, of which the field factor is summed up in H and the needle factor in m—pole-strength. Introducing this total force into (249), we obtain the magnetic moment M of the compass:

$$M=2l.m.H.\sin\theta. \quad . \quad . \quad . \quad . \quad (250)$$

To resolve the arm: the resultant force $m.H$ remaining unchanged in amount, it may be applied to the branches $\widehat{NF}$ and $\widehat{SE}$ into which $\widehat{NS}$ is resolved at right angles to the direction of the forces, that is, the magnetic moment M becomes

$$M=(m.H)(\widehat{NF})+(m.H)(\widehat{SE}). \quad . \quad . \quad (251)$$

By Fig. 425, we have

$$\sin\theta = \frac{\widehat{NF}}{\widehat{NO}} = \frac{\widehat{SE}}{\widehat{SO}}; \text{ hence } \widehat{NF} = \widehat{SE} = l.\sin\theta. \qquad (252)$$

Substituting this in (251), it becomes

$$M = 2l.m.H.\sin\theta, \quad . \quad . \quad . \quad . \quad (253)$$

which is the same as equation (250), as it should be.

254. The natural magnetic field modified by various causes.—Were the compass controlled by the natural field alone, it would point forever to the magnetic north; but such is seldom the case: the influence of the ship itself is an artificial modification—ever present—ever varying. Being a magnet, when she pitches or rolls it gives rise to ether waves that beat upon the compass, and if timed to its period, may set up considerable oscillation of the needle. Then, on board, there are equipments that excite other commotions—dynamos and motors that send out magnetic waves; instruments for wireless telegraphy that generate electromagnetic waves; and circuits that create strong magnetic fields—all which, conspiring or conflicting, affect the compass: from whatever source they come, they are ether waves whose varied manifestations have been described in Vol. I.

Firing the battery often radically changes the magnetic character of a ship, which in turn has a reflex action on the deviations.

There are also natural sources of modification of the Earth's magnetic field: experiment has shown that oxygen is magnetic, and hence air also; it has been calculated that one cubic meter of oxygen would act on a compass like 0.54 gram of iron, and one cubic meter of air like 0.11 gram of iron; also, that in its magnetic character the entire

atmosphere equals a rind or peel of iron 0.1 millimeter thick enveloping the Earth: it can easily be imagined how the incessant motion—the varied wrinkling of this skin—might set up disturbing ripples. Again: the relative condition of Earth and Air, electrically considered, is that of positive and negative charge—the former negative, the latter positive, with increasing potential the higher we ascend; the dense stratum of non-conducting air that intervenes between the rare regions and the Earth completes the analogy of the whole to a huge Leyden jar: in this stratum the potential has been observed to vary 40 volts in the height of a foot, and sometimes much more in unsettled weather; with this extreme and rapid change vertically, something akin must occur *horizontally*—quick differences of potential in small areas—and this might readily be felt by the compass.

Finally: instances are on record where the poles of the compass have been reversed by lightning—or neutralized—or turned 90°, so that the *N*-point of the card was east or west: it is therefore essential to examine the compass after a thunder-storm.

It has also been observed that lightning magnetizes anew masses of iron and steel in the ship.

Add to the foregoing *disturbances* the counter-strains—the magnets and soft iron that are disposed about it to reduce waywardness, and it may truly be said that the compass, like a mettlesome colt, is bound and curbed until scarcely a free motion is left it in the natural field destined for its abode.

255. The true, the magnetic, and the compass, course.—However complex may be the composition of the field around the compass, this always points in the direction of the *resultant* force; and from this direction all *compass* courses are reckoned: but it is often necessary to know their equivalents in angles reckoned from other initial

lines, and hence we have the *true* course counted from the meridian through the geographical poles, and the *magnetic* course counted from the closed curve through the magnetic poles.

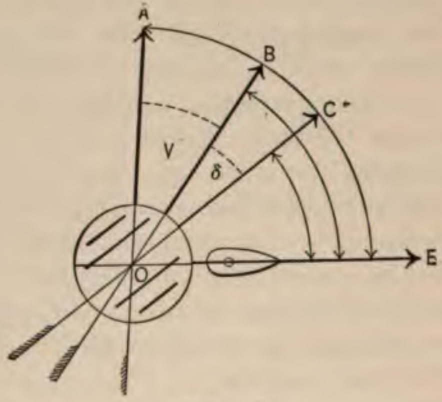


FIG. 426.

Consider Fig. 426: $\widehat{AO}$ is the true meridian and $\widehat{BO}$ the magnetic, forming the angle $AOB = V$, the variation; $\widehat{CO}$ is the direction of the resultant of all the forces to which the compass is subject, making with $\widehat{BO}$ the angle $BOC = \delta$, the deviation: it is evident that $\widehat{CO}$ may lie between $\widehat{BO}$ and $\widehat{AO}$, or on the other side of the latter line; or, that $\widehat{BO}$ and $\widehat{CO}$ may *both* be on that side, occupying the relative positions of the figure, or a transposed one.

Then E being the direction of the ship's head, EOC is the compass course; EOB, the magnetic course; and EOA, the true course. Regarding directions east of the true meridian as plus (+) and those west as minus (−) the several courses may be expressed thus:

The true course is the angle formed by the ship's track with the meridian through the poles of the Earth: the

magnetic course = true course ± the variation; the compass course = true course ± the variation ± the deviation proper to that compass course.

RULE I. Given magnetic course; required true course: look from the center of the compass at the given point; if the variation is easterly, add its amount to the right of that point; if westerly, to the left.

RULE II. Given true course; required magnetic course: look from the center of the compass at the given point; if the variation is easterly, add its amount to the left of the given point; if westerly, to the right.

RULE III. Given compass course; required magnetic course: use Rule I, substituting for variation the deviation proper to the given compass course.

RULE IV. Given magnetic course; required compass course: use Rule II, substituting for variation the deviation proper to the given magnetic course; observing, however (since the compass course is not known and hence not the deviation proper to it), that when the deviation is large, and changes greatly in amount from point to point, the process must be repeated, using the new deviation proper to the new compass course, and thus approximating until a correct result is reached.

For the mechanical indication of these courses, the Indicator represented in Fig. 427 was devised in 1882: it is a variation of an instrument long known to seamen.

It consists of three pieces of thick cardboard, two circular and one square; the graduation on each is printed from copper plates; the cards are placed one on top of the other—the upper is red, five inches in diameter; the middle white, seven inches in diameter; and the lower blue, eight inches; the latter is fixed in a box, and the other two turn about it on the brass screw in the center, which is so made that both circular cards and a brass arm may be clamped separately or together.

GENERAL RULE FOR USE OF THE COURSE INDICATOR: Set the north point of the white card (with reference to

[illegible] Course Indicator, set to Example given on page 711. [illegible] points on fixed *blue* card; middle degree-marks (in [illegible] and points on *white* revolving circle; and inner degree- [illegible] *red* revolving circle.

[illegible] the blue one) in the direction and by [illegible] variation; similarly, set the north point

of the red card (with reference to the north point of the white one) in the direction and by the amount of the deviation: then any one of the three courses—the true, the magnetic, or the compass course—being given, revolve the arm until the *beveled* edge is upon that course, when the other two are seen on the same edge.

To illustrate: suppose a ship heading W. by N., deviation 7° W., variation 23° E.; required the true and magnetic courses: set the *N*-point of the white circle at 23° E. of the *N*-point of the blue card, and, while holding it there, set the red circle with *its N*-point at 7° W. of the *N*-point of the white circle; clamp the two circles and move the brass arm until its *beveled* side is on W. by N. of the *red* circle; then the corresponding divisions in line on the beveled edge will be found to be: on the *red* circle, the *compass* course W. by N. = N. 79° W. = [281°]; on the *white* circle, the *magnetic* course = N. 86° W. = [274°]; and on the *blue* card, the *true* course = N. 63° W. = [297°].

Suppose at the same time, that the ship were in the vicinity of land, where it was required to fix her position by bearings, while heading W. by N., and that to plot these on a chart, the corresponding magnetic or true bearings were required: all that is necessary is to turn the brass arm to the compass bearing on the red card and the corresponding magnetic and true bearings will be seen in line along the beveled edge on the white and blue cards respectively. The following is an example:

First set of bearings:

by Compass = S. 46° E. = [134°];
corresponding Magnetic = S. 53° E. = [127°];
and True = S. 30° E. = [150°].

Second set of bearings:

by Compass = S. 24° W. = [204°];
corresponding Magnetic = S. 17° W. = [197°];
and True = S. 40° W. = [220°].

In all the preceding, the figures within brackets, as [220°], denote the several courses and bearings, *if the card were graduated to degrees continuously from 0° at North to 360°*, and this simplicity would be most desirable—for it is entirely specific without any additions of N., S., E., or W.

When bearings are frequently taken by compass, as on entering port, their ready conversion into true or magnetic equivalents for plotting is a matter of the first importance; and it is in such cases that the Indicator is most useful: many things then engage the navigator's attention—*he* is liable to err, however capable and careful, but the *mechanism*—never. Let any one propose to himself the question solved above so quickly—ship heading W. by N., deviation 7° W., variation 23° E., what are the true and magnetic bearings corresponding to compass bearings S. 40° E. and S. 24° W.?—and he will realize the facility of the mechanical over the mental process, with the unfailing correctness of the former, in addition.

All courses and bearings should be by the Standard Compass, whose deviations are accurately determined; and the steering compass should be compared with the standard on every change of course.

250. The Compass a reliable guide.—From what has been set forth in Chapter II. it will be seen that not only do the needles receive the greatest care to ensure enduring strength, but also that the various parts constituting their bowls are accurately made and fitted, one to another, and [illegible] a whole.

[illegible] combined product of the skill and intelligence [illegible] makers and the technical knowledge and minute [illegible] of the Superintendent of Compasses, the in[illegible] worthy of the fullest confidence.

[illegible] ship of war need ever lay blame to it for "running [illegible] on shoals or being otherwise hazarded"; and

indeed its record justifies this assertion; for it has now these many years been guiding ships of every class—gun-boats, cruisers, battleships—of wood, of iron, and of steel—in every sea; through all kinds of weather; in peace and in war—and I have yet to learn of a single mishap justly chargeable to it, or a defect that could lessen reliance upon it in any contingency.

And it is well that those who use it should know the details of its manufacture, for it tends to inspire them with confidence in its indications in many a critical juncture.

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PART THIRD.

THE SHIP A MAGNET.

CHAPTER XVI.

THE MAGNETIC CHARACTER IMPRESSED ON A SHIP WHILE BUILDING.

257. The Earth is a huge *spherical* magnet and the compass-needle a small *cylindrical* one ; these facts have been established in Vol. I: the ship—the third of the bodies whose intimate connection forms the subject of this Treatise—will now be shown to be equally a magnet.

258. **Acquisition of the magnetic state illustrated by temperature changes.**—Suppose several bodies possessed of different degrees of heat and surrounded by air or water of uniform temperature, but different from that of any of the bodies: heat will be communicated from one to another and from all to the medium until the same temperature pervades all; and the amount of interchange will depend upon the original difference of temperature of the bodies and medium.

In Vol. I it was indicated that Heat is probably a result of motion—a vibration of the molecules of matter, caused by ether waves from a source of heat beating upon the matter: degrees of heat and cold, then, are really only different rates of vibration; and changes of temperature only an interchange of these rates of vibration between bodies.

But the ether is the seat of other waves than those that produce heat—waves (whatever their origin) that give rise to manifestations called electric and magnetic;

and these, also, beat upon matter and impress upon it a characteristic condition, just as heat waves make it hot.

All bodies are not equally receptive of heat: a metal will acquire and conduct it readily, while asbestos scarcely at all; and between these there are materials of every degree of conductivity. The same is true of Light: some bodies are wholly opaque, others beautifully transparent; the ether waves that produce light are absorbed by the first, and transmitted by the second; and between these extremes there are many shades of opacity and translucency.

And so, too, with the magnetic condition—it impresses itself with ease upon wrought iron, but finds bismuth as adamant to its touch; and with matter of other kinds it meets a varying reception.

The magnetic condition of the ether is analogous to its thermal state—one of eternal change with time and place: if we traverse the Earth from Equator to Pole, we find a varying magnetic intensity as well as different degrees of heat and cold; and each condition invests bodies susceptible to it with an intensity proportional to its own amount and their receptivity. Hence, a ship acquires the magnetic condition of the region she is in to the extent that the iron in her structure will admit—that is, her magnetic character is ever changing, and what we deal with is the *resultant* of her original nature and terrestrial conditions.

Four influences enter into the formation of a ship's magnetic character: 1st, the quality of the iron and steel used in construction; 2d, the amount of manipulation it undergoes; 3d, the direction in which the keel is laid; and 4th, the strength of the terrestrial magnetic field.

259. **Iron used in ship-building and its permeability to magnetism.**—Porosity of matter is a more or less loose condition of its particles that allows a liquid to filter through

it—ease of entry and ease of exit: an analogous condition of iron for magnetism is called its Permeability; in any particular mass of the metal, it is the *ratio* of its magnetization to the magnetizing force—a numerical quantity, since both the cause and its effect can be expressed by numbers. Magnetization, however, does not increase in a constant ratio to field intensity: when this is feeble, the metal acquires but little magnetism; when strong, the magnetization is great and rapid; while a further increase of field adds but little to the amount already acquired—the tendency is toward a limit. This point is explained in Art. 191.

The name Iron is used here in the generic sense, and therefore includes all its varieties—wrought and cast—pure and mixed with carbon to form steel of varied hardness.

The mixture of certain ingredients with iron lessens its susceptibility to magnetism: it is said that 12 per cent. of manganese, and even a less quantity of antimony, combined with steel, will make this metal non-magnetic; and that arsenic produces the same effect on nickel. Besides iron, nickel and cobalt are sufficiently receptive of magnetism to be injurious in close proximity to the compass.

Every mass and particle of iron on the Earth's surface acquires in some degree the intensity of the natural field surrounding it; and this without violence of any kind to the metal, but quietly, by induction alone—by the mere fact that the metal is in the midst of the Earth's magnetism. The steel rails that afford transit from seaboard to interior, the trestle-work upon which the elevated trains traverse the metropolis, the heavy castings in a foundry, the massive forgings in a machine-shop, even the little scraps upon a neglected heap, have one and all magnetic features that distinguish them from other metals—features

entirely analogous to those of a steel magnet, and acquired from the gentle, steady impress of terrestrial magnetism alone.

Mechanical work upon iron—such as hammering, bending, twisting, boring, drawing into wire, heating and suddenly chilling, tensile stress and compression—will affect the degree of magnetization: after any of these processes, the magnetism possessed will differ from what it would have been before them; and in the hull, armament, and equipment of a ship of war, there are masses of iron that have undergone one or other of these processes—the iron of her structure will be differently affected by the Earth's magnetism.

Soft iron affords a standard of magnetic permeability, and should therefore be defined: *pure* Swedish iron is "made soft by soaking in a blood-red fire for some hours, and then cooling very slowly by burying in the hot ashes, or allowing the fire to go out." It is susceptible of the highest magnetization; but this is only transient.

Let us consider a metallically pure cylinder of soft iron that has not been hammered, and conceive it free of magnetism: hold it in the line of Dip, and instantly the upper end becomes a *south*-pole and the lower a *north*-pole—*that is, the* FORMER (a blue pole) *will attract the north point of a compass-card, and the* LATTER (a red pole) will *repel it;* this is what occurs north of the Magnetic Equator—the converse will be the case south of it; while *on* that line, neither end, with the cylinder vertical, will exhibit these characteristics, because no vertical component of terrestrial magnetism exists there to produce them.

Reverse the cylinder as quickly as we may, *and the magnetism also reverses*, so that the upper and lower poles are the same as before.

Hold it horizontally in the meridian, and the end toward the north becomes a red pole, while that toward the south

a blue pole. Revolve it slowly or rapidly in azimuth, and the foci of magnetic polarity also move with the fidelity of a shadow, until, when the cylinder points east and west, all the side facing the north is pervaded by red magnetism, and that facing the south by blue magnetism. Of course, the colors are here used to express the oppositeness of polarity, not an actual fact.

Now, let us conceive the hull of a ship to be like the cylinder, of metallically pure soft iron and as susceptible to magnetic induction in her ever-changing course as the cylinder is when turned round. Then as the ship heads north (in north latitude), the bow will become the center of red polarity, and the stern that of blue. As she gradually changes course to the eastward, so will the red focus shift to the port bow, the blue focus to the starboard quarter, and the neutral line dividing them (which, while the ship headed north, was athwartships) will now become a diagonal from starboard bow to port quarter. When the ship heads east, all the starboard side is pervaded with blue polarity, the port with red, and the neutral line takes a general fore-and-aft direction. Continuing to change course to the southward, the poles and neutral line continue their motion in the opposite direction, until at south the conditions at north are repeated, but this time it is the stern that is red and the bow blue.

At west the conditions at east prevail, only that it is the starboard side that has red polarity, and the port side blue. In south magnetic latitude, the reverse of all the preceding would occur.

And this transitory induction in both the cylinder and the ideal ship is solely the effect of the Earth's magnetic field.

But, to consider this in connection with an *actual* ship: the iron of no vessel, its armament, or equipment is metallically pure; nor has it acquired shape without much hammering; moreover, it cannot be made an abstraction from

a magnetic state. By the varied processes of building, the whole structure has become as permanent a magnet as a steel bar, with the poles and neutral line located according to the magnetic direction in which the ship lay on the stocks, in conformity to the places they occupied in the ideal vessel just described. Therefore, she is not now as susceptible to the mild induction of the Earth as the soft-iron cylinder and ideal hull are, although the straining while on a passage and the buffeting of the waves *do* assist the inducing tendency; besides, once that the magnetic impress has been made—as in building—it does not move with the facility it did in the soft-iron cylinder, so that the transitory induction finds a tenacious occupant of the vessel, and must adapt itself accordingly: it is the resultant of both the temporary and permanent magnetism we always find, and not the individuality of either.

Time is an important element in the acquisition and intensity of this transient magnetism; for the longer a ship steers on a given course, or swings at anchor in the same *general* direction, the greater will be its amount: it is of prime importance in navigation; the *characteristic* magnetic features of a ship are not very mobile, but those that acquire temporary lodgement are treacherous and changeable to a degree that necessitates constant vigilance to prevent disaster.

Instead of attributing the loss of vessels to improbable influences upon the compass, it were more reasonable to ascribe it to the changed condition of her magnetism by temporary induction during the passage, and which has not been discovered or kept account of by frequent azimuths previous to closing in with the land. Suddenly, a course the Captain thought perfectly safe, carries the ship upon a shoal or rock, and the fault is laid upon the compasses, whereas they but obeyed the magnetic influences that became altered during a long passage.

260. Experimental proof of the ship being a magnet.— Occasional reference has been made in this Treatise to the SCORESBY—Fig. 428. In 1883, when the Compass Office was under the Bureau of Navigation, I received authority from Rear-Admiral J. G. Walker, U.S.N., then chief of the Bureau, to have this little vessel made at the Washington Navy Yard: it was designed to illustrate experimentally the mathematical theory of the deviations, the magnetism of ships, and other related matters. It is still in use for these purposes, and also for showing officers ordered as navigators of ships the practical methods of compensating compasses: indeed it is the final *experimental* arbiter of all questions in this branch of enquiry. For convenience of reference, it was given a name—that of the able investigator who accomplished so much in this field of research.

A few changes have been made in it since 1883, so that the description here given applies to the original design, with which the experiments described in this Part of the Treatise were made.

Dimensions of the SCORESBY: length, 6 ft. 9 in.; beam, 3 ft., 2 in.; height of upper deck from floor, 3 ft. 2 in. The stem, keel, and stern-post form one piece—a stout bronze casting. From strong bronze cross-pieces attached to the keel near the bow and stern, four heavy bronze screws rose and supported the three wooden decks; the middle and lower decks could be raised and lowered at will. The vessel was pivoted at the stern in a wooden socket screwed to the floor; this socket was in the center of a large brass circle (also fixed to the floor) and upon which a bronze wheel at the bow turned, affording motion in azimuth.

There was a contrivance for heeling the vessel to any angle up to 45°, and also for giving it a deep and rapid rolling motion.

Thus the SCORESBY, like a ship at compass-buoys,

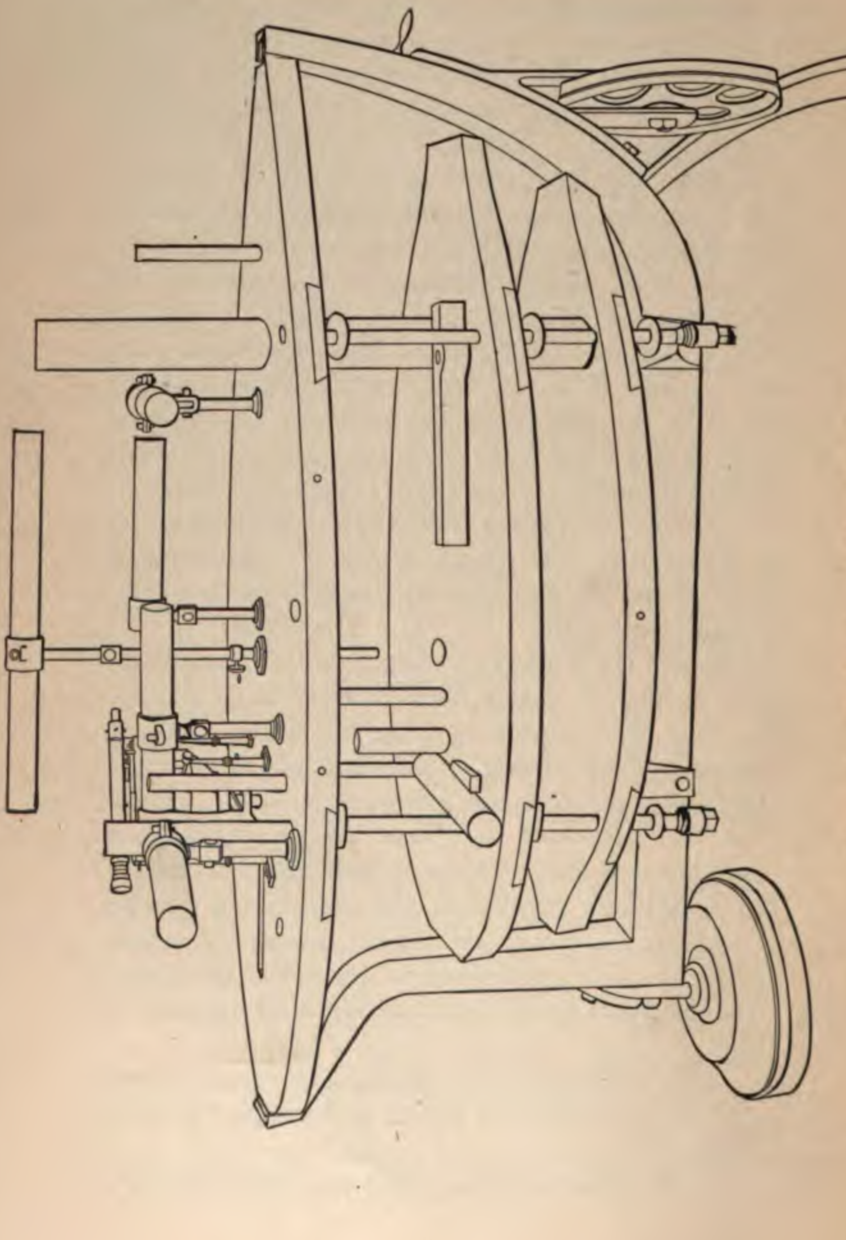

FIG. 428.—The Experimental Vessel SCORESBY.

could be "swung" either on an even keel or heeled; and with iron tubes and magnets suitably disposed about its decks to represent beams, smoke-stack, engines, propeller-shaft, boat-davits, masts, yards, battery, and armor,

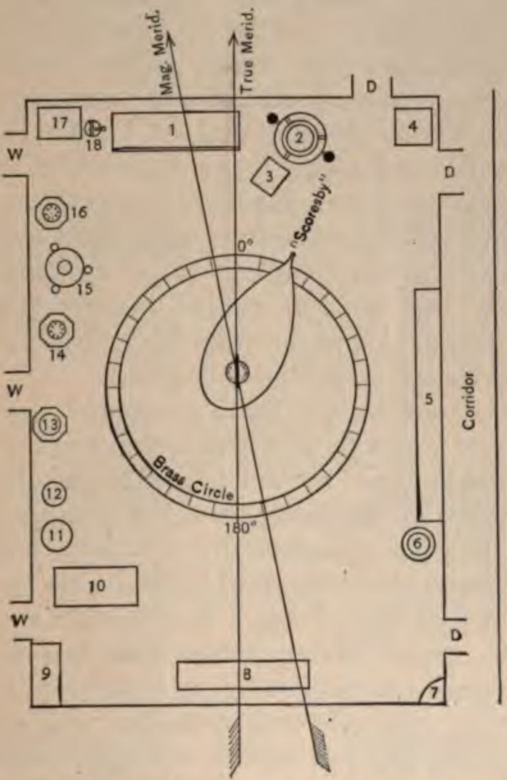

FIG. 429.—Office of Superintendent of Compasses in 1883.

the varied disturbing effects of these upon the compass could be shown: in fact, there is scarcely any magnetic investigation of a *real* battleship that cannot be carried out in miniature on the SCORESBY. There were several strong bronze arms for holding tubes and magnets in any position or in any direction; Fig. 428 shows *one* combina-

tion of the soft-iron rods and steel magnets that physically represent the mathematical theory of the deviations.

The Scoresby was mounted, as in Fig. 429, in the Office of Superintendent of Compasses—then on the second floor of the Navy Department—a spacious room, well lighted by three large windows looking west; its dimensions are: length, 29 feet; width, 20 feet; height, 15 feet, all in clear: materials of construction—iron, brick, granite, and wood. The numbered articles in the room were: binnacle with appliances for compensating compasses; hand dynamo machine; case for instruments and magnetic records; globe to illustrate magnetic theories of the Earth; U. S. monitor compass; case for instruments and blanks; Fox Dip Circle on gimbal table; types of U. S. Navy compasses; compass-testing instrument; chart table —magnetic charts; and standard alidade.

As already stated, the Earth's magnetic field converts every piece of iron into a magnet—the opposite polarities being separated by a neutral line; the direction of this line in the iron will generally be at right angles to the line of Dip at the place; theoretically, it should be so exactly, but sometimes circumstances intervene to prevent it. The Line of Dip, then, being a fixed direction in space, if the iron be turned about, the neutral line will occupy different positions *in the iron*—each indicative of the direction of the iron with reference to the line of Dip.

Plate *R* exhibits the results of experiments made to show the distinctive magnetic features an iron vessel acquires by being built on a particular heading.

For these experiments two rectangular wrought-iron plates were provided: each was 5 feet long, 2 feet 6 inches wide, and ¼ inch thick; both were devoid of permanent magnetism; wooden boards of like size were also prepared. A framework of shelves was made, so that a compass could be supported at different heights and at intervals along

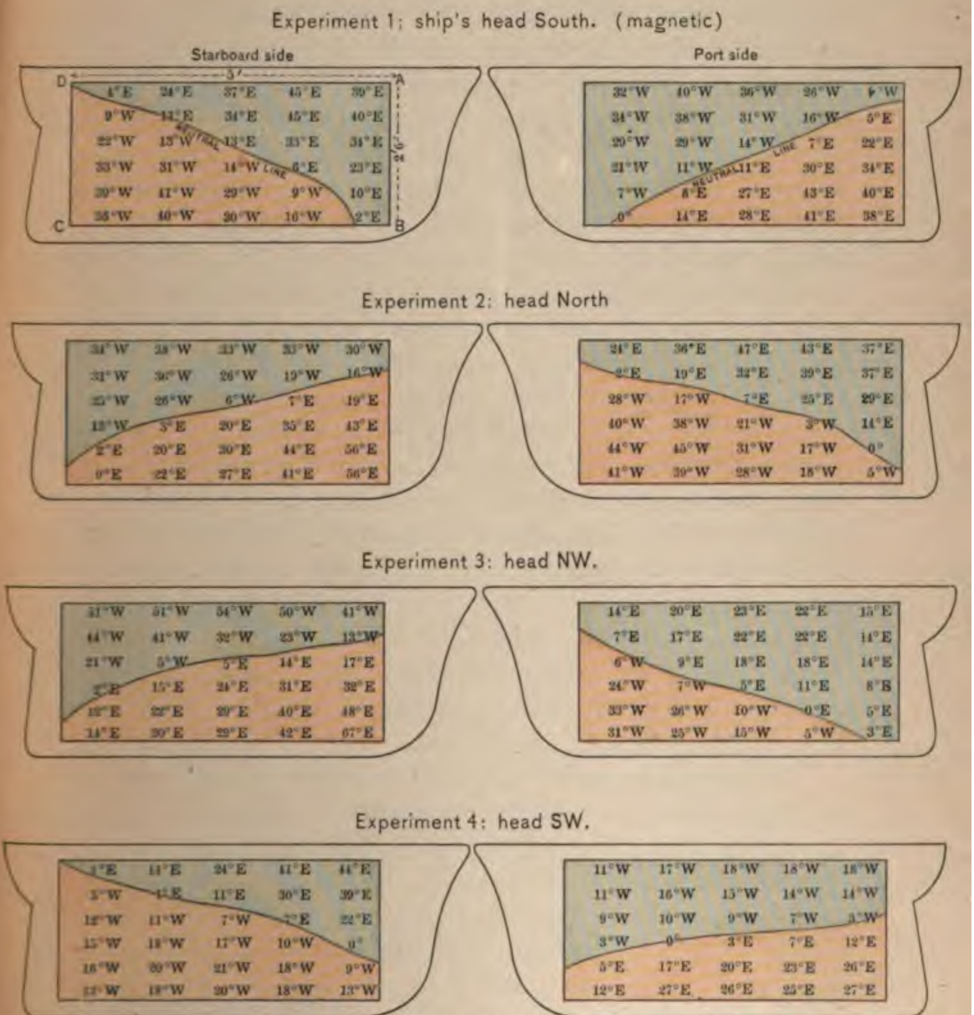

PLATE R.—MAGNETIC CHARACTER OF SHIPS.

(To face p. 726.)

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each height—a kind of aërial dry-dock—for making a magnetic survey of the SCORESBY. The vessel was successively swung into the magnetic headings South, North, Northwest, and Southwest, and blocked on each while the observations were made: first, the wooden boards were attached to the vessel's sides in a vertical position, and thirty observations made on each side with a compass at the regular intervals indicated by the deflections recorded on each figure of Plate *R*; the magnetic effect of the room alone for each of the above headings of the SCORESBY was thus obtained; second, the iron plates were then attached in precisely the same way that the boards had been, and after resting several days on each of the same magnetic headings as before (during which time the plates were frequently tapped all over with a wooden mallet) the observations of deflection were repeated on each heading at the same stations. The distance of the compass from the boards and plates was in all cases thirteen and one half inches.

A comparison of the deflections when the boards were attached with those when the plates were substituted, gave the magnetic condition of the plates alone—just as a comparison of the observations of a dry-dock, with and without a ship in it, gives the effect of the latter.

Using colors to represent polarity, let *blue* stand for that which will attract the north point of a compass-card, and *red* for that which will repel it: then in north magnetic latitude an iron ship built head south should have blue polarity pervading the upper forward body of the hull, and red polarity the lower after body, the neutral line extending diagonally from rail aft to keel forward, and making with the horizontal an angle equal to the complement of the Dip at the place where the ship was built; any inclination of the keel to the horizontal while building will produce a like change in the *direction* of the neutral line.

Now to examine the results portrayed in Plate *R*, in the light of these theoretical considerations: the two figures illustrating each experiment represent the starboard and port sides opened out from aft as if hinged at the bow, so that an exterior view of the whole ship may be had at once. In Experiment 1, the Dip on the SCORESBY was 65°—hence the neutral line should decline 25° from the horizontal, and this is the inclination; moreover, the polarity is in conformity to theory.

In a ship built head north, the neutral line should extend from the bow diagonally to the keel aft, with the upper after-body blue, and the lower forward-body red—and Experiment 2 shows this to be the case. The plates, however, undoubtedly retained some vestige of the magnetic features acquired in Experiment 1, to affect the theoretical direction of the neutral line, for the inclination is 16°, whereas it should be 22°, as the Dip on the SCORESBY, head north, was 68°.

If, at the same navy-yard, a series of iron ships were built with their heads on the points between north and west, according to theory it would be found: that the neutral line on each, as we went from north toward west, was gradually shifted from a diagonal on both sides of the ship whose head was north, to a line parallel to the keel on the ship whose head was west; and that the area covered by each polarity on both sides would undergo a corresponding contraction or expansion, until, in the case of the ship head west, we would find, either that all of the starboard side had red polarity and all of the port side blue, or, if both polarities existed in each side, they would appear only as a narrow band of blue beneath the rail on the starboard side with all below red, while on the port side there would be a narrow band of red just above the keel with all else blue. And the *tendency* to this state is evident in Experiment 3. An entirely similar tendency is apparent

in Experiment 4, only, that while Experiment 3 indicates traces of its northerly origin in the changing direction of the neutral line, Experiment 4 equally shows its southern relationship—both as theory requires.

From these experiments it is fair to infer that ships built in the eastern semicircle would equally exhibit features indicative of the particular azimuth of their heads while building.

As to the direction most favorable for making a magnetic survey, a few remarks, based upon theoretical considerations, which have been borne out by personal experience, will be offered.

The inductive action of the Earth will variously alter the permanent magnetism of the hull according to the azimuth in which the ship is surveyed, but to some extent in all azimuths; so that it is never the permanent effect of the magnetism of the hull we obtain, but this, somewhat modified by the transient magnetism due to the particular azimuth of the head during survey.

The best results will be obtained with a ship lying north and south, because the disturbing force of the hull then acts at right angles to the deflected needle and small changes are accurately measured.

The value of direction of ship's head steadily decreases until we reach east or west, which is not only the worst, but one in which it is all but impossible to deduce any decided results, on account of the disturbing force acting in the prolongation of the needle.

Experiments were made with the SCORESBY heading both west and east as follows: after completing Experiment 2, the vessel was swung to head west and remained so seven days, the iron plates being hammered at intervals during this period; the survey was then made, but no decided characteristics appeared—the two polarities were

irregularly commingled and the deflections small on both plates.

A month later the experiment was repeated, with a similar result. Then the plates were taken off and placed in a north and south direction, where they remained without being hammered at all for two months.

Then the SCORESBY was turned to head east, and the iron plates attached to her sides as in former experiments: for six days they were repeatedly tapped all over with the wooden mallet, the vessel all the time heading east; the survey was then made, and disclosed approximately what theory required—red polarity on the port side, and blue on the starboard; only a very small strip of blue just below the rail forward on the port side, and a small area of red on the starboard side, aft, above the keel, broke the continuity of color on each side; and in these contracted areas the deflections, though few in number, were comparatively large in amount, whereas over the large blue area of the starboard, and the like large extent of red on the port side, where the deflections were large in number, the amount in each case was very small. The experiment was sufficiently decided to establish the features of a ship built head east, and these were in accord with theory.

261. The means of ascertaining the magnetic character of a ship.—The magnetic character of a ship may be investigated by swinging for deviations—by intensity experiments with oscillating needles—and by deflection observations along her sides; but each gives only a partial insight—a profile, plan, or sectional view, *as it were*, of the condition of affairs: it requires all combined to afford that picture in perspective that presents the matter completely.

Intensity experiments should be made throughout the region of the compass-site, and they will be described in

the next chapter; swinging for deviations belongs to Part Fifth and will be treated there. The dock survey will now be explained.

The meaning attached to *north*, *south*, *red*, and *blue*, in Arts. 93 and 172, and in the preceding chapter—as representing polarity—holds equally here and throughout this Treatise.

The steps of a dry-dock afford stations for making the observations; the vertical height of each step above the keel, intervals along its length, and horizontal distances from the ship's side constitute the coördinates for plotting the values observed: by Fig. 430, however, it will be seen that all parts of the hull are not equally distant from the steps, so that this variability of the third coördinate precludes a very accurate comparison of the *intensity* of the magnetism at different points; but it is its distribution that is sought by this enquiry—qualitative, rather than quantitative, results; and for them the method is entirely satisfactory and appropriate: even more, if we are sure that the action on the *N*-point of the compass at any station is attractive, or repulsive, it matters little whether the deflection be a degree or two more or less; the fact that it *is* attractive or repulsive decides the *kind* of polarity, and that is mainly the point we are in quest of.

As the compass is placed at different points along each step, it indicates the balance of power between Ship and Earth; the *relative* positions of the poles of both and their effect vary with time and place, and hence—as dependent upon their resultant—the indicated magnetic character of the ship varies also: for instance, if built head north, the ship will exert her strength to most advantage if surveyed with her head in that direction; but if surveyed head south, the Earth's inductive effort tends to weaken

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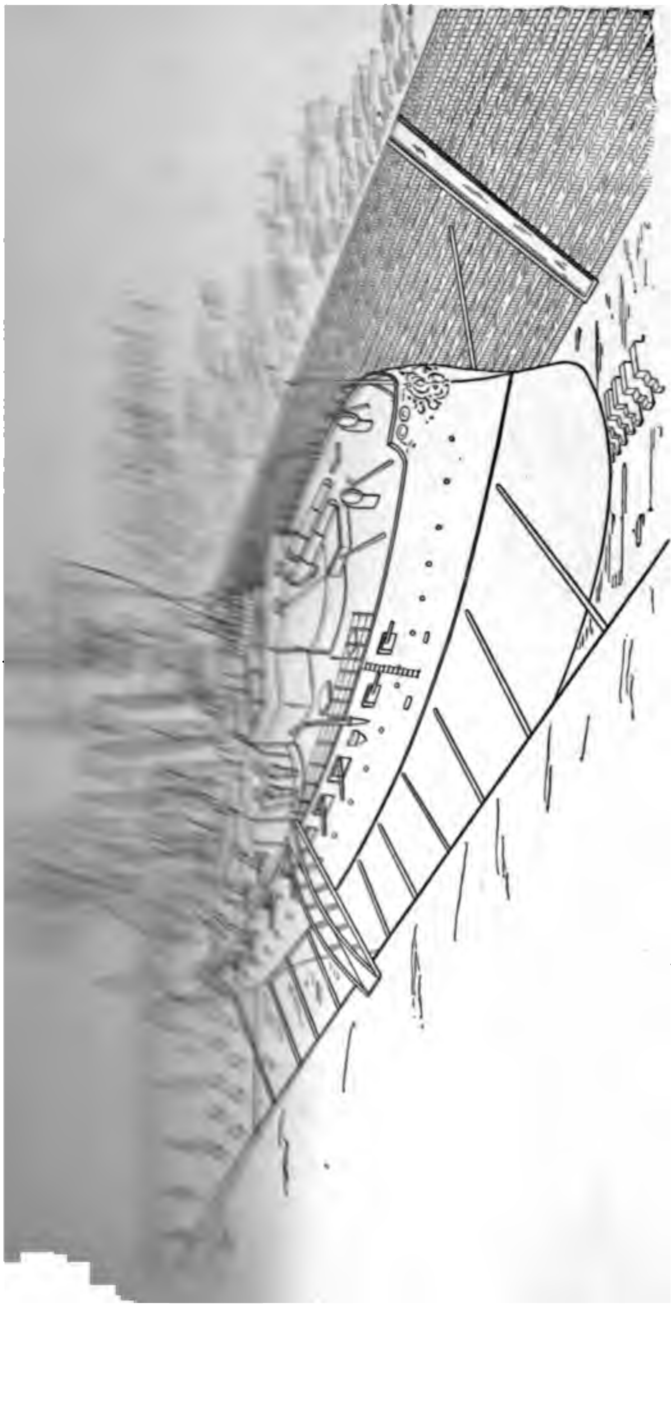
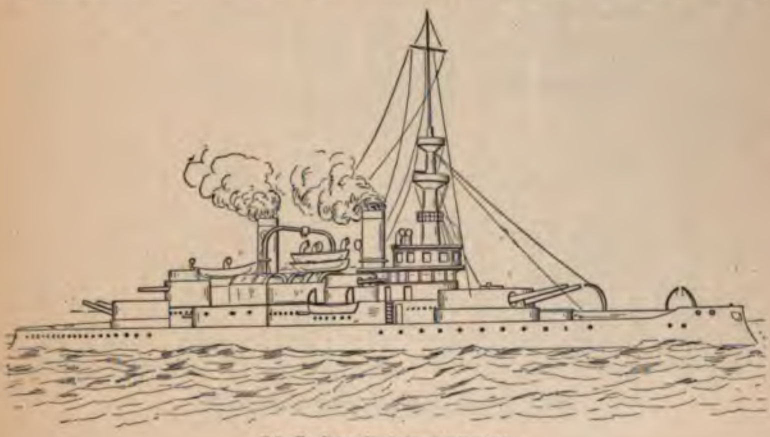


FIG. 439.—A Battleship in Dry-dock at the New York Navy Yard.

the ship's red polarity and strengthen the blue; this, in north magnetic latitude, and the converse in south.

The conditions under which a survey may be made are so many that slightly different magnetic features *may* be expected as a result of each—not the set expression for all that would denote a rigid impress.

262. Magnetic survey of the U. S. Battleship Oregon.—In order to illustrate the details of a survey, that of the OREGON will be selected: it was made in April, 1897, by the navigator, soon after the ship was finished and commissioned, and while she was in dry-dock at the U. S.



U. S. Battleship OREGON.

Naval Station, Puget Sound, in the State of Washington. The height of each step of the dock was 33 inches; the distance between the observation stations along the steps, 25 feet; and the number of steps used, 14: this gave 210 observations of deflection on each side of the ship—a very thorough portrayal of her magnetic condition.

Observations were made at the same points when the dock was empty as well as when the ship was in it, and the effect of local masses of iron in the dock construction thus determined; and it will be observed by Table 52

[illegible]

[illegible] BATTLESHIP OREGON IN [illegible] STATION ON PUGET SOUND, [illegible] APRIL, 1897. SHIP'S HEAD

[illegible] and repulsive (R) effort [illegible] is given in degrees of de- [illegible]

[illegible] Keel of Ship.

Starboard Side.

[illegible]	Compass Readings. Dock Empty.	Compass Readings. Ship in Dock.	Deflections Difference bet Cols. [illegible] and 4.
	[illegible]	(4)	[illegible]
	N [illegible] W	N 26° W	[illegible]
	N [illegible] W	N 35 W	[illegible] R
	N [illegible] W	N 51 W	[illegible] R
	N [illegible] W	N 66 W	45 R
	N [illegible] W	N 66 W	[illegible] R
	N [illegible] W	N [illegible] W	[illegible] R
	N [illegible] W	N 46 W	24 R

[illegible] Step [illegible]

	N [illegible] W	N [illegible] W	[illegible]
	N [illegible] W	N 20 W	[illegible] R
	N [illegible] W	N 40 W	[illegible]
	N [illegible] W	N [illegible] W	44 R
	N [illegible] W	N [illegible] W	[illegible] R
	N [illegible] W	N [illegible] W	42 R
	N [illegible] W	N [illegible] W	[illegible] R

[illegible] Step [illegible]

	N [illegible] W	N [illegible] W	[illegible]
	N [illegible] W	N [illegible] W	[illegible] R
	N [illegible] W	N [illegible] W	[illegible] R
	N [illegible] W	N [illegible] W	[illegible] R
	N [illegible] W	N 43 W	[illegible] R
	N [illegible] W	N [illegible] W	20 R
	N [illegible] W	N [illegible] W	[illegible] R

TABLE 52.—*Continued.*

STEP *H*: 66 INCHES ABOVE STEP *F*.

PORT SIDE.					STARBOARD SIDE.				
No. of Station.	Distance bet. Stations. Feet.	Compass Readings. Dock Empty.	Compass Readings. Ship in Dock.	Deflections: Difference bet. Cols. 3 and 4.	No. of Station.	Distance bet. Stations. Feet.	Compass Readings. Dock Empty.	Compass Readings. Ship in Dock.	Deflections: Difference bet. Cols. 3 and 4.
(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
4	50	N. 32° W.	N. 46° W.	14° *A*	3	50	N. 37° W.	N. 12° W.	25° *A*
8	50	N. 32 W.	N. 28 W.	4 *R*	7	50	N. 15 W.	N. 15 W.	0
12	50	N. 18 W.	N. 10 W.	8 *R*	11	50	N. 31 W.	N. 45 W.	14 *R*
16	50	N. 31 W.	N. 11 W.	20 *R*	15	50	N. 20 W.	N. 41 W.	21 *R*
20	50	N. 22 W.	N. 3 W.	19 *R*	19	50	N. 29 W.	N. 42 W.	13 *R*
24	50	N. 30 W.	N. 13 W.	17 *R*	23	50	N. 20 W.	N. 39 W.	19 *R*
28	50	N. 18 W.	N. 3 W.	15 *R*	27	50	N. 18 W.	N. 30 W.	12 *R*

STEP *J*: 66 INCHES ABOVE STEP *H*.

4	50	N. 27° W.	N. 44° W.	17° *A*	3	50	N. 39° W.	N. 9° W.	30° *A*
8	50	N. 32 W.	N. 35 W.	3 *A*	7	50	N. 14 W.	N. 9 W.	5 *A*
12	50	N. 21 W.	N. 27 W.	3 *A*	11	50	N. 29 W.	N. 25 W.	4 *A*
16	50	N. 28 W.	N. 28 W	0	15	50	N. 18 W.	N. 29 W.	11 *R*
20	50	N. 24 W.	N. 18 W	6 *R*	19	50	N. 20 W.	N. 28 W.	8 *R*
24	50	N. 28 W.	N. 18 W.	10 *R*	23	50	N. 20 W.	N. 30 W.	10 *R*
28	50	N. 16 W.	N. 6 W.	10 *R*	27	50	N. 20 W.	N. 28 W.	8 *R*

STEP *L*: 66 INCHES ABOVE STEP *J*.

4	50	N. 32° W	N 38° W.	6° *A*	3	50	N. 26° W.	N. 6° W.	20° *A*
8	50	N. 30 W.	N. 38 W	8 *A*	7	50	N. 15 W.	N. 6 W.	9 *A*
12	50	N. 17 W.	N. 26 W	9 *A*	11	50	N. 36 W.	N. 29 W.	7 *A*
16	50	N. 31 W	N. 33 W	2 *A*	15	50	N. 19 W.	N. 17 W.	2 *A*
20	50	N. 22 W	N. 22 W.	0	19	50	N 31 W.	N. 31 W.	0
24	50	N. 33 W.	N. 27 W.	6 *R*	23	50	N. 20 W.	N. 22 W.	2 *R*
28	50	N. 14 W.	N. 8 W.	6 *R*	27	50	N. 18 W.	N. 25 W	7 *R*

STEP *N*: 66 INCHES ABOVE STEP *L*.

4	50	N 28° W.	N. 45° W	17° *A*	3	50	N. 21° W.	N. 6° W.	15° *A*
8	50	N. 27 W.	N. 39 W.	12 *A*	7	50	N. 20 W.	N. 9 W.	11 *A*
12	50	N. 18 W.	N 27 W.	9 *A*	11	50	N. 20 W.	N. 13 W.	7 *A*
16	50	N. 23 W.	N. 33 W	10 *A*	15	50	N. 21 W.	N. 12 W.	9 *A*
20	50	N. 18 W.	N 23 W.	5 *A*	19	50	N 21 W	N. 15 W.	6 *A*
24	50	N. 24 W.	N. 25 W.	1 *A*	23	50	N. 27 W	N. 27 W.	0
28	50	N. 24 W.	N. 20 W.	4 *R*	27	50	N. 17 W.	N. 20 W.	3 *R*

how necessary this was: for whereas the magnetic direction of the steps was N. 23° 34′ W., individual points of different steps, according to col. (3), varied extremely and irregularly from this direction.

To plot the entire number of observations would so encumber the diagram as to destroy its clearness; therefore, both in Fig. 431 and Table 52, only alternate steps, and alternate stations on these, are given; but with the

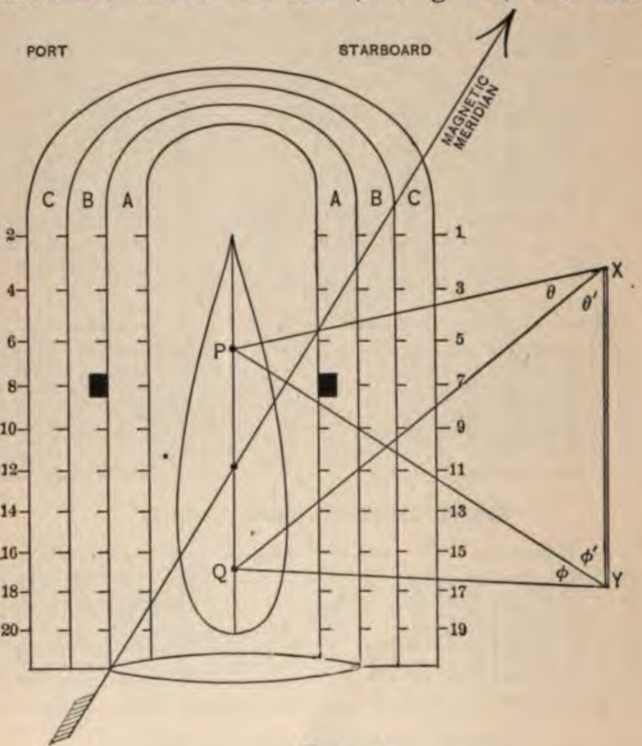


FIG. 432.

letters and numbers (for each) of the original survey: it is almost needless to say that the outline of the plan and sides of the OREGON are not strictly those of the ship herself—they suffice for illustrative purposes, however.

The general procedure of a survey is this: let Fig. 432

represent a plan, and Fig. 433 a section of a vessel and dock. The direction of the magnetic meridian is first determined; or rather—what we really wish to know—the angle that the keel of the ship, and consequently the direction of the steps (as the keel may practically be considered parallel to them), makes with the meridian. Several methods are available for this, but the following is simple and will suffice: in Fig. 432 find a line $\overline{XY}$, from the extremities of which reciprocal bearings differ exactly 180°; measure this line, note its magnetic bearing, and measure the angles ϕ, ϕ', and θ and θ' to any points in the midship line of the vessel; from these data the required magnetic direction of the keel may be found either by computation or construction.

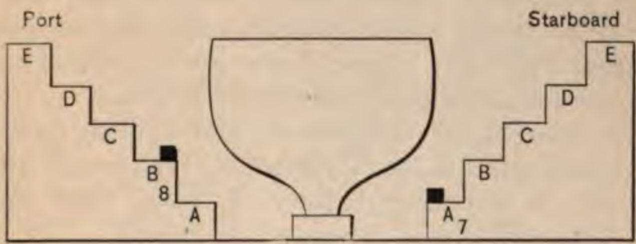


FIG. 433.

On each of the steps A, B, C, . . . , measure distances of 20 to 25 feet, and mark the points with chalk—odd numbers on the starboard side, even on the port, and the same letters on both sides. Carry a compass in its box successively to each numbered station; place the box with its side parallel to the edge of the step, as at A_7 and B_8, in both figures, with the keel-line of the compass in the direction of the bow, and when the card has come to rest, observe and record the degree-mark that is in coincidence with the keel-line.

A comparison of the observations with the magnetic direction of the steps will give the deflections of the com-

pass caused by the ship (provided no iron exist in the dock construction)—those in one direction denoting red polarity, and those in the other, blue: these results are to be represented on side views of the ship.

When the dock is clear, observations are to be taken at each numbered station to determine the effect of any iron that may lie hidden in its construction; then a comparison of the two series — with and without the ship—gives the effect of the latter.

When the foregoing observations of deflection are supplemented by others with oscillating needles, it is evident that we have explored the ship magnetically in precisely the same way that both a bar-magnet and the Earth are, as described in Vol. I.

The OREGON is a battleship of the first class, 348 feet long, 69 feet beam, 24 feet draught, and of 10,200 tons displacement: she has a very thick armor belt along the water-line for about two thirds of the length.

The main battery consists of four 13-inch guns and eight 8-inch guns—all in turrets, and four 6-inch guns in broadside; a secondary battery of twenty-eight guns is variously disposed.

Evidently, in this huge and complicated structure every kind and grade of iron and steel exist.

She was built at the Union Iron Works, San Francisco, California, with head S. 23° E. magnetic; and after launching, was finished with the head in the same direction: so that the whole amount of hammering during construction was to impress upon her—*indelibly*, if such were ever possible—the characteristic features of a ship built head toward the southward.

Now, to compare the real with the ideal: in Fig. 431 the sides are represented opened out from the keel, as if hinged at the rail, to exhibit the exterior of the hull at one view; and it is seen that the neutral line extends

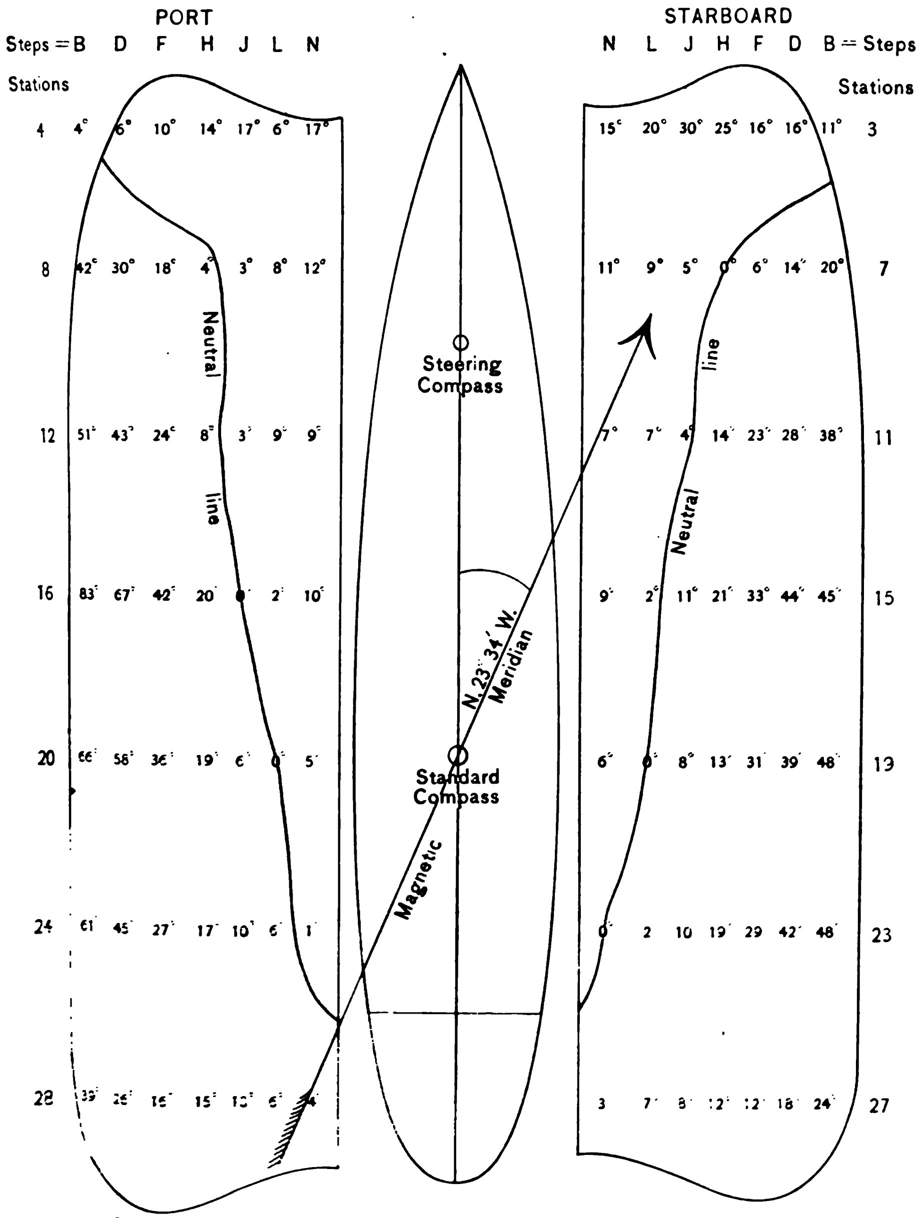


FIG. 43[illegible].—Magnetic Survey of the U. S. Battleship OREGON.

To face p. 738.

diagonally from the keel at the bow to the rail near the stern, that the whole upper forward-body is of blue polarity, and the entire lower after-body of red—all in strict conformity to theory and its illustrative experiment, number 1 in Plate *R*.

The two conditions under which the theory is thus tested, could not be more dissimilar—the simple iron plates of the miniature SCORESBY, and the complicated entanglement of varied kinds of steel in the huge OREGON—and yet the identity of magnetic features in both (head toward the southward) could not be more complete. Again: that these features are mobile in the soft iron of the SCORESBY is shown by the fact that when swung to head north, as in Experiment 2, Plate *R*, and the sides lightly tapped all over, they assumed quite another aspect—in fact, the very opposite of what they had with head south; but that the OREGON has the rigidity of a steel magnet is shown by the survey in the diametrically opposite direction to that in which she was built—it made no perceptible change in her magnetic character.

Fig. 431 is the pictorial representation of Table 52, and each is self-explanatory. One point, however, should be noticed: the deflections in col. (5) are, as a rule, differential—that is, the compass-card is generally deflected in the *same* direction both with the dock empty and with the ship in it, but by different amounts, and it is this difference that indicates the polarity, by being either attractive or repulsive *with the ship in dock*.

263. Characteristics disclosed by magnetic surveys in general.—It would be tedious to give the individual surveys of many ships—better state some of the facts they establish by their number and variety.

During the first passage after completion, much loose magnetism is shaken out of every ship: it is the surcharge found in all magnets, even hard-tempered compass-needles.

Indirectly, it is a source of great danger; for, when taken into account, as it must be in the first compensation of compasses, its counteracting means—the magnets—remain placed as at first, while the necessity for them has in part disappeared: the compasses are now *over*-compensated, and often by considerable amounts—they are *forced* into error—to indicate courses that are neither known nor intended.

But this surcharge once gone, the real, enduring, magnetic character of the ship appears, and is generally that which theory assigns to it.

The distinctive impress of this character is derived from the *direction* of the *ship's head* while building; and a typical curve of deviations results from each particular direction, so that, given either—direction of head, or curve—the other may be inferred.

Many things occur to a ship to affect her magnetism: extensive repairs that necessitate much hammering; firing the battery; stranding; violent concussions of any kind; steering the same average course for several days; lying in one direction for some time, as at a wharf, in dry-dock, or at anchor; great change of geographical position—all these, in varied degree according to the quality of iron in the structure, modify the characteristic magnetic features of a ship which are then reflected anew, as in a mirror, by a dock survey or curves of deviation; these faithfully exhibit every change that occurs.

In some ships the magnetic character is so clear-cut and steadfast as to be but slightly varied by ordinary influences—such a one may go from New York to Montevideo and show but small changes in her deviations—the mild field of the Earth affects her scarcely more than it would the hard steel of a bar-magnet: again, other ships are so plastic that they take every fleeting impress—it requires only a week's heading on a single course to modify

their features so much that only a few traces of the original lines remain—the deviations of such ships would, in consequence, change so much as to be positively dangerous.

The magnetic character of any ship—its exact degree of rigidity or pliability—can become known only by frequent swingings for deviations: hence the necessity for partial ones—the daily azimuths on several of the courses that may be used, especially in the vicinity of land—they are *an absolute essential to safety*, and cannot be too positively insisted upon.

That the ship is a magnet, one need but point to the OREGON for ocular proof—there are the two regions of opposite polarity, separated by a neutral line: and as the OREGON is, so is every iron and steel ship—only that the relative location of the poles and neutral line will be different according to the magnetic azimuth and hemisphere in which she was built.

CHAPTER XVII.

SELECTING A SITE FOR THE STANDARD COMPASS.

264. **Conditions midst which a compass is placed on a ship.**—When two magnets as unequal in size as a steel ship and a compass-needle are brought together, it is evident that the one may ever lead the other captive; and yet it is the pigmy that must guide the giant: in order to do so, however, it must have by vantage of position what it lacks in strength. The compass may occupy every degree of magnetic surroundings in a ship—from one in which it is completely dominated, to one in which it retains much freedom to point to the Earth's magnetic pole: it cannot wholly escape the influence of the two grand areas of opposite polarity existent in the hull of the ship, but neither need it be placed within the grip of their greatest intensity; nor subject to any one of a hundred small but concentrated foci of magnetism in individual masses of iron in the structure.

To discover these pitfalls, large and small, to place the compass where vicious influences will affect it least, and to determine the kind and degree of those it must unavoidably have, is the object of examining magnetically all possible sites for it.

Little scope is allowed for locating the steering compass—the position of the pilot-house or conning-tower decides the fate of that; but it is far otherwise with the standard compass. Two requisites should govern in its location: first, that it be the best—magnetically—that

the ship affords; and second, that it command the largest possible view of the horizon for taking bearings.

Consider the case of the OREGON, as probably realizing the fulfillment of these conditions as much as circumstances would admit: the standard compass is not far from the neutral line, where the two large areas of opposite polarity affect it least; it is well removed both horizontally and vertically from the magnetic poles of the ship—influenced only by the diffuse and weakened portions of their fields; it is elevated on a framework which commands much of the horizon; and is beyond the grasp of concentrated poles of smoke-stacks, masts, turrets, guns, boat-cranes, and other masses.

The steering compass, on the other hand, is less favorably placed: it is within easy reach of the blue pole of the ship, whose every roll and pitch send out ether waves to beat upon the compass, and, if synchronous with *its* period, become as timed impulses to a swing—setting up violent motion.

The futility of trying to shut out the injurious effects of a ship by iron barriers is so well known that it need not be dilated upon; all such devices equally screen the compass from the beneficent influence of the Earth—strip it of directive power: the matter is explained in Art. 173 and in other parts of this Treatise.

Wood is the best material for the immediate surroundings of a compass; but if metal *must* be used, as for stanchions, railings, ventilators, etc., then every such article within 25 feet should be of copper, or brass, or manganese-steel, as it is said that this is non-magnetic.

It would be well, too, if pilot-houses and conning-towers could be made of metals that did not destroy so much of the usefulness of the *steering* compass.

265. The time and mode of examining a compass site.—When the magnetic character of a ship has been ascer-

tained by survey, and while she is still in dock, is the time to make observations for the installation of the compasses, and for determining their magnetic environment: the ship is then perfectly steady, and the oscillations and deflections of delicate needles can be observed with accuracy, which is not the case when riding at anchor—however quietly, or lying alongside a wharf—however smooth the water. But observations under these latter circumstances are not to be contemned: the conditions are often favorable enough to afford results worthy of qualified confidence when an opportunity to obtain better cannot be had. The horizontally mounted needle furnished ships will suffice for observations of deflection and of horizontal intensity, and a small dip circle, those of dip and vertical intensity.

Observations should be made with these instruments as nearly as possible in the exact place the compass-card will occupy, and also at points in the same horizontal plane about 3 feet and 6 feet respectively, both forward and abaft the compass, and to starboard and port of it; identical observations are to be made in a parallel horizontal plane about 2 feet below, and a similar set in a plane 2 feet above, that of the compass-card, and at points corresponding to those in the middle plane: from all these the magnetic field around the compass can be mapped out, and its varying intensity seen at a glance. The same should be done for the steering compass.

It will be found convenient to have an open framework made, with a movable shelf, for supporting the instruments; the frame can be easily moved to the required distances around the compass, and the shelf placed successively in the three planes of observation.

At each point, the frame should be carefully aligned to the direction of the keel, whose magnetic direction having been previously determined, then gives that of the

edge of the frame and its shelf: when the side of the box containing the horizontal needle is aligned with the edge of the shelf, the deflection of the needle from the magnetic meridian becomes known, that is, the deviation at that point; a long ruler attached to the *side* of the box will conduce to accuracy of alignment.

266. Exploring the magnetic field around the compass.—The observations to be taken at each point are: *first*, the deviation; *second*, the time in which the horizontal needle makes ten (10) oscillations (and in this connection see Art. 230, and Tables 36 and 37); *third*, the Dip—that is, the maximum angle the direction the dipping needle will make with the horizontal plane, noting at the same time whether the needle swings in the vertical plane through the magnetic meridian or outside it, and if the latter, by what angle (to be read on the horizontal graduation of the dip circle); and *fourth*, the time in which the dipping needle makes ten (10) oscillations through a moderately small arc in the *vertical* plane at right angles to the one in which the angle of dip was observed—said plane to be found by turning the dip circle through 90° of the horizontal circle from the position it occupied when indicating the dip.

The observations for dip, and for horizontal and vertical intensity, are to be repeated on shore in a spot entirely free from magnetic masses: such a spot can be found by taking two sets of reciprocal bearings on lines at right angles to each other with an azimuth circle and 7½-inch compass; and if the lines are about 100 feet long, it should not suffice to take the reciprocal bearings at their extremities alone, but also at every 20 feet of their length. The Variation should be determined in this spot by a Time-azimuth of the Sun; and the magnetic heading of the ship might be determined by a like observation on board.

The theory upon which the oscillations of a needle

indicate the intensity of a magnetic force is explained in Parts First and Second. In the case under consideration, it is not the absolute force of either Ship or Earth that is required, but the relative value of the former to the latter; therefore it is optional to assign any standard for the Earth: it is customary to call it *unity*. The magnetic forces aboard and ashore being to each other as the inverse squares of the periods of oscillation of the *same needle* in both places, we then obtain that of the ship in terms of the Earth's by assuming the latter unity, and making the proportion between the periods of oscillation on ship and shore for the horizontal needle, and also for the vertical needle. A comparison of all the ship's observations with those ashore will give the values of the Deviation, Dip, Horizontal and Vertical Forces of the Ship for the particular heading she had in dock: in the same locality, these will vary with every new heading; and it would afford a better knowledge of the matter if all the observations could be repeated with the ship in the diametrically opposite direction—that is, docked stern first, instead of head first in the same place. Indeed, like swinging for deviations, the more points upon which the observations are made, the fuller the information afforded; but, for large ships, to make it on even the cardinal points would involve so much labor and variety of docking opportunities, that it seems almost hopeless to ever attain such completeness. The partial examination on one heading, however, reveals the fact whether it is the large magnetic field of the hull alone we have to deal with, or the concentrated but often powerful pole of some individual mass of iron.

Other information, also, is derived from observations on one heading: 1st, if begun while the ship is on the stocks and continued at intervals until completed, they exhibit the gradual forming of her magnetic character and its variations; 2d, if conducted in different parts of the ship

while in dry-dock, they indicate which is the best magnetic place for the compasses; 3d, when made on the site of the standard compass, they afford the means of obtaining a Table of Deviations; and 4th, they enable the navigator to compensate that compass with magnets.

Results under the third and fourth headings are only rough approximations; but still cases may arise when even such are useful: they will be explained in Part Fourth, and should be employed only when accurate methods are unavailable; and *when* used, they should not be depended on longer than the first opportunity to replace them by reliable work.

In securing the binnacles to the deck, it is important that they be placed so that the keel-line of the compass shall be exactly in the fore-and-aft mid-ship line: the trace of the central vertical plane through the keel is generally marked by the Constructor on the hatchways while building; if not, he has data and appliances for readily doing it; and with this to work from, it needs only a theodolite, or compass, azimuth-circle, and tripod, and the knowledge possessed by any officer who has to deal with the matter, to set the binnacles properly in place.

In order, however, to indicate a mode of procedure, the following is quoted from an officer who has had much experience in the matter—Commander Diehl, U. S. Navy:

"The hatches are brought up from the keel in plumb lines during construction of the ship; determine middle points of the hatches on the upper deck, and place a vertical staff on each point; run a line through all the middle points; at selected positions on this line erect straight-edges perpendicular to the keel, and by means of these place the binnacles on the mid-ship line so that this shall divide them symmetrically and be traced as a chalk mark on the upper rim or surface of each binnacle—a guide for further work. Level each binnacle by means of a spirit-level.

"Lower the heeling-magnet well down into its tube, place the compass in the binnacle, and proceed to center as follows: Raise the heeling-magnet in its tube, and if any deflection of the card results, move the compass to one side or the other by means of its centering screws, until no movement or deflection of the card occurs while the heeling-magnet is raised and lowered; the compass is then centered; remove the heeling-magnet and cause the binnacle to assume the same inclination as the deck.

"Place an azimuth-circle on the compass, and sight successively on the straight-edges erected forward and abaft the binnacle: the readings should coincide with the keel-lines of the compass, and should differ 180°; if not, the binnacle should be moved until this is attained."

CHAPTER XVIII.

PHYSICAL REPRESENTATION OF THE THEORY OF THE DEVIATIONS.

267. The magnetic make-up of a ship analogous to that of a steel bar.—A theory of magnetism was stated in Arts. 191 and 192: briefly, it was, that a steel magnet is composed of molecules, either permanently magnetized or girded by electric circuits—either conception explains observed facts; that the alignment of the molecular axes develops the magnetic condition, and their heterogeneous mixture the neutral state. By analogy we may consider the steel ship a magnet whose distinctive character is not confined to the hull alone, but depends also upon a thousand individual masses of iron distributed throughout the structure: engines—shafts—smoke-stacks—turrets—conning-towers—guns—masts—hoisting cranes—hatch coamings—beams—ventilators—stanchions:—these, and innumerable minor masses, are so many separate magnets whose efforts variously affect the general result.

As with a bundle of bar-magnets—when placed with like-poles together, they exert a certain force; but if one, two, or more be reversed, the effect is thereby weakened—so with the iron masses in a ship: some contribute to the prevalent magnetism, others contravene it.

Provided it is not within the concentrated field of one of these masses that we place the compass, we may consider their individualities merged in the two grand regions of magnetic polarity that pervade the ship; and thus—

ultimately—have only the poles of those regions to deal with.

268. The magnetic effect of a ship replaced by that of permanent magnets and soft-iron rods.—Consider Fig. 434: the compass at O is supposed to be affected by the magnet-

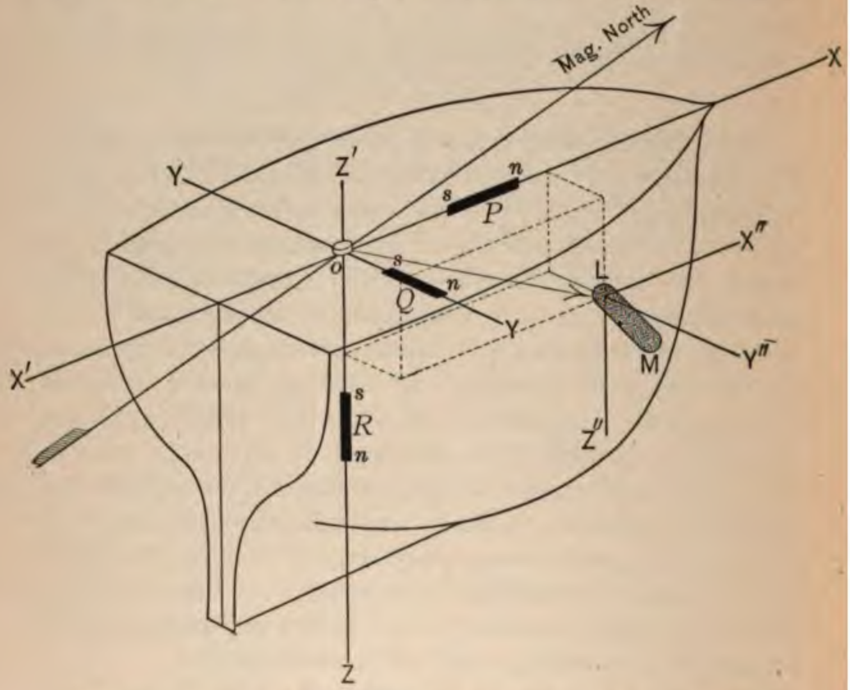


FIG. 434.—The Ship's Magnetism resolved longitudinally, transversely, and vertically.

ism of the forward part of the structure only, whose focus is at L, so that the line $\overline{OL}$ represents the resultant of all the forces that cause deviation.

If we conceive the ship divided by three planes—the first horizontal, the second and third vertical, the former through the keel, the latter transverse to it, and all passing through the compass-pivot—we thus form eight solid angles

around the pivot; in any two of these, the two foci of the ship's polarity *may* exist; though, as a matter of fact, they will scarcely ever be found above the horizontal plane: in Fig. 434, the acting pole is in the solid angle O–XYZ. In any particular case, the location becomes definite for either reference or mathematical treatment, by designating directions toward the bow, to starboard, and downward as positive (+), and those toward the stern, to port, and upward as negative (−), the compass being the origin of coördinates, through which the traces OX, OY, and OZ of the respective planes pass as axes. Of course the compass may be moved aft so as to bring it within the influence of the other pole alone, located somewhere in the after-body of the ship.

Continuing the analogy of the magnetic composition of a ship to that of a steel bar, in the latter the molecules are presumably iron of the same kind, equally hard, and identically magnetized; but in the ship all grades and kinds of the metal enter, and therefore we have to deal with all degrees of magnetization. The theory, however, does not take cognizance of such varied and indeterminate conditions; it is based on but two—that all the iron in a ship is divided between the *hard* and the *soft*, the former difficult to magnetize, but retentive of what it acquires; the latter easy to magnetize, but without hold upon what enters.

Experiment—as in swinging ship, or in a magnetic survey, or in oscillating needles for force—deals directly with the iron as it exists in the ship, and therefore discloses the actual state of the case; but the mathematical deductions based on imaginary conditions—for none of the iron of a ship is absolutely hard or absolutely soft—give results that do not strictly agree with experiment: yet the hypothetical classification so nearly covers the actual state, that no difference that would invalidate the mathematical treatment of the subject has ever been found.

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Now, the [illegible]elds; so that for the pass (if me[illegible]nsidered of constant bination of [illegible] across it, or reversed the ship; b[illegible]herefore, for the hard- been substit[illegible] we may substitute to truly rep[illegible]itable strength, in the be employe[illegible]ffect will be the same. tempered s[illegible] the same strength and the soft[illegible]hip turns, and in degree heated to r[illegible]d at the same distance

In Fig. [illegible]ert, above or below, with pole a[illegible]ill depend upon which tion LM of [illegible] magnets—[illegible] to represent a ship's position p[illegible]e starboard bow; if planes of t[illegible] matter, P should be

Then t[illegible] if toward the compass-m[illegible] be transferred to between [illegible] magnets from one axis but also, [illegible] distances from between [illegible] altogether as their *rem*[illegible] magnetic effect of of the re[illegible] it may near end[illegible] between [illegible] signs *plus* (+)

The r[illegible] of the mag- when p[illegible]compass, all these L-$X''Y''$ [illegible]

LX'', L[illegible] of the mag- and *ea*[illegible]s components [illegible] that a single

PLATE S.—RODS a, d, AND g.

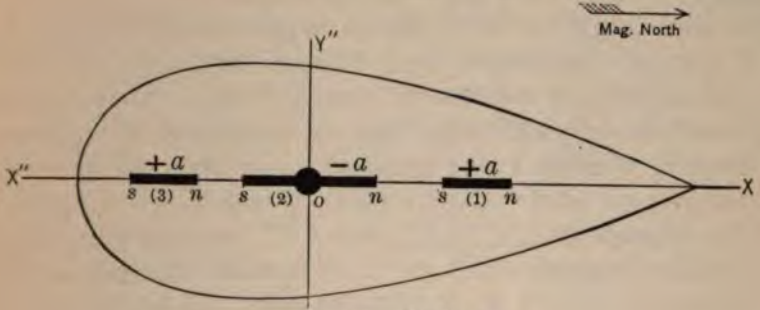

FIG. 435.—Rod a.

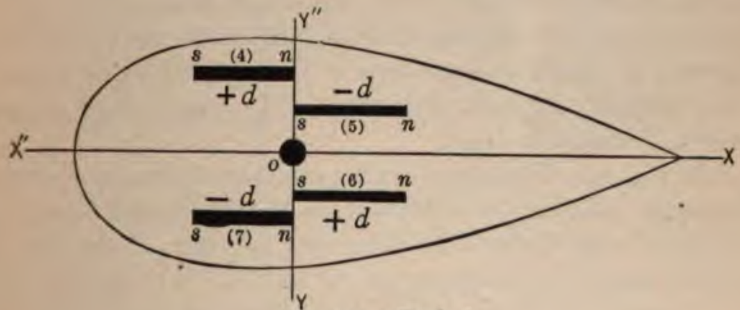

FIG. 436.—Rod d.

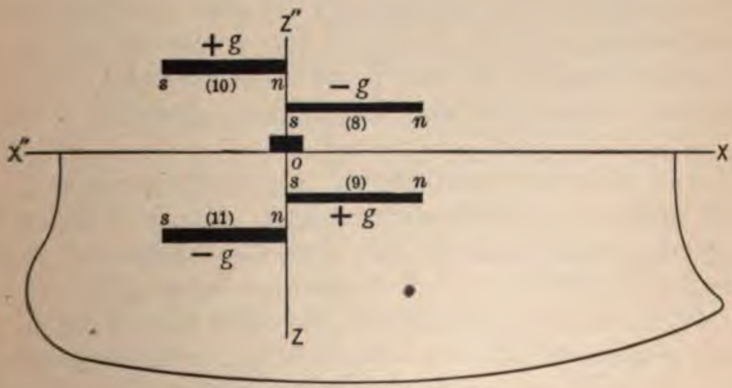

FIG. 437.—Rod g.

combination of hard- and soft-iron magnets, since such is the composition of their resultant $\overline{OL}$.

To a degree far more than is required in the treatment of compass deviations, steel magnets probably maintain their identity in moderately varying fields; so that for the present investigation they may be considered of constant strength, whether in the meridian, or across it, or reversed with respect to the Earth's poles: therefore, for the *hard*-iron magnets at $\overline{LX''}$, $\overline{LY''}$, and $\overline{LZ''}$, we may substitute three bar-magnets, P, Q, and R, of suitable strength, in the coördinate axes X, Y, Z, and the effect will be the same. These magnets will practically be of the same strength in all azimuths through which the ship turns, and in *degree* will produce the same effect if placed at the same distance forward or abaft, to starboard or to port, above or below, the compass; in *kind*, the effect will depend upon which pole is brought to bear.

In Fig. 434, P, Q, and R are placed to represent a ship's magnetic pole in the direction of the starboard bow; if the pole were toward the starboard quarter, P should be shifted abaft the compass to the axis $\overline{OX'}$; if toward the port quarter, Q must at the same time be transferred to $\overline{OY'}$; and thus, by changing the magnets from one axis to another and moving them to different distances from the compass, or doing away with one or two altogether as conditions require, we may produce the magnetic effect of the *hard* iron of the ship, in whatever direction it may be or however great its intensity.

And with due regard for the algebraic signs *plus* $(+)$ and *minus* $(-)$, as indicating both the polarity of the magnets and directions in the ship from the compass, all these physical conditions may be treated mathematically.

When we come to represent the other part of the magnetic system at $\overline{LM}$—the *soft-iron* factor, or its components in $\overline{LX''}$, $\overline{LY''}$, and $\overline{LZ''}$—by rods, we find that a single

PLATE U.—RODS *c*, *f*, and *k*.

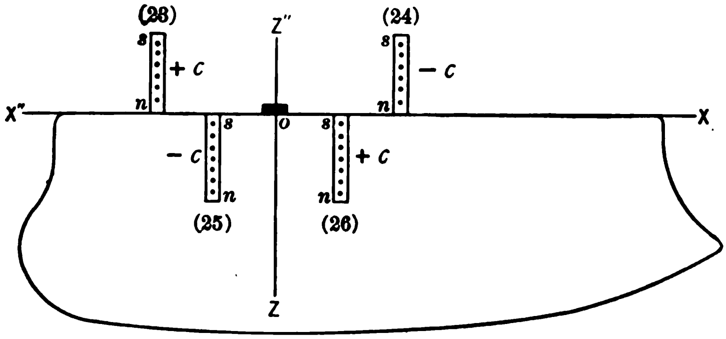

FIG. 441.—Rod *c*.

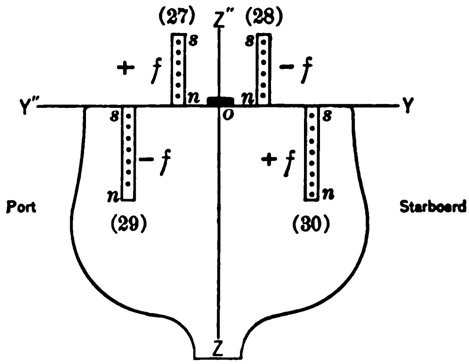

FIG. 442.—Rod *f*.

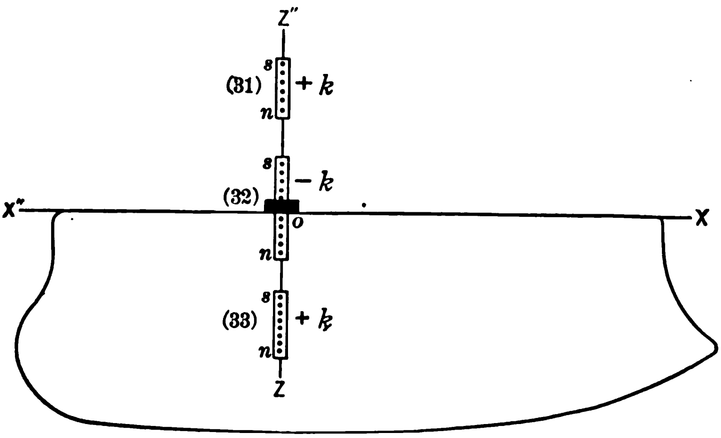

FIG 443.—Rod *k*.

one for each axis will not suffice; for, unlike permanent magnets, soft iron exhibits extremes of magnetic condition in various positions: in the meridian a rod of it is strongly magnetic—across the meridian, it fails of effect; entirely forward or abaft the compass, one pole acts—extending continuously above or below it, another pole comes into play; and so on through many possible changes.

Thus, we have to consider soft iron, first, with regard to *direction*—that is, whether fore-and-aft, or athwartships, or vertical; and, second, as to *position*—that is, whether entirely forward or wholly abaft the compass, or to starboard or port of it, or above or below, or extending continuously in any three planes.

The rods requisite to fulfill these conditions naturally divide into three categories representative of *direction*—the fore-and-aft, $\overline{LX}''$; the athwartship, $\overline{LY}''$; and the vertical, $\overline{LZ}''$: and in each the rods may have every variety of *position*.

Plates S, T, and U illustrate every possible case. Plate S has rods representative of all the *longitudinal* soft iron: in Fig. 435, rod a stands for it when in the middle section of the ship, as (1) wholly forward of the compass, (2) continuously above or below it, and (3) entirely abaft; in Fig. 436, rod d represents it when on each side of the compass whether wholly forward and toward either bow as (5) and (6), or entirely abaft and toward each quarter as (4) and (7); in Fig. 437, rod g represents it when above and below the compass, and entirely forward as (8) and (9), or entirely abaft as (10) and (11). Similarly, Plate T shows the position of *transverse* soft iron: in Fig. 438, rod b represents it when wholly forward or abaft the compass and on each side of the longitudinal section; in Fig. 439, rod e when in the vertical plane through the compass, and whether wholly on either side, or unbroken in one extended mass; and in Fig. 440, rod h when above or below

PLATE U.—RODS *c*, *f*, and *k*.

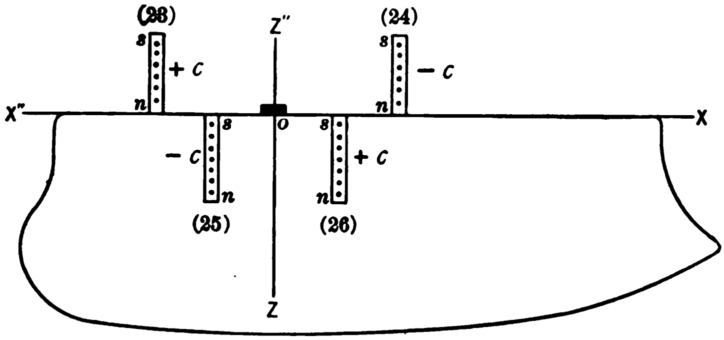

FIG. 441.—Rod *c*.

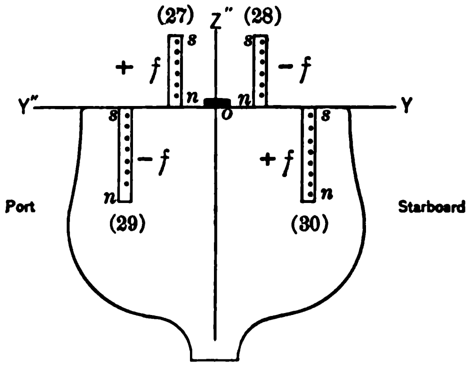

FIG. 442.—Rod *f*.

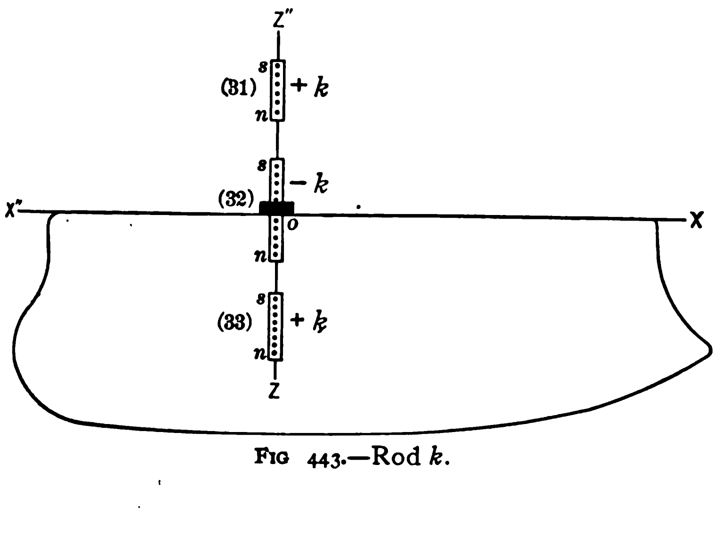

FIG 443.—Rod *k*.

on each side. In like manner, Plate U exhibits the varied location of *vertical* soft iron: in Fig. 441, rod c represents it in the longitudinal section both forward and abaft the compass when abutting on the horizontal plane through it; in Fig. 442, rod f represents analogous positions in the transverse plane; and in Fig. 443, rod k represents positions directly above or below, or through, the compass.

Thus, the number of rods representing all the soft iron in the hull, armament, and equipment of the ship is *nine;* and there can be no more, since its every possible direction and position are represented.

Of course, it is not likely that the constituency of *every* representative rod will exist in the ship; for example, instances of h (19) and (20), and k (31) and (32), do not readily suggest themselves to the mind; nevertheless, should they arise, their representatives are already in the arena to assert their strength; when absent, these representatives having no foundation, reduce to naught; and so, either way, the problem is complete—covers the contingencies both of accession and defect.

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$$B = P + c; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

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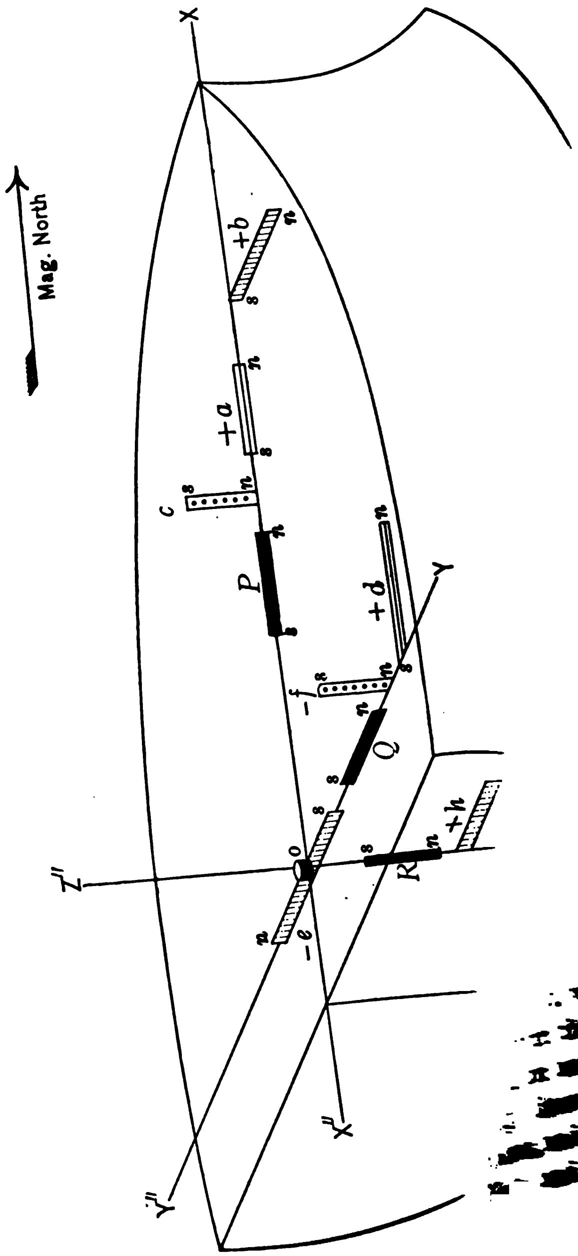
Mag. North
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Y
Z''
X''
Y''
+b
+a
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OY (Figs. 442 and 444), the
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$$C = Q + f; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

positions of C with the corresponding
rom its action are shown in Figs. 448,
51. They need no further explanation
dependence of C upon the course.

ures, while the ship heads north, the devia-
; as she swings to the east, the needle returns
decrements to the normal direction and attains
p heads east; at south there is another maxi-
at west another minimum. Thus the effect
eight when the course is 0°, and at its lowest
s 90°, which supplies at once the required relation,
the azimuth, or

$$C . \cos \zeta' . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

returns by constant decrements toward the meridian, and reaches it as the vessel finishes a semicircle; let the ship complete the circle, and the needle will but describe a path symmetrical with the first on the opposite side of the meridian.

To represent the path of the needle graphically, unfold the circumference of the compass-card and straighten it into the line $NESW$ of Fig. 446: distances along this line

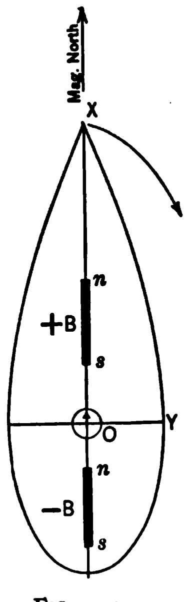


FIG. 445.

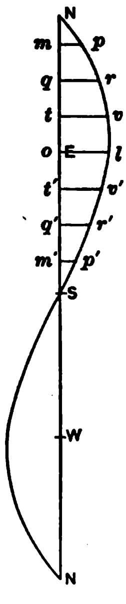


FIG. 446.

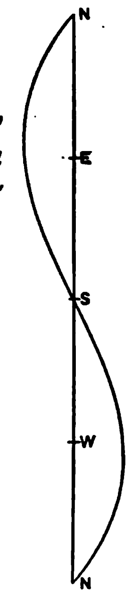


FIG. 447.

from the origin N, as $\overline{Nm}$, $\overline{Nq}$, $\overline{Nt}$, will represent azimuths of the ship while swinging; from the end of each erect an ordinate as $\overline{mp}$, $\overline{qr}$, $\overline{tv}$, equal, on any scale of parts, to the deviation for that heading; draw a curve through the extremities of all the ordinates, and it is a wavy line having maxima at East and West, and minima at North and South.

The curve is thus seen to be a function of the azimuth, swelling or flattening as that increases; more than this, it is zero when the azimuth is 0°, and steadily grows to a maximum when the course is 90°—the characteristic feature of the sine of an angle: denote the compass course or azimuth by ζ', and then the value of B for different azimuths will be

$$B.\sin \zeta'. \quad \ldots \ldots \ldots \quad (2)$$

If, in Fig. 445, the n-pole of $+B$ or the s-pole of $-B$ had been presented to the compass, then repulsion would be the action between the near ends of B and the needle, and as the ship swung to the eastward, the needle would deviate, first, toward the west, and the curve representative of its path would be that of Fig. 447.

In the transverse axis $\overline{OY}$ (Figs. 442 and 444), the magnet Q and the rod f are the counterparts of P and c in the longitudinal axis $\overline{OX}$: represent their joint effect by C, plus (+) when to starboard of the compass and minus (−) when to port; then

$$C=Q+f; \quad \ldots \ldots \ldots \quad (3)$$

and all the possible positions of C with the corresponding curves resulting from its action are shown in Figs. 448, 449, 450, and 451. They need no further explanation than to state the dependence of C upon the course.

In both figures, while the ship heads north, the deviation is greatest; as she swings to the east, the needle returns by constant decrements to the normal direction and attains it as the ship heads east; at south there is another maximum; and at west another minimum. Thus the effect is at its height when the course is 0°, and at its lowest when it is 90°, which supplies at once the required relation, cosine of the azimuth, or

$$C.\cos \zeta'. \quad \ldots \ldots \ldots \quad (4)$$

B and C form the two sides of a right-angled triangle; their combined effect—the hypothenuse—is obtained by the expression

$$\sqrt{B^2+C^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which is called the semicircular deviation: according to the different positions of B and C, and their relative strength,

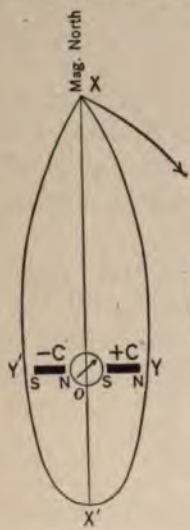


FIG 448.

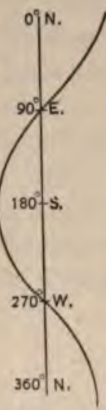


FIG. 449.

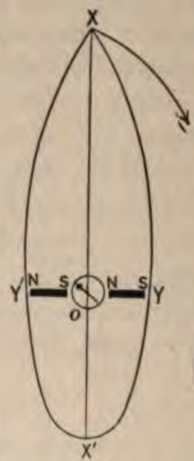


FIG. 450.

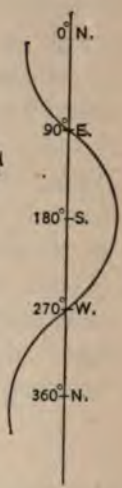


FIG. 451.

this resultant will point toward either bow, or either quarter, and the angle that its direction makes with the midship line (the compass being the center of angular measure, which is reckoned from 0° to 360°) is known as the STARBOARD ANGLE; this is shown in Fig. 452, where B is plus and C minus, and the former the stronger: the ship's semicircular force in this case lies in the direction of the port bow, with a starboard angle of about 340°.

271. The Quadrantal Deviation.—The rods a and e (Figs. 435, 439, and 444) represent soft iron IN the longitudinal and transverse sections respectively; on account

of this analogy of position (each with reference to its proper axis) they may be considered jointly: let one particular combination of the two be that shown in Fig. 453.

Both *a* and *e* are magnetized by the Earth's horizontal component. While the ship's keel is in the meridian, the magnetic strength of *a* is concentrated near its ends, but

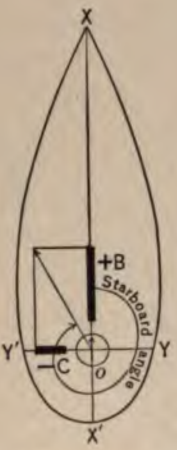


FIG. 452.

produces no deviation, because it is in line with the needle; *e*, on the other hand, is favorably located for a strong pull, but its magnetism is diffused along its sides, equally at each end and of no effect.

As the ship swings out of the meridian, *a* causes the needle to deviate in the same manner that $+B$ did, with this difference, however, that while B retained its power intact during the swinging, *a* gradually loses its hold; but on account of the direction it successively takes with reference to the needle's direction, the combined effect, due to its leverage and magnetism, increases up to a course of 45°, when it is a maximum; from 45° to 90°, *a* gradually loses its sway over the needle, which returns, and

coincides with the meridian as the ship reaches 90°. But, meantime, this effect of a is modified by the action of e.

On the ship swinging out of the meridian, the polarities of e gradually shift toward its ends, where they become concentrated in foci by the time she heads east, the end

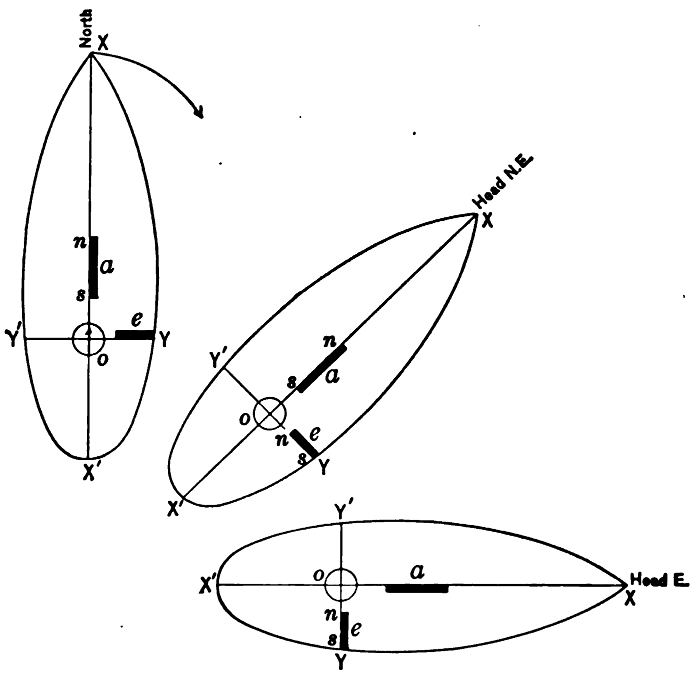


FIG. 453.

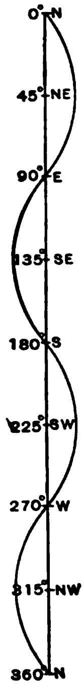


FIG. 454.

nearest the compass becoming a north pole and therefore repellent of the north end of the needle, while at the same time the end of a nearest the compass is a south pole and attracts the north end of the needle: the resultant effect during the swinging is therefore a difference of these two; represent it by D, that is,

$$D = a - e. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The sign of D will depend on the relative strength of a and e.

With the ship's head east, the magnetism of *a* is diffused along its sides—that of *e* concentrated at its ends; and as the ship continues swinging, they successively interchange these conditions of polarity according to which one coincides with the meridian and which is transverse to it.

To follow their resultant *D* around the circle: from 90° to 180°, the curve traced between 0° and 90° is repeated, but now on the opposite side of the meridian; from 180° to 270° we have the same curve as from 0° to 90°; and between 270° and 360°, the same as from 90° to 180°—in fine, the wavy line of Fig. 454. Its general contour is that of alternate maxima and minima with symmetrical branches on each side of a central line; as each of these characteristic points occurs twice in a semicircle the effect of *D*, considered as a function of the course, may be represented by

$$D.\sin 2\zeta' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The rods *b* and *d* (Figs. 436, 438, and 444) represent soft iron entirely outside the three sectional planes of the ship, with the possibility of it abutting on those planes; both rods depend on the Earth's horizontal intensity for their strength, and these two facts—similarity of position and source of magnetism—admit of treating them together.

Let Fig. 455 represent the particular combination to be examined.

The analogy of this case to that of *a* and *e* (Fig. 453) is so striking that the explanation of that may almost be paraphrased: while the ship is in the meridian, *b* is magnetic along its sides and produces little effect; *d*, on the contrary, has its poles at the ends and exerts its power to most advantage. As the ship swings through the first quadrant, the polarities of both rods shift—that of *b* toward the ends and that of *d* toward the sides, until at east their conditions have been interchanged—the magnetism of *b* now being at its ends, strong and effective,

that of d diffused over its sides and weak; and so, alternately, they continue round the circle, the corresponding action on the needle being the deviation delineated in Fig. 456.

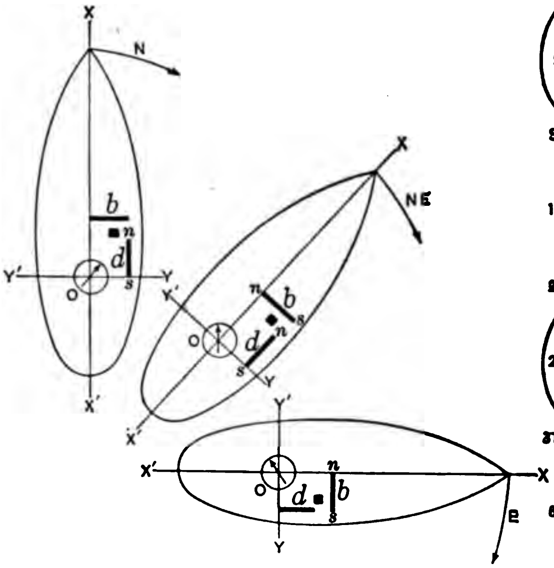


Fig. 455.

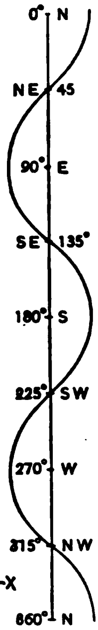


Fig. 456.

It is seen to be similar to Fig. 454—only, that its maxima and minima occur at the points of minima and maxima of Fig. 454—one the exact converse of the other.

Represent the joint effect of b and d by E, and since the acting ends of the rods antagonize each other, the resultant will be

$$E = b - d. \qquad \qquad (8)$$

This resultant has maxima when the course is 0°, 90°, 180°, and 270°; and minima at 45°, 135°, 225°, and 315°—that

is, *two* maxima and two minima in each semicircle: the cosine of double the angle, then, is the proper link to connect the effect of b and d with the course, that is,

$$E.\cos 2\zeta' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

This is analogous to (7).

As with B and C, the resultant of D and E is given by the expression

$$\sqrt{D^2+E^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

This produces the quadrantal deviation.

For the same ship, it is theoretically unchangeable with time or place, and numerous analyses of deviations confirm the theory for all practical purposes.

The reason of the invariability is this: the quadrantal deviation is caused by induction of the Earth's horizontal component in soft iron, laid horizontally; it is also this component that gives direction to the compass needle: therefore, any change in the disturbing force is accompanied by an equal change in the directive power, and hence their ratio is a constant.

272. The constant deviation.—Suppose the positions of b and d to be those of Fig. 457, and that the rods are of equal magnetic strength, so that when each comes to act, it will exert the same pull on the needle; then as the ship swings round the circle, the needle will be kept at an unvarying deflection—a constant deviation.

Again: the axis of the magnetic system may not coincide with the zero-line of the card as in Fig. 458, and this likewise contributes to a constant deviation. Other errors of similar nature may exist, and all such are represented by A—*the* constant deviation.

273. The total deviation.—Let δ represent the sum of the deviations arising from the several sources; then col-

lecting the expressions found for them in (2), (4), (7), and (9), we have

$$\delta = A + B.\sin\zeta' + C.\cos\zeta' + D.\sin 2\zeta' + E.\cos 2\zeta', \quad . (11)$$

which is the mathematical formula of the deviations derived from consideration of the physical aspect of the problem.

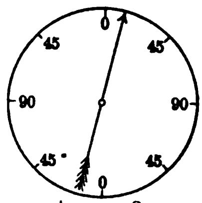


FIG. 458.

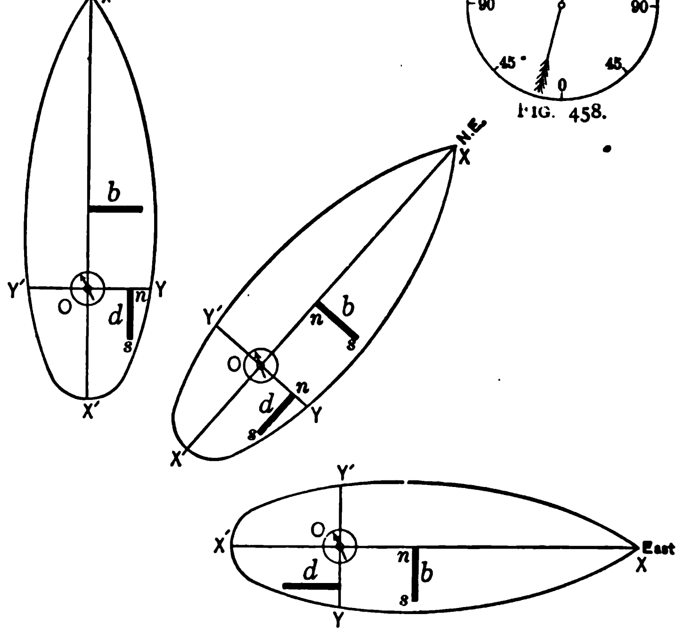


FIG. 457.

Each component, *by itself*, produces a symmetrical curve; it is the combined action of all—their superposition—that gives rise to the irregularity so often seen; and according to the varying strength of each component,

we shall have every variety of the general type of curve. Moreover, by its algebraic sign, which appears in the analysis of a table of deviations, we may infer the location (with reference to the compass) of each part of the disturbing force. The quantities A, B, C, D, and E in eq. (11) are called the Magnetic Coefficients.

In this representation of the deviations, the identity of the curves with those illustrative of wave-motion in Vol. I is apparent; indeed the identity continues throughout all the forms both of waves and deviations, and hence it is that Fourier's Theorem constitutes the basis of the mathematical treatment of the deviations: for it expresses, analytically, every variety of wave whether in the ether of space that disturbs the needle, or in the water of ocean that buffets the ship. It is a theorem based on trigonometrical functions, and by eq. (11) these are seen to express the several components of the total deviation.

In order not to convey an incorrect impression, it must be stated that formula (11) is incomplete.

The phenomenon of the deviations is a periodic one, all whose phases are covered by Fourier's Theorem: this takes cognizance, not only of the constant, semicircular, and quadrantal components—summed up in eq. (11)—but also of all others; the sextantal, which has six maxima and six minima in a complete swing; the octantal, which has eight such distinctive points; and so on—in fact, an infinite series of the form

$$\left.\begin{aligned}\delta = A + B.\sin\zeta' + C.\cos\zeta' + D.\sin 2\zeta' \\ + E.\cos 2\zeta' + F.\sin 3\zeta' + G.\cos 3\zeta' \\ + H.\sin 4\zeta' + K.\cos 4\zeta' +, \text{etc.}\end{aligned}\right\} . \quad (12)$$

This series converges rapidly, and for deviations of the ordinary amount, its first five terms generally cover all the errors that need practically be considered.

274. The heeling error.—Thus far, the disturb.ng force has been treated with the ship on an even keel; but when listed to either side, a new and additional source of deviation arises—the *heeling error;* this is caused by magnetic influences, some of which are illustrated by Fig. 459—that is, the magnet R and the rods g, h, k, and e: the subject will be treated fully in Part Fourth.

When the ship heels, the rod e inclines also, and acquires magnetism from both the horizontal and vertical components of the Earth's intensity, with poles near its ends; which pole acts on the needle will depend on the position of e as shown in Fig. 439 of Plate T.

Rod h is in every respect the analogue of e, and its various positions are given on Fig. 440: the polar action may readily be inferred from each case.

Rod g is magnetized by the Earth's horizontal intensity, and most strongly when the ship is in the meridian, with poles at the ends; it is evident that its pull upon the needle, with the ship heeled, will be greatest on both northerly and southerly courses, and that it will lose its grip entirely on both easterly and westerly courses, because its magnetism is then distributed over the sides of the rod—equally at each end—and therefore ineffective: Fig. 437 exhibits all the positions of g, and the direction in which the acting pole will draw the needle may easily be inferred from an inspection of each case.

Rod k is magnetized by the Earth's vertical component; its poles are always at the ends, and the different positions it may have, with the corresponding action in each, are indicated on Fig. 443.

A magnet whose axis lies *vertically* above or below the pivot of a compass will not deflect the needle; but when the ship heels, the magnet inclines also and will cause deviation: the magnet R and rod k are in this situation,

and may conspire or conflict in their action—it depends on which pole of R is uppermost.

Indeed, it may be said that as each component has its distinctive polarity, derived from different sources and governed by diverse laws—R being a permanent steel magnet; the rods variable soft ones; g magnetized by the Earth's horizontal intensity only; k by the vertical component chiefly; e and h partly by both; the poles of

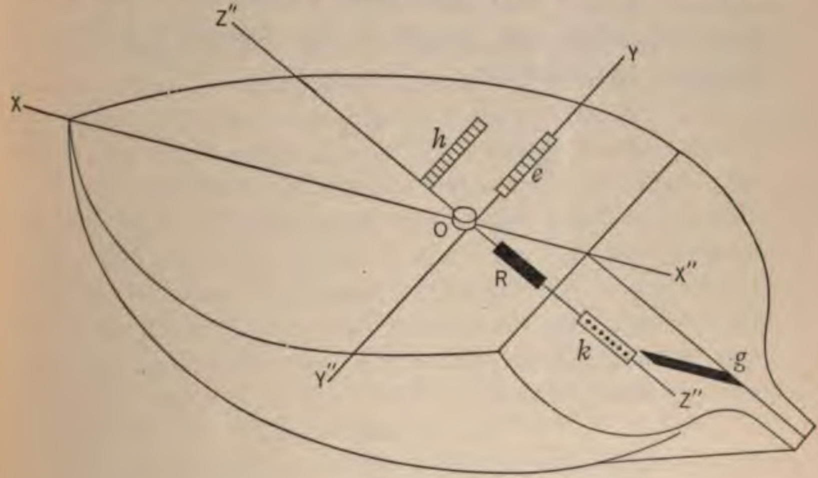


FIG. 459.

k always at its ends, the polarity of e, g, and h sometimes toward the ends and sometimes toward the sides—all this variety must have a resultant whose effect alone is experienced: it may attract the needle to windward or repel it to leeward.

275. The complete information relative to the ship's magnetism.—In Art. 266 it was stated that if oscillation experiments with horizontal and vertical needles be made with the ship on one heading, her magnetic force for that heading would be obtained; if this be done for two opposite headings, or for a number of equidistant ones, the mean

of all gives the mean horizontal force to north denoted by λ, and the mean vertical force denoted by μ: both these quantities are essential in certain calculations.

By swinging ship we obtain Tables of Deviations; analysis of these discloses the components of the disturbing force; further separation of each component exhibits the various representatives of the hard and soft iron of the ship: and all this information in connection with a magnetic survey and oscillation experiments for force, afford an intelligent exposé of the magnetic influences acting on the compass.

CHAPTER XX.

EXPERIMENTAL ILLUSTRATION OF THE DEVIATIONS, AND THEIR COMPENSATION.

276. Instruments and arrangements for the experiments.—The following experiments were made with the SCORESBY, Fig. 428, mounted as shown in Fig. 429, in the Office of Superintendent of Compasses when it was on the second floor of the Navy Department in 1883. Considering the circumstances that contributed to magnify error—the contracted space of the room and the materials of which it was constructed; instrumental imperfections; the close proximity to the compass of both the iron and magnets used; the size of the compass-needle, the rods, and bars employed—the results obtained were in close accord with theoretical values.

The representatives of the soft-iron rods consisted of a number of wrought-iron tubes, each 14 inches long, 2 inches external diameter, and $1\frac{7}{8}$ inches internal diameter; they were found by experiment to be absolutely free from permanent magnetism.

The compass used in all the experiments, unless otherwise stated, is seen mounted on the upper deck, Fig. 428, in the midst of the tubes; it was a liquid compass of the $7\frac{1}{2}$-inch type, and bore the number 6216, by which it will be referred to hereafter; its card was of the new form then recently adopted; its graduation was to 20′ of arc, on a gold rim fastened to the top of the card.

There was but one magnet, however, attached cen-

trally in a diameter of the card (Fig. 401) instead of the four magnets of the same type of compass issued to ships: this magnet was 3½ inches long, weighed 740 grains, and consisted of a bundle of fine wires of hardened steel; the pressure of the card on the pivot, in liquid, was about 50 grains.

For observing the meridian-line traced on the north wall of the room, Fig. 429, while swinging ship, a telescope specially made for the purpose, was used on top of the compass—Fig. 428: it magnified eight diameters, had cross-hairs, and a prism for reflecting-up the graduation of the card, so that the meridian-line and division of the card in coincidence could be seen together; although the card was divided only to 20′, still 5′ could easily be estimated.

Previous to making an experiment, all iron and magnets were removed from the SCORESBY and its vicinity to the far limits of the room, and left there in the same position throughout the entire series. The vessel was then carefully swung, stopping at each point to let the needle come to rest before taking the bearing of the meridian-line.

A table of deviations made from these readings on every point then included both the varying parallactic angle and the influence of all iron in the room that affected the compass. Such a table was obtained first with the ship upright, and, secondly, while heeled to the side and by the amount that was subsequently used in the heeling experiments.

These two tables, upright and heeled, were verified every day, before making an experiment, by swinging the vessel on the points to be used. Then, to ascertain the effect of either a tube or magnet, it was placed in the desired position and direction by means of one of the bronze arms, and the vessel carefully swung, resting ample time on each heading by compass, and the bearing of the meridian-line was then observed and recorded.

The table of deviations thus obtained was compared with that when neither iron nor magnet was on the vessel, and in succession the effect of each of the nine soft-iron tubes and the three steel magnets was thereby determined.

Although *tubes* were actually employed, still they are called *rods* throughout the experiments to accord with the technical name used in this investigation.

277. The deviations produced by soft-iron rods in different positions.—Each experiment is illustrated by a diagram showing the position of the rod on the SCORESBY and by a curve of the deviations it produced; the table accompanying each experiment is self-explanatory.

In the figures representing the curves, each small square is 1°; and whether the curve is of natural size or enlarged, is stated in the description of the experiment.

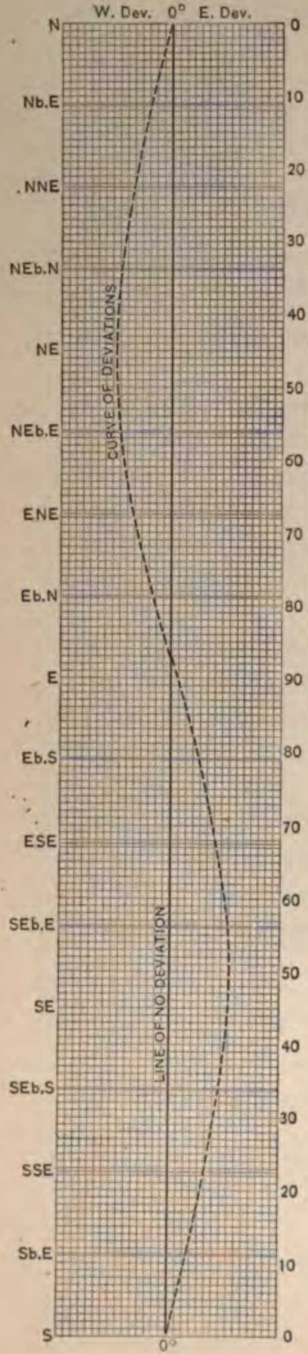


FIG. 463.

TABLE 54.

Heading of SCORESBY by Compass No. 6216.	Bearing of True Meridian and Other Lines on Wall, *without* Tube on Vessel.	Bearing of Lines Designated in Column 2, *with* Tube Placed as in Fig. 462.	Dev Prod Ro
(1)	(2)	(3)	(
N.	N. 44° 10′ W.	N. 44° 10′ W.	0°
N. by E.	N. 44 20 W.	N. 43 50 W.	0
N. NE.	N. 5 40 E.	N. 6 25 E.	0
NE. by N.	N. 4 55 E.	N. 5 50 E.	0
NE.	N. 4 05 E	N. 5 20 E	1
NE. by E.	N. 3 30 E.	N. 4 40 E.	1
E. NE.	N. 3 0 E.	N 4 0 E.	1
E. by N.	N. 3 0 E.	N. 3 20 E.	0
E.	N. 3 0 E.	N. 2 50 E.	0
E. by S.	N. 3 0 E.	N. 2 20 E.	0
E. SE.	N. 3 20 E	N. 2 10 E	1
SE. by E.	N. 3 40 E.	N. 2 20 E.	1
SE.	N. 4 10 E.	N. 2 50 E.	1
SE. by S.	N. 4 40 E.	N. 3 30 E.	1
S. SE.	N. 5 20 E.	N. 4 30 E.	0
S. by E.	N. 6 0 E.	N. 5 40 E.	0
S.	N. 38 10 W.	N. 38 10 W.	0

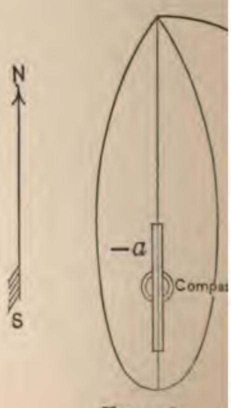


FIG. 462.

EXPERIMENT 2: ROD $-a$.

Vessel upright and swung through eastern semicircle, resting two minutes on each point. Tube: same 28-inch one used in Experiment 1: placed horizontally in vertical plane through keel, *above* compass; middle point over pivot; axis ten (10) inches above plane of needle and parallel thereto.

A propeller shaft, or an iron keel or deck, would be represented by $-a$.

The results of this experiment could scarcely be in closer accord with theory—Table 54: the curve of Fig. 463 is enlarged six times.

From the relative positions of the tube and azimuth telescope, it was not always possible to observe the meridian line traced on the north wall of the room; so that in this and other experiments, lines that were suitably traced on the different walls had to be observed.

In the present experiment, this was the case when the vessel headed north, north by east, and south. This practice in nowise affected the results.

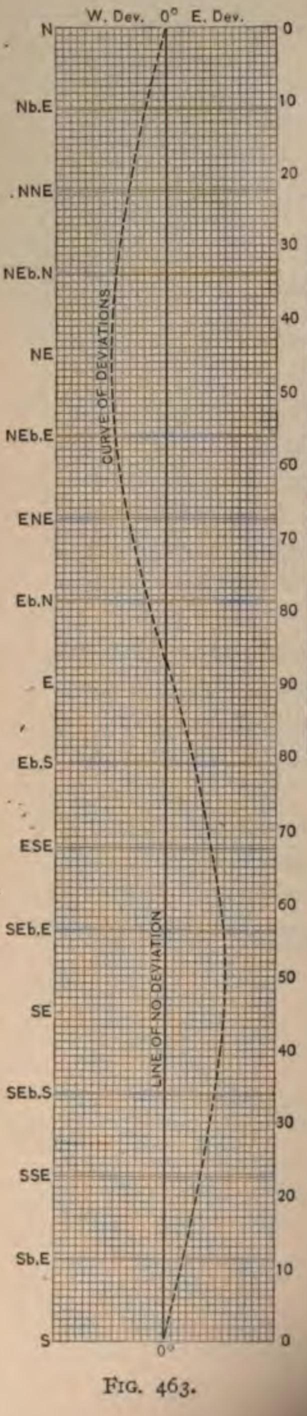


FIG. 463.

TABLE 5

Heading of SCORESBY by Compass No. 6216.	Bearing of True Meridian and Other Lines on Wall, *without* Tube on Vessel.	Bear[illegible] De[illegible] Col[illegible] Tu[illegible]	[illegible]
(1)	(2)	[illegible]	(4)
N.	N. 44° 10′ W.	[illegible]	[illegible] 45′ E.
N. by E.	N. 44 20 W.	[illegible]	[illegible] 0 E.
N. NE.	N. 5 40 E.	[illegible]	[illegible] 45 E.
NE. by N.	N. 4 55 E.	[illegible]	[illegible] 35 W.
NE.	N. 4 05 E.	[illegible]	[illegible] 20 W.
NE. by E.	N. 3 30 E.	[illegible]	[illegible] 0 W.
E. NE.	N. 3 0 E.	[illegible]	[illegible] 20 W.
E. by N.	N. 3 0 E.	[illegible]	[illegible] 55 W.
E.	N. 3 0 E.	[illegible]	[illegible] 40 W.
E. by S.	N. 3 0 [illegible]	[illegible]	[illegible] 55 W.
E. SE.	N. 3 20 [illegible]	[illegible]	[illegible] 50 W.
SE. by E.	N. 3 40 [illegible]	[illegible]	[illegible] 30 W.
SE.	N. 4 10 [illegible]	[illegible]	[illegible] 55 W.
SE. by S.	N. 4 40 [illegible]	[illegible]	[illegible] 10 W.
S. SE.	N. 5 20 [illegible]	[illegible]	[illegible] 20 W.
S. by E.	N. 6 [illegible]	[illegible]	[illegible] 15 E.
S.	N. 38 [illegible]	[illegible]	[illegible] 45 E.

ern semicircle,

eriment 1; placed
edle and at right
el; nearest point 7
l.
al size.
s —Table 55—should be
west: while they corre-
e said that there *is* a trace
tube when the vessel heads

south, all the side of the tube
magnetism and all toward the
tracts the south end of the needle
rly deviation; as the ship swings
change occurs in the distribution
the tube, the end *m* becoming a red
a blue one; the latter repelling the
edle, and thus producing westerly de-
hip heads north, where the converse of
south are repeated. The fact that the
SW. quarter are less than those in the
the maximum falls at W. by N., is easily

el swings from south toward the westward,
rections of tube and needle constantly change
ner that the distant pole *m* of the tube, which
ole, repels the north end of the needle, thus
the deviations produced by the south pole *t*.
of affairs continues until *m* comes into the pro-
of the needle, which is the case when the vessel
by N. Beyond this point, and until the vessel
rth, the pole *m* is on the other side of the needle,
erefore, aids *t* in its action; hence the difference
n the deviations in the two quadrants. Drawing
s of figures like Fig. 464 on points of the western
cle will elucidate this.

TABLE 56.

Heading of SCORESBY by Compass No. 6216.	Bearing of True Meridian Line on Wall, *without* Tube on Vessel.	Bearing of True Meridian Line on Wall, Tube Placed as in Fig. 466.	Deviations Produced by Rod + d.
(1)	(2)	(3)	(4)
S.	N. 6° 45′ E.	N. 0° 30′ E.	6° 15′ E.
S. by W.	N. 7 30 E.	N. 2 05 E.	5 25 E.
S. SW.	N. 8 05 E.	N. 3 50 E.	4 15 E.
SW. by S.	N. 8 45 E.	N. 5 50 E.	2 55 E.
SW.	N. 9 10 E.	N. 7 40 E.	1 30 E.
SW. by W.	N. 9 40 E.	N. 9 15 E.	0 25 E.
W SW.	N. 10 0 E.	N. 10 20 E.	0 20 W.
W. by S.	N. 10 25 E.	N. 10 40 E.	0 15 W.
W.	N. 10 40 E.	N. 10 20 E.	0 20 E.
W. by N.	N. 10 50 E.	N. 9 20 E.	1 30 E.
W. NW.	N. 10 40 E.	N. 7 35 E.	3 05 E.
NW. by W.	N. 10 30 E.	N 5 50 E.	4 40 E.
NW.	N. 10 10 E.	N. 4 0 E.	6 10 E.
NW. by N.	N. 9 40 E.	N. 2 30 E.	7 10 E.
N. NW.	N. 9 0 E.	N. 1 20 E.	7 40 E.
N. by W.	N. 8 15 E.	N. 0 45 E.	7 30 E.
N.	N. 7 20 E.	N. 0 25 E.	6 55 E.

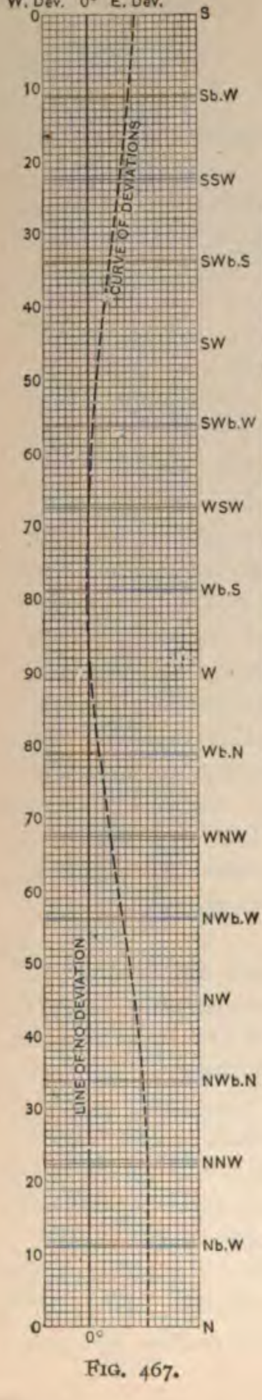


FIG. 467.

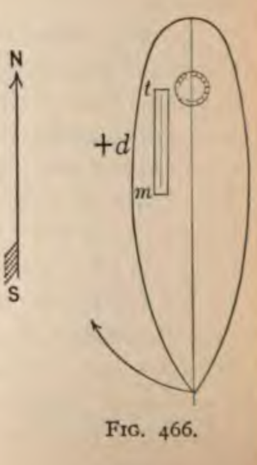


FIG. 466.

. 4: ROD $+d$.

··ing through western semicircle,
..h point.

one used in Experiment 1; placed in plane of needle and parallel to a keel; after-end t of tube abutting through compass-pivot; nearest point s pivot—Fig. 466.

g. 467 made from Table 56 is of natural y, the deviations in the two quadrants etrical, with maxima at north and south, on at west. The differences are due to analogous to those explained in Experi-h the vessel's head south, the distant pole the effect of the near pole t; this continues inging of the ship brings m into a line trans- needle, which occurs near the west point: here ribution of the tube's magnetism occurs, whose on the needle is apparent in the irregularity of ations.

the vessel continuing to swing through the north-quarter, *both* ends of the tube conspire to produce on, whereas they conflicted in the southwest quar-ence the want of symmetry.

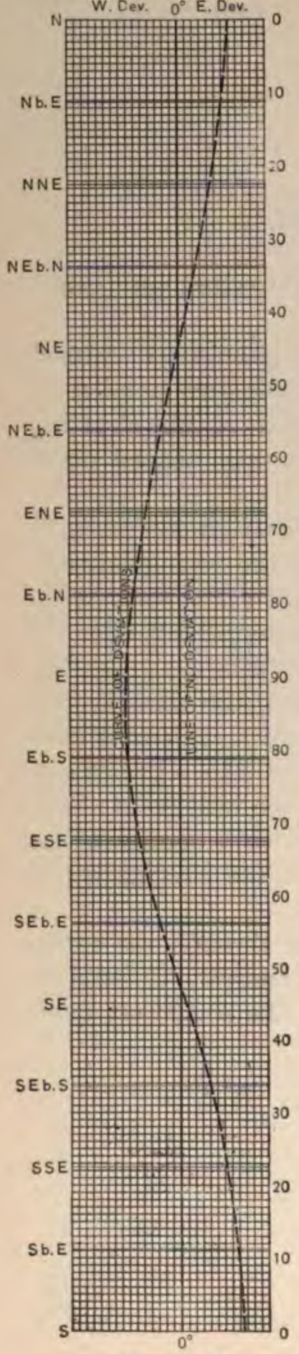


FIG. 469.

EXPERIMENT 5: RESULTANT OF $+b$ AND $+d$.

TABLE 57.

Heading of SCORESBY by Compass No. 6216.	Bearing of True Meridian Line on Wall, *without* Tube on Vessel.	Bearing of True Meridian Line on Wall, Tube Placed as in Fig. 468.	Deviations Produced by Mass of Soft Iron Placed as in Fig. 468.
(1)	(2)	(3)	(4)
N.	N. 7° 20′ E	N. 0° 35′ E.	6° 45′ E
N. by E.	N. 6 20 E.	N. 0 35 E.	5 45 E.
N. NE.	N. 5 40 E.	N. 1 20 E.	4 20 E.
NE. by N.	N. 4 55 E.	N. 2 20 E.	2 35 E
NE.	N. 4 05 E.	N. 3 50 E.	0 15 E.
NE. by E.	N. 3 30 E.	N. 5 50 E.	2 20 W.
E. NE.	N. 3 0 E.	N. 7 30 E.	4 30 W.
E. by N.	N. 3 0 E.	N. 9 10 E.	6 10 W.
E.	N. 3 0 E.	N. 10 05 E.	7 05 W.
E. by S.	N. 3 0 E.	N. 10 20 E.	7 20 W.
E. SE.	N. 3 20 E.	N. 9 20 E.	6 0 W.
SE. by E.	N. 3 40 E.	N. 7 0 E.	3 20 W.
SE.	N. 4 0 E.	N. 4 0 E.	0 0
SE. by S.	N 4 40 E.	N. 0 50 E.	3 50 E.
S. SE.	N. 5 20 E.	N. 1 0 W.	6 20 E.
S. by E.	N. 6 0 E.	N. 1 40 W.	7 40 E.
S.	N. 6 40 E.	N 1 10 W.	7 50 E.

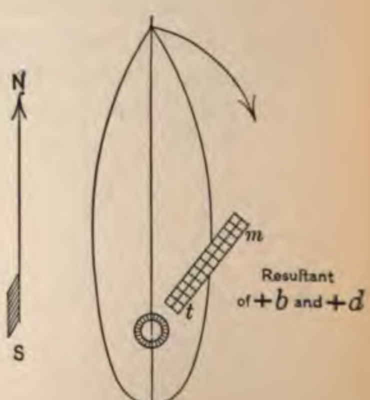


FIG. 468.

EXPERIMENT 5: RESULTANT OF $+b$ AND $+d$.

Vessel upright and swung through eastern semicircle, resting two minutes on each point.

Tube: same 28-inch one used in Experiment 1, and which was also used as $+b$ and $+d$ in Experiments 3 and 4; it was placed horizontally, the axis in plane of needle and inclined 45° to the vertical *plane* through keel; nearest point 7 inches from compass-pivot—Fig. 468.

The curve of Fig. 469 is of natural size; the small deviation at NE., Table 57, is but a trifling difference from the theoretical curve, and the slight want of symmetry on corresponding points is readily accounted for by considering how the distant pole *m* affects the action of the near pole *t* in different parts of the semicircle. Attention is called to the reasoning connected with Figs. 453 to 456—which was prior to the experiment—and how it is borne out by this experiment and those with $+b$ and $+d$ separately: the combination or superposition of the two dissimilar curves of Experiments 3 and 4 could scarcely be more accurately accomplished, with each preserving its characteristic features.

EXPERIMENT 6: CONSTANT DEVIATION.

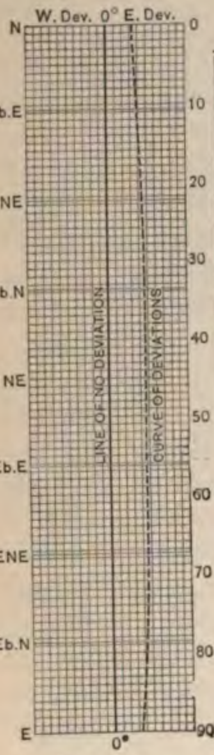


FIG. 471.

TABLE 58.

Heading of SCORESBY by Compass No. 6216.	Bearing of True Meridian Line on Wall, *without* Tubes *b* and *d* on the Vessel.	Bearing of True Meridian Line on Wall, Tubes *b* and *d* Placed as in Fig. 470.	Deviations Produced by Tubes $+b$ and $-d$ Placed as in Fig. 470.
(1)	(2)	(3)	(4)
N.	N. 7° 20′ E.	N. 11° 10′ E.	3° 50′ W.
N. by E.	N. 6 20 E.	N. 10 40 E.	4 20 W.
N. NE.	N. 5 40 E.	N. 10 0 E.	4 20 W.
NE. by N.	N. 4 55 E.	N. 9 40 E.	4 45 W.
NE.	N. 4 05 E.	N. 9 0 E.	4 55 W.
NE. by E.	N. 3 30 E.	N. 8 20 E.	4 50 W.
E. NE.	N. 3 0 E.	N. 7 40 E.	4 40 W.
E. by N.	N. 3 0 E.	N. 7 10 E.	4 10 W.
E.	N. 3 0 E.	N. 6 50 E.	3 50 W.

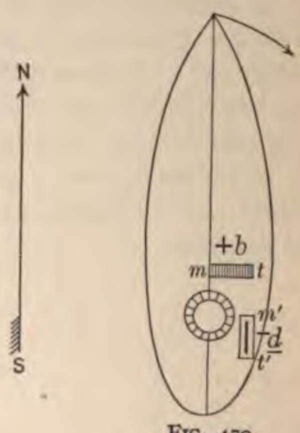


FIG. 470.

EXPERIMENT 6: CONSTANT DEVIATION.

Vessel upright and swung through NE. quadrant, resting two minutes on each point.

The two parts of the 28-inch tube used in Experiment 1 (and also in all subsequent experiments up to the present one) were disconnected and placed as in Fig. 470; that is, tube $+b$ transverse to vertical plane through keel, and tube $-d$ parallel to that plane: both tubes horizontal and their axes in plane through needle; end m of b abutting on vertical plane through keel, and nearest point 7 inches from compass-pivot; end m' of d abutting on transverse vertical plane through compass-pivot, and nearest point 7 inches from pivot.

The deviations of col. (4), Table 58, are sufficiently uniform to prove the correctness of the theory: the differences from strictly equal values may readily be ascribed to the size of the tubes, any little inequality in their form, weight, or quality, or any defect of placing them symmetrically, with reference to the needle. The curve of Fig. 471 is of natural size.

EXPERIMENT 6: CONSTANT DEVIATION.

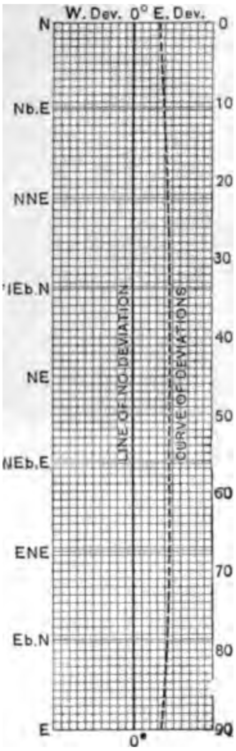
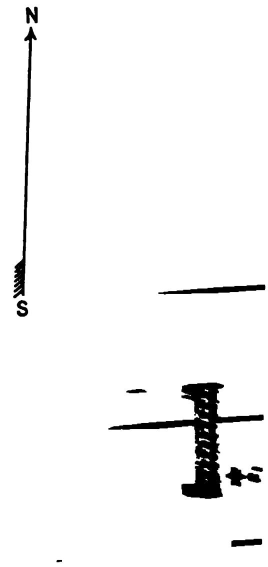


FIG. 471.

TABLE 58.

Heading of SCORESBY by Compass No. 6216.	Bearing of True Meridian Line on Wall, *without* Tubes *b* and *d* on the Vessel.	Bearing of Tru Meridian Lin on Wall, Tub *b* and *d* Plac as in Fig. 47
(1)	(2)	(3)
N.	N. 7° 20′ E.	N. 11° 10′
N. by E.	N. 6 20 E.	N. 10 40
N. NE.	N. 5 40 E.	N. 10
NE. by N.	N. 4 55 E.	N. 9 4
NE.	N. 4 05 E.	N. 9
NE. by E.	N. 3 30 E.	N. 8
E. NE.	N. 3 0 E.	N. 7
E. by N.	N. 3 0 E.	N. 7
E.	N. 3 0 E.	N. 6



EXPERIMENT 7: ROD + c.

Vessel upright and swung through eastern semicircle, resting two minutes on each point.

Tube: one of the 14-inch tubes used in Experiment 6; placed forward of compass, vertically, in vertical plane through keel, upper pole in horizontal plane of needle, and nearest point of tube ten (10) inches from compass-pivot—Fig. 472. This rod represents the smoke-stack, ventilators, stanchions, or any vertical soft iron whose upper pole is near the horizontal plane of the compass.

The curve of Fig. 473 is enlarged twice.

The difference between the deviations on corresponding points of the two quadrants (Table 59) is not readily explained: by raising the tube 14 inches, that is, until the lower pole came into the plane of the needle, it was found that the curve was nearly symmetrical. The observations in the unfavorable position are given, however, to show the *identity of effect* of the rod c and magnet P, as stated in Art. 270.

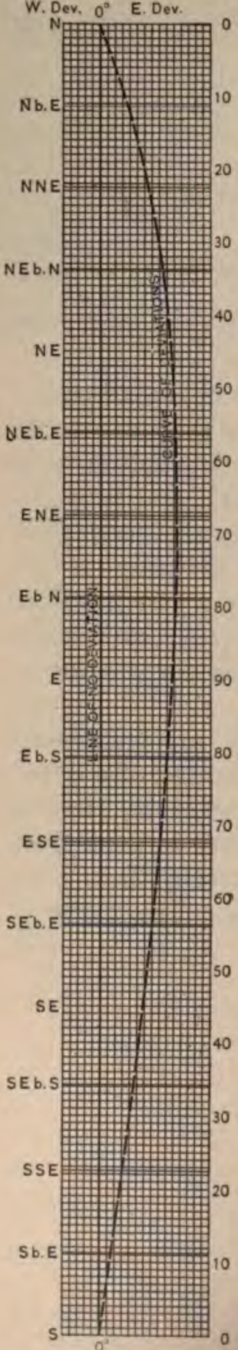


FIG. 473.

TABLE 59.

Heading of SCORESBY by Compass No. 6216.	Bearing of True Meridian Line on Wall, *without* Tube on Vessel.	Bearing of True Meridian Line on Wall, Tube Placed as in Fig. 472.
(1)	(2)	(3)
N.	N. 7° 20′ E.	N. 7° 20′ E.
N. by E.	N. 6 20 E.	N. 4 20 E.
N. NE.	N. 5 40 E.	N. 2 20 E.
NE. by N.	N. 4 55 E.	N. 0 50 E.
NE.	N. 4 05 E.	N. 0 40 W.
NE. by E.	N. 3 30 E.	N. 1 25 W.
E. NE.	N. 3 0 E.	N. 2 0 [illegible]
E. by N.	N. 3 0 E.	N. 2 10 [illegible]
E.	N. 3 0 E.	N. 1 50 [illegible]
E. by S.	N. 3 0 E.	N. 1 25 [illegible]
E. SE.	N. 3 20 E.	N. 0 50 [illegible]
SE. by E.	N. 3 40 E.	N. 0 1[illegible]
SE.	N. 4 0 E.	N. 1 [illegible]
SE. by S.	N. 4 40 E.	N. 2 [illegible]
S. SE.	N. 5 20 E.	N. 4 [illegible]
S. by E.	N. 6 0 E.	N. 5 [illegible]
S.	N. 6 40 E.	N. 6 [illegible]

EXPERIMENT 8: ROD $+e$.

Vessel upright and swung through western semicircle, resting two minutes on each point. Tube: the same one used in Experiment 1, that is, a 28-inch tube, formed by screwing together the two 14-inch tubes of which it was originally composed; placed horizontally, axis in plane of compass-pivot and also in plane transverse to vertical section through keel; wholly to starboard of compass, as rod $+e$, and nearest point 7 inches from compass-pivot—Fig. 474.

The curve of Fig. 475 is made from Table 60 and is enlarged twice; it corresponds beautifully to theory. The rod $+e$ may represent a deck beam cut amidships for a hatch.

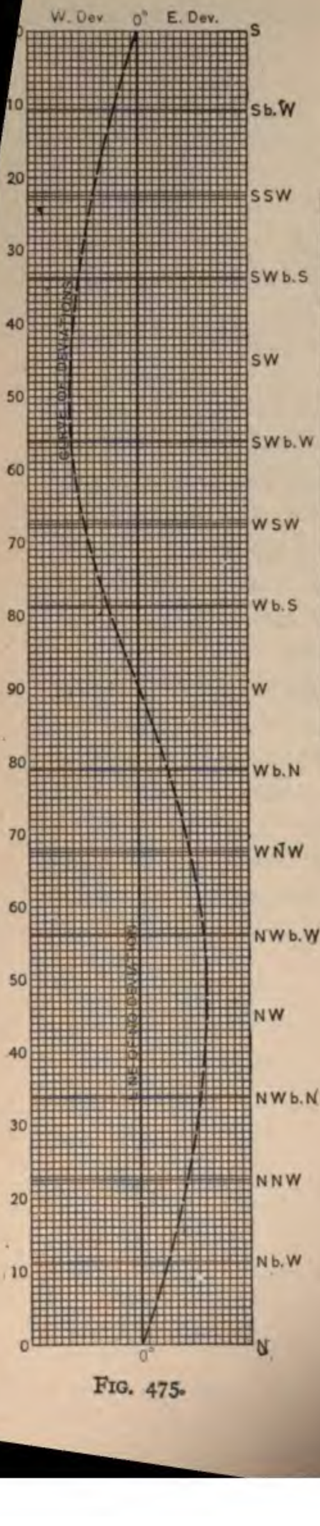


FIG. 475.

TABLE 60.

Heading of SCORESBY by Compass No 6216.	Bearing of True Meridian Line on Wall, *without* Tube on Vessel	Bearing of Meridian L on Wall, T Placed as Fig 474
(1)	(2)	(3)
S.	N. 6° 45′ E.	N. 7° 15
S. by W.	N. 7 30 E.	N. 9 20
S. SW.	N. 8 05 E.	N. 11 20
SW. by S.	N. 8 45 E.	N. 12 50
SW.	N. 9 10 E.	N. 13 50
SW. by W.	N. 9 40 E.	N. 14 10
W. SW.	N. 10 0 E.	N. 13 40
W. by S.	N. 10 25 E.	N. 12 20
W.	N. 10 40 E.	N. 10 40
W. by N.	N. 10 50 E.	N. 8 40
W. NW	N. 10 40 E.	N. 7 0
NW. by W.	N. 10 30 E.	N. 6 0
NW.	N. 10 10 E.	N. 5 40
NW. by N.	N. 9 40 E.	N. 5 [illegible]
N. NW.	N. 9 0 E.	N. 6 [illegible]
N. by W.	N. 8 15 E.	N. 6 [illegible]
N.	N. 7 20 E.	N. 7 [illegible]

EXPERIMENT 9: ROD $-e$.

Vessel upright and swung through eastern semicircle, resting two minutes on each point.

Tube: same one used in Experiment 1, 28 inches long; placed horizontally, at right angles to vertical plane through keel, middle point over compass-pivot; axis of tube 11 inches above plane of needle—Fig. 476. The curve of Fig. 477 is enlarged six times.

As in Experiment 8, the deviations are in strict conformity to theory—Table 61.

This rod represents iron deck beams.

EXPERIMENT 10; ROD—f.

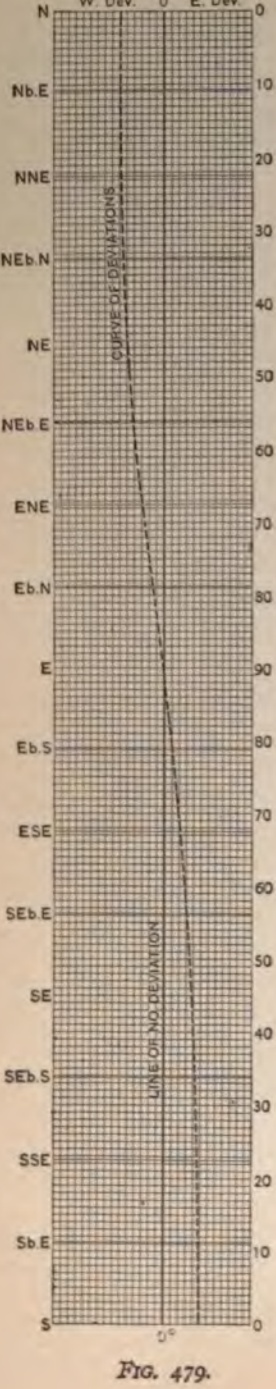


FIG. 479.

TABLE 62.

Heading of SCORESBY by Compass No. 6216.	Bearing of True Meridian Line on Wall, *without* Tube on Vessel.	Bearing of True Meridian Line on Wall, Tube Placed as in Fig. 478.	Deviations Produced by Rod, $-f$ Placed as in Fig. 478.
(1)	(2)	(3)	(4)
N.	N. 7° 20′ E.	N. 13° 0′ E.	5° 40′ W.
N. by E.	N. 6 20 E.	N. 12 20 E.	6 0 W.
N. NE.	N. 5 40 E.	N. 11 10 E.	5 30 W.
NE. by N.	N. 4 55 E.	N. 10 0 E.	5 05 W.
NE.	N. 4 05 E.	N. 8 30 E.	4 25 W.
NE. by E.	N. 3 30 E.	N. 7 10 E.	3 40 W.
E. NE.	N. 3 0 E.	N. 5 30 E.	2 30 W.
E. by N.	N. 3 0 E.	N. 4 10 E.	1 10 W.
E.	N. 3 0 E.	N. 3 0 E.	0 0
E. by S.	N. 3 0 E.	N. 1 50 E.	1 10 E.
E. SE.	N. 3 20 E.	N. 1 10 E.	2 10 E.
SE. by E.	N. 3 40 E.	N. 0 40 E.	3 0 E.
SE.	N. 4 0 E.	N. 0 30 E.	3 30 E.
SE. by S.	N. 4 40 E.	N. 0 25 E.	4 15 E.
S. SE.	N. 5 20 E.	N. 0 40 E.	4 40 E.
S. by E.	N. 6 0 E.	N. 1 10 E.	4 50 E.
S.	N. 6 40 E.	N. 1 50 E.	4 50 E.

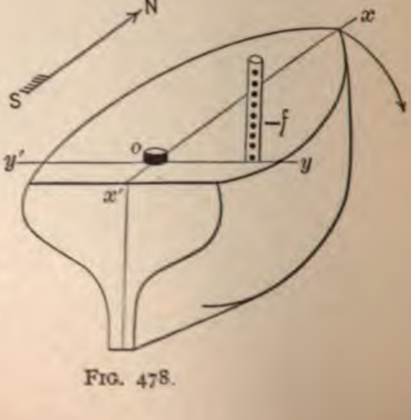


FIG. 478.

EXPERIMENT 10: ROD—*f*.

Vessel upright and swung through eastern semicircle, resting two minutes on each point. Tube: same used in Experiment 1, 28 inches long; placed vertically to starboard, in transverse section through compass-pivot; lower pole in plane of needle, and nearest part of tube ten (10) inches from pivot—Fig. 478.

The curve of Fig. 479, made from Table 62, is of natural size.

This rod exemplifies the effect of boat-davits, or cranes, or generally, any vertical soft iron on one or both sides of the ship.

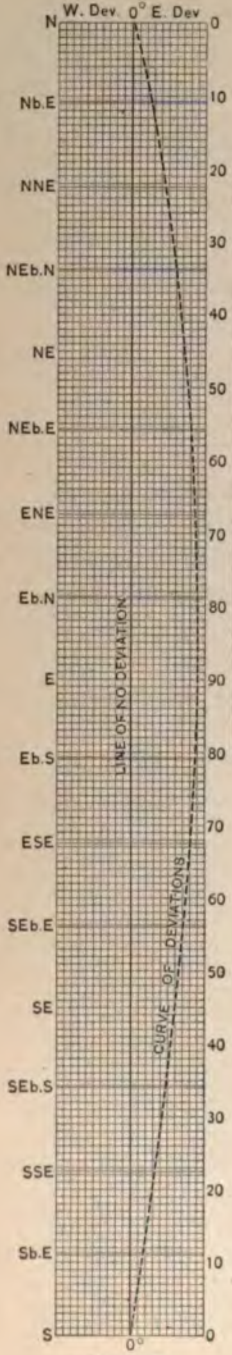


FIG. 481

EXPERIMENT II: STEEL MAGNET $+P$.

TABLE 63.

Heading of SCORESBY by Compass No 6216.	Bearing of True Meridian Line on Wall, *without* Magnet on Vessel	Bearing of True Meridian Line on Wall, *with* Steel Magnet $+P$, Placed as in Fig 480.	Deviations Produced by Steel Magnet $+P$, Placed as in Fig. 480.
(1)	(2)	(3)	(4)
N.	N 7° 20′ E.	N. 7° 20′ E.	0° 0′
N. by E.	N. 6 20 E.	N. 3 20 E.	3 0 E.
N. NE.	N. 5 40 E.	N. 1 30 E.	4 10 E.
NE. by N.	N. 4 55 E.	N. 0 50 W.	5 45 E.
NE.	N. 4 05 E.	N. 2 40 W.	6 45 E.
NE. by E.	N. 3 30 E.	N. 4 05 W.	7 35 E.
E. NE.	N. 3 0 E.	N. 5 20 W.	8 20 E
E. by N.	N. 3 0 E.	N. 6 0 W.	9 0 E.
E.	N. 3 0 E.	N. 5 40 W.	8 40 E.
E. by S.	N. 3 0 E.	N 5 15 W	8 15 E.
E. SE.	N. 3 20 E.	N. 4 20 W.	7 40 E.
SE. by E.	N. 3 40 E.	N. 3 20 W.	7 0 E.
SE.	N. 4 0 E.	N 1 30 W.	5 30 E.
SE. by S	N. 4 40 E.	N. 0 15 E.	4 25 E.
S. SE	N. 5 20 E.	N. 2 20 E.	3 0 E.
S. by E.	N. 6 0 E.	N. 4 30 E.	1 30 E.
S.	N. 6 40 E.	N. 6 40 E.	0 0

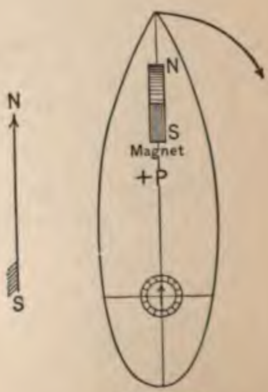


FIG 480.

278. The deviations produced by permanent magnets.—Experiments 11 and 12 were made with hard-steel magnets, well seasoned.

Attention is directed to the identity of the curves of Figs. 473 and 481—the former produced by a soft-iron rod $(+c)$ and the latter by a permanent magnet $(+P)$; thus bearing out the statement of Art. 270—that these dissimilar magnets produce like results: also, in the same article, the analogous case of Q and f, the statement there made in regard to them, and its fulfillment by the curves of Figs. 479 and 483; if f had been placed to port of the compass in Fig. 478, as Q is in Fig. 482, the two curves would be alike in every respect—they are the converse of each other only because they are on opposite sides of the compass.

EXPERIMENT 11: STEEL MAGNET $+P$.

Vessel upright and swung through eastern semicircle, resting two minutes on each point.

The steel magnet used was a solid cylindrical rod, $9\frac{1}{2}$ inches long and $1\frac{1}{4}$ inches in diameter; placed forward of the compass in midship line, axis in horizontal plane through needle, south pole toward compass, and nearest part 33 inches from its pivot—Fig. 480.

The curve of Fig. 481 is of natural size.

This magnet stands for the hard iron of the ship, when blue polarity pervades the forward body and red the after body.

EXPERIMENT 12: STEEL MAGNET + Q.

TABLE 64.

Heading of SCORESBY by Compass No. 6216.	Bearing of True Meridian Line on Wall, *without* Magnet in Place.	Bearing of True Meridian Line on Wall, *with* Steel Magnet, $+Q$ Placed as in Fig. 482.	Deviations Produced by Steel Magnet, $+Q$ Placed as in Fig. 482.
(1)	(2)	(3)	(4)
N.	N. 7° 20′ E.	N. 3° 10′ W.	10° 30′ E.
N. by E.	N. 6 20 E.	N. 3 40 W.	10 0 E.
N. NE.	N. 5 40 E.	N. 3 50 W.	9 30 E.
NE. by N.	N. 4 55 E.	N. 3 40 W.	8 35 E.
NE.	N. 4 05 E.	N. 3 0 W.	7 05 E.
NE. by E.	N. 3 30 E.	N. 1 50 W	5 20 E.
E. NE.	N. 3 0 E	N. 0 30 W.	3 30 E.
E. by N.	N. 3 0 E.	N. 1 40 E	1 20 E
E.	N. 3 0 E.	N. 3 20 E.	0 20 W.
E. by S.	N. 3 0 E.	N. 5 20 E.	2 20 W.
E. SE.	N. 3 20 E.	N. 7 20 E.	4 0 W.
SE. by E.	N. 3 40 E.	N. 9 0 E.	5 20 W.
SE.	N. 4 0 E.	N. 11 0 E.	7 0 W.
SE. by S.	N. 4 40 E.	N. 12 40 E.	8 0 W.
S. SE.	N. 5 20 E.	N. 14 0 E.	8 40 W.
S. by E.	N. 6 0 E.	N. 15 10 E.	9 10 W.
S.	N. 6 40 E.	N. 16 0 E.	9 20 W

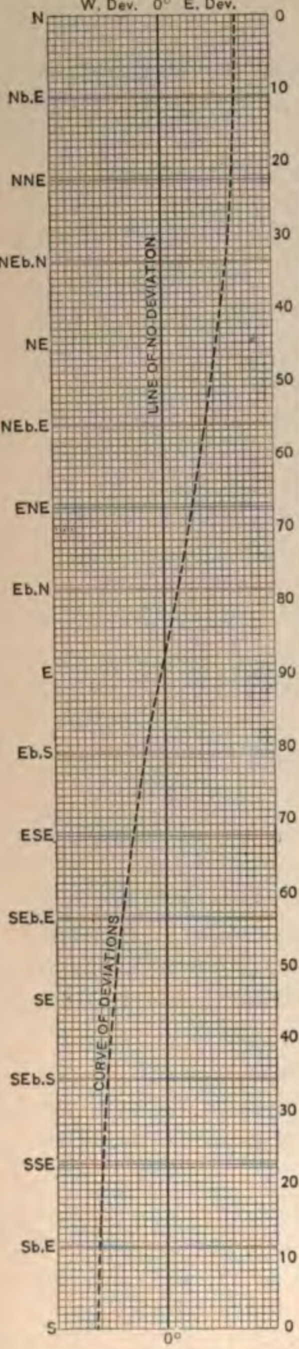


FIG. 483.

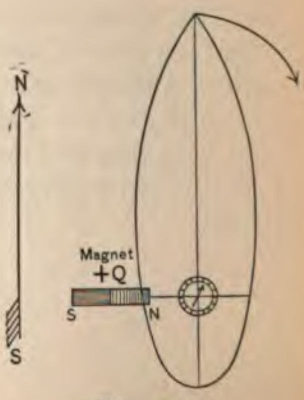


FIG. 482.

EXPERIMENT 12: STEEL MAGNET $+Q$.

Vessel upright and swung through eastern semicircle, resting two minutes on each point. Steel magnet: the same used in Experiment 11; placed in transverse section through compass, on its port side; axis in horizontal plane of needle; north pole toward compass; and nearest part 25 inches from its pivot—Fig. 482.

The curve of Fig. 483, made from Table 64, is of natural size.

This experiment illustrates the action of the hard iron of the ship when pervaded by red polarity on her port side and blue on the starboard.

279. **Components of the heeling error.**—Experiments 13 to 16, inclusive, illustrate the heeling error; they all have the same characteristic—the largest deviations are on northerly and southerly courses, with none on easterly and westerly, as shown by the curves of Figs. 487 to 491: the fact is in marked contrast with the deviations produced by the various rods and magnets with the ship on an even keel, where maxima and minima occur at different points of the circle.

In these four experiments (13 to 16), the compass of previous experiments was replaced by one of the four-needle type in daily use as a standard.

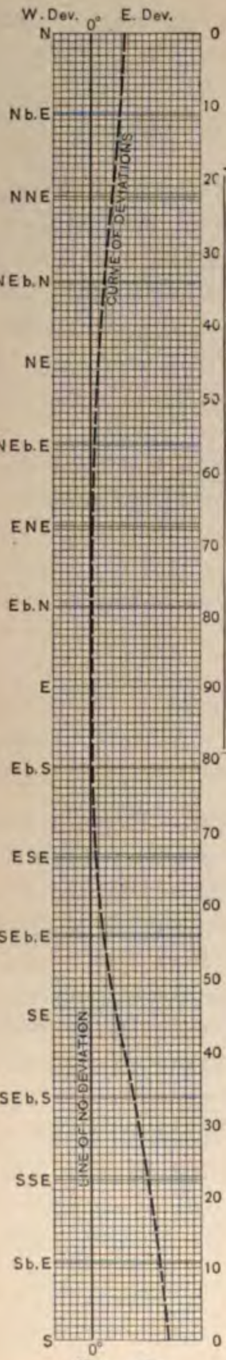


FIG. 485.

TABLE 65.

Heading of SCORESBY by Compass No. 6211.	Bearing of True Meridian and Other Lines Traced on Walls of Room *without* Tube g in Place.	Bearing of True Meridian and Other Lines on Walls of Room, $-g$ Placed as in Fig. 484.	Deviations Produced by $-g$ Placed as in Fig. 484.
(1)	(2)	(3)	(4)
N.	N. 47° 0′ W.	N. 47° 45′ W.	0° 45′ E.
N. by E.	N. 46 45 W.	N. 47 15 W.	0 30 E.
N. NE.	N. 46 15 W.	N. 46 45 W.	0 30 E.
NE. by N.	N. 45 45 W.	N. 45 45 W.	0 0
NE.	N. 45 0 W.	N. 45 0 W.	0 0
NE. by E.	N. 0 0 E.	N. 0 15 E.	0 15 W.
E. NE.	N. 0 15 E.	N. 0 45 E.	0 30 W.
E. by N.	N. 1 0 E.	N. 1 15 E.	0 15 W.
E.	N. 1 30 E.	N. 1 45 E.	0 15 W.
E. by S.	N. 2 15 E.	N. 2 30 E.	0 15 W.
E. SE.	N. 37 30 E.	N. 37 15 E.	0 15 E.
SE. by E.	N. 37 30 E.	N. 37 15 E.	0 15 E.
SE.	N. 37 45 E.	N. 36 30 E.	1 15 E.
SE. by S.	N. 46 0 E.	N. 45 15 E.	0 45 E.
S. SE.	S. 34 45 E.	S. 36 0 E.	1 15 E.
S. by E.	S. 35 45 E.	S. 37 30 E.	1 45 E.
S.	S. 36 15 E.	S. 38 0 E.	1 45 E.

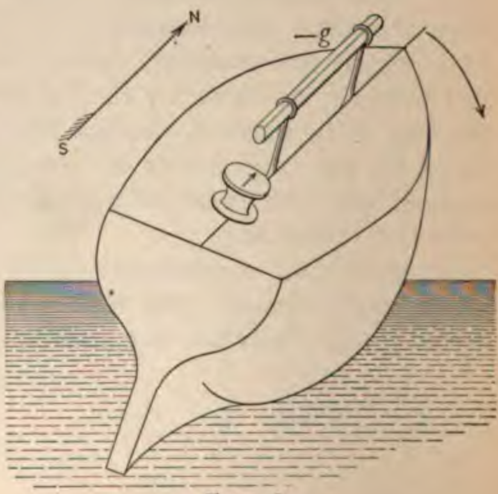


FIG. 484.

EXPERIMENT 13: ROD $-g$.

The vessel was heeled 20° to starboard during the observations of cols. (2) and (3) of Table 65, and swung through the eastern semicircle, resting two minutes on each point. Tube: the same used in Experiment 1, 28 inches long, placed with its axis in the longitudinal section through keel and parallel to latter; above plane of card and forward of compass, nearest end 13 inches from compass-pivot—Fig. 484.

The curve of Fig. 485 is enlarged six times.

The induced magnetism of the tube and its leverage are at their best when the ship heads north or south.

Heading north, heeled to starboard, and tube placed as above described, the end of the tube nearest the compass has blue magnetism and attracts the north end of the card to the low side, producing easterly deviation; as the vessel swings into the northeast quarter, the magnetism of the tube is distributed over its sides, the leverage lessens, and both become ineffectual at east; the swinging continuing, the *ends* of the tube again acquire opposite polarity, that nearest the compass this time being red and attracting the south end of the card to the low side, thus producing easterly deviation as before: in the observations of Table 65 this general procedure is apparent; the irregularities and discrepancies are due to instrumental and other unavoidable errors, the smallness of the quantities to be measured, and the difficulties of making the observations.

The rod g, if below, would represent the effect of a propeller shaft or iron keel.

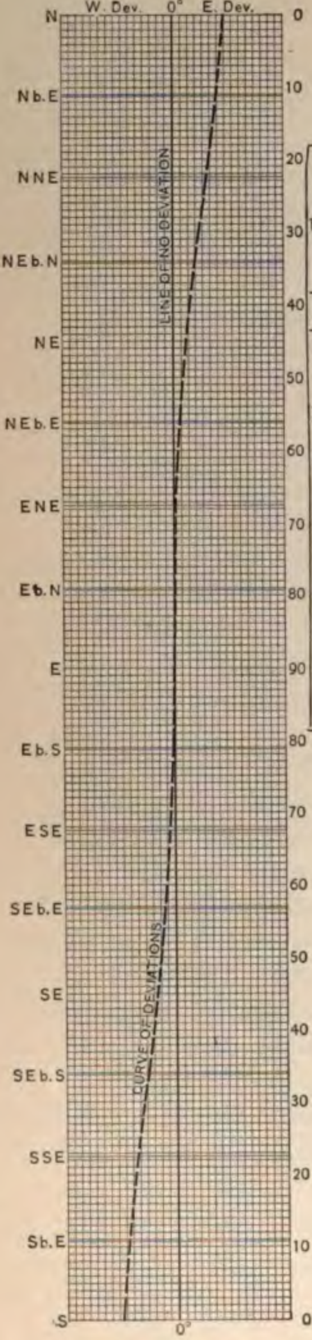


FIG. 487.

EXPERIMENT 14: ROD + h.

TABLE 66.

Heading of SCORESBY by Compass No. 6211.	Bearing of True Meridian and Other Lines Traced on Walls of Room, *without* Tube h in Place.	Bearing of True Meridian and Other Lines on Walls of Room, + h Placed as in Fig. 486.	Deviations Produced by + h Placed as in Fig. 486.
(1)	(2)	(3)	(4)
N.	N. 47° 0′ W.	N. 48° 15′ W.	1° 15′ E.
N. by E.	N. 46 45 W.	N. 47 45 W.	1 0 E.
N. NE.	N. 46 15 W.	N. 47 0 W.	0 45 E.
NE. by N.	N. 45 45 W.	N. 46 15 W.	0 30 E.
NE.	N. 0 15 W.	N. 0 30 W.	0 15 E.
NE. by E.	N. 0 0 W.	N. 0 15 W.	0 15 E.
E. NE.	N. 0 15 E.	N. 0 0 W.	0 15 E.
E. by N.	N. 1 0 E.	N. 0 45 E.	0 15 E.
E.	N. 42 0 W.	N. 42 0 W.	0 0
E. by S.	N. 2 15 E.	N. 2 15 E.	0 0
E SE.	N. 37 0 E.	N. 37 15 E.	0 15 W.
SE. by E	N. 3 45 E.	N 4 0 E.	0 15 W.
SE.	N. 37 45 E.	N. 38 0 E.	0 15 W.
SE. by S.	N. 46 0 E	N. 46 15 E.	0 15 W.
S. SE.	S. 34 45 E.	S. 34 0 E.	0 45 W.
S. by E.	S. 35 45 E.	S. 34 45 E.	1 0 W.
S.	S. 36 15 E	S. 35 0 E	1 15 W.

FIG. 486.

EXPERIMENT 14: ROD $+h$.

Vessel heeled 20° to starboard and swung through eastern semicircle, resting two minutes on each point.

Tube: same used in Experiment 1, 28 inches long; placed as in Fig. 486, its axis in section transverse to keel through compass-pivot; parallel to plane of deck; above, and on port side of compass, nearest end 13 inches from its pivot. The tube is now magnetized by a component of the Earth's total intensity that makes an angle with the horizon, the lower end of the tube—that nearest the compass—becoming a red pole, and the upper a blue.

With the ship heading north, the north point of the card will be repelled, and easterly deviation produced; as the ship swings into the northeast quarter, the leverage of the tube's magnetic force becomes less, and at east fails entirely of effect, as it is then in line with the needle; the swinging continuing, the leverage of the tube's magnetic force increases—the tube is now to the eastward of the compass—still repelling the north point of the card and there is westerly deviation: all which is shown in Table 66, and illustrated by Fig. 487, where the curve is enlarged six times.

Were the rod h below the compass, it would represent a deck beam cut for a hatchway.

TABLE 67.

Heading of SCORESBY by Compass No. 7860.	Bearing of True Meridian and Other Lines on Walls of Room, *without* Tube k in Place.	Bearing of True Meridian and Other Lines on Walls of Room, $+k$ Placed as in Fig. 488.	Deviations Produced by $+k$ Placed as in Fig. 488.
(1)	(2)	(3)	(4)
N.	N. 1° 30′ E.	N. 5° 0′ E.	3° 30′ W.
N. by E.	N. 41 15 E.	N. 44 15 E.	3 0 W.
N. NE.	N. 40 15 E.	N. 43 15 E.	3 0 W.
NE. by N.	N. 39 15 E.	N. 42 0 E.	2 45 W.
NE.	N. 38 15 E.	N. 40 45 E.	2 30 W.
NE. by E.	S. 62 0 W.	S. 63 45 W.	1 45 W.
E. NE.	S. 63 0 W.	S. 64 0 W.	1 0 W.
E. by N.	S. 24 45 E.	S. 24 0 E.	0 45 W.
E.	S. 25 30 E.	S. 25 45 E.	0 15 E.
E. by S.	S. 27 0 E.	S. 28 0 E.	1 0 E.
E. SE.	S. 28 15 E.	S. 30 0 E.	1 45 E.
SE. by E.	N. 40 0 W.	N. 42 0 W.	2 0 E.
SE.	S. 31 30 E.	S. 34 30 E.	3 0 E.
SE. by S.	S. 61 45 W.	S. 58 45 W.	3 0 E.
S. SE.	S. 60 15 W.	S. 57 0 W.	3 15 E.
S. by E.	S. 58 45 W.	S. 55 30 W.	3 15 E.
S.	S. 57 15 W.	S. 53 30 W.	3 45 E.

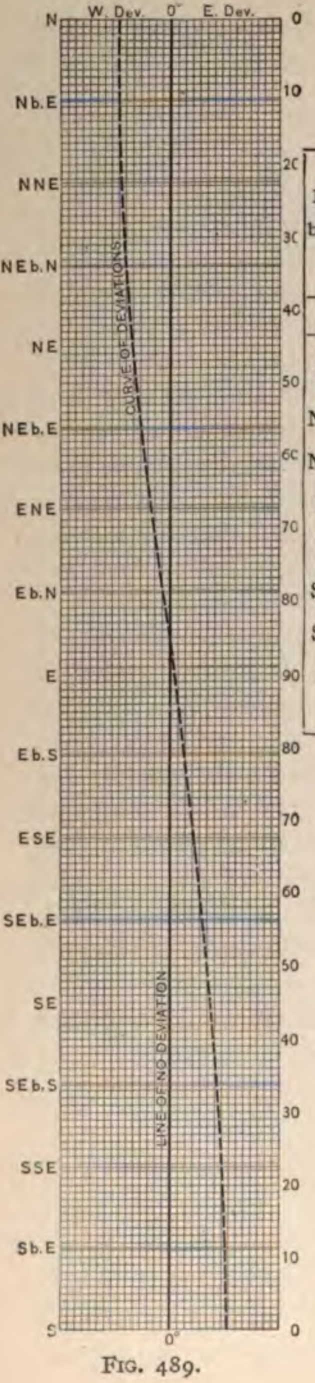


FIG. 489.

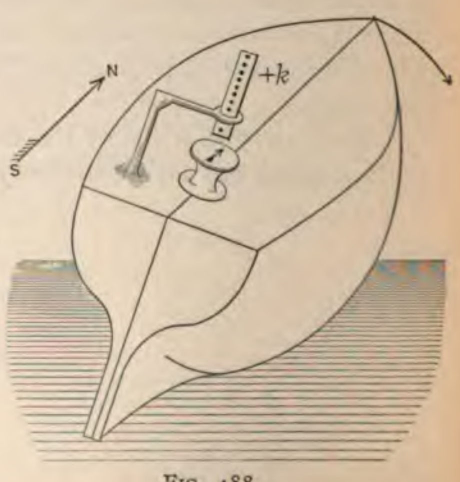


FIG. 488.

EXPERIMENT 15: ROD $+k$.

Vessel heeled 20° to starboard and swung through eastern semicircle, resting two minutes on each point.

Tube: the same used in Experiment 1, 28 inches long; placed as in Fig. 488, entirely above the compass; axis in prolongation of compass-pivot; nearest end 13 inches from that pivot.

In this case the magnetism of the tube is due to a component of the Earth's total intensity that is inclined to the vertical, the lower end of the tube being a red pole, and the upper a blue: the maximum leverage of the magnetic force occurs with the ship's head north or south, gradually decreasing to zero at east and west. When the ship's head is north, the lower pole of the rod is thrown by the heeling to the eastward of the needle, and repels the north end of the needle, producing westerly deviation: this continues, with gradually decreasing effect, as the ship swings through the northeast quarter, until she heads east, when the rod being in a plane through the needle, its effect is zero. The vessel continuing to swing through the southeast quarter, the lower end of the rod passes on the other side of the needle, and again repels its north end, producing easterly deviation, which gradually increases to a maximum at south—Table 67.

The curve of deviations, Fig. 489, which is enlarged twice, is almost perfectly symmetrical.

Were k below the compass, its blue pole would act upon the needle, causing deviation to the high side.

A rod, $-k$, would cause deviation to the low side.

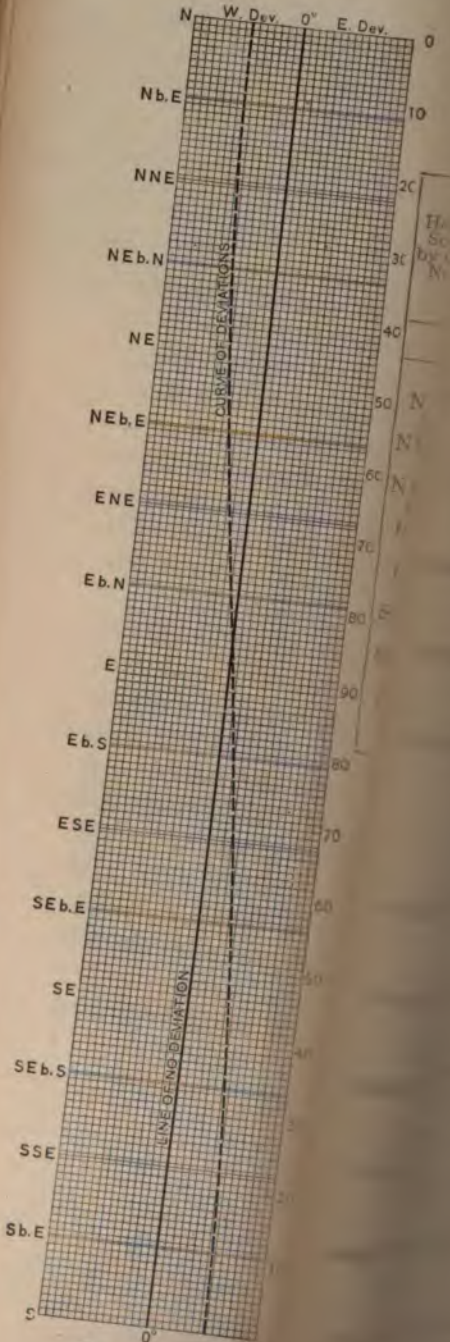


FIG. 489.

TABLE 68.

[illegible]	Bearing of True Meridian and Other Lines on Walls of Room, with Magnet R Placed as in Fig. 490.	Deviations Produced by Magnet R Placed as in Fig. 490.
[illegible]	(1)	(2)
[illegible] E.	N. 3° 0′ W.	4° 30′ E.
[illegible] E.	N. 37 0 E.	4 15 E.
[illegible] E.	N. 36 0 E.	4 15 E.
[illegible] E.	N. 35 30 E.	3 45 E.
[illegible] E.	N. 35 30 E.	2 45 E.
[illegible] 0 W.	S. 60 15 W.	1 45 E.
[illegible] 0 W.	S. 62 0 W.	1 0 E.
[illegible] 45 E.	S. 25 0 E.	0 15 E.
[illegible] 30 E.	S. 25 15 E.	0 15 W.
[illegible] 0 E.	S. 25 30 E.	1 30 W.
[illegible] 15 E.	S. 26 0 E.	2 15 W.
[illegible] 0 W.	N. 37 30 W.	2 30 W.
[illegible] 30 E.	S. 28 0 E.	3 30 W.
[illegible] 45 W.	S. 66 0 W.	4 15 W.
[illegible] 15 W.	S. 65 15 W.	5 0 W.
[illegible] 45 W.	S. 64 0 W.	5 15 W.
[illegible] 15 W.	S. 62 30 W.	5 15 W.

EXPERIMENT 16: STEEL MAGNET *R.*

Vessel heeled 20° to starboard and swung through tern semicircle, resting two minutes on each point.

Magnet, a short steel tube which had been made a manent magnet by an electric current several months viously: placed entirely above the compass, axis coinent with prolongation of compass-pivot; south pole rest compass, and 6 inches from it—Fig. 490.

The difference in the deviations on corresponding points the two quadrants, Table 68, is probably due to a want exact coincidence of the axis of the magnet with the s of the compass. The curve of Fig. 491 is enlarged ce.

It will be observed that the needle is drawn to the low e in both quadrants: were the magnet below the comss—south pole uppermost—the needle would be drawn the high side in both quadrants; and the magnet in this se would represent the action of the hard iron of a vessel ose after body was pervaded by blue polarity and forward dy by red polarity—the characteristic features of a ship ilt head north magnetic.

280. Experimental compensation of the deviations.—le *principle* underlying compensation of the deviations very simple and will be illustrated by the following three ses: in each, the deviation was first produced by a disrbing magnet—then the SCORESBY was swung for a ble of Deviations—and finally the compensating marials were applied. There are different methods of perming the latter operation which will be described in rt Fifth, but the principle is the same in all, and it is *t* which is sought to be made clear.

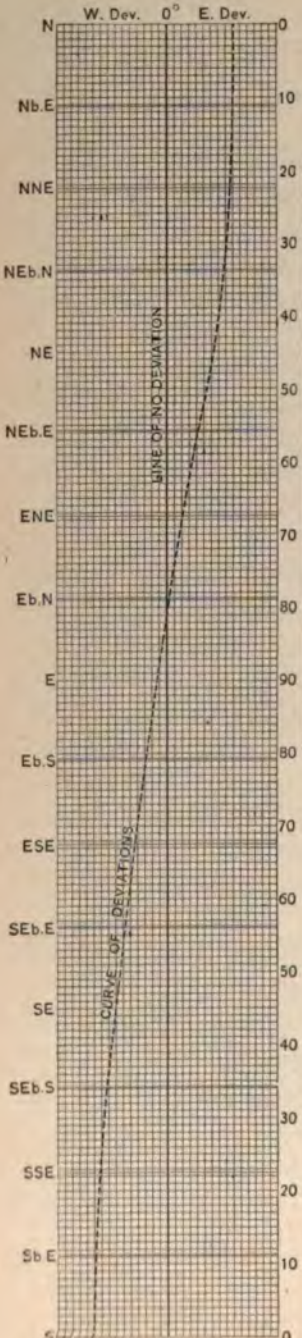


FIG. 491

EXPERIMENT 16: STEEL MAGNET *R*.

TABLE 68.

Heading of SCORESBY by Compass No 7860.	Bearing of True Meridian and Other Lines on Walls of Room, *without* Magnet *R* in Place.	Bearing of True Meridian and Other Lines on Walls of Room, *with* Magnet *R* Placed as in Fig. 490	Deviations Produced by Magnet *R*, Placed as in Fig. 490.
(1)	(2)	(3)	(4)
N.	N. 1° 30′ E.	N. 3° 0′ W.	4° 30′ E.
N. by E.	N. 41 15 E.	N. 37 0 E.	4 15 E.
N. NE.	N. 40 15 E.	N. 36 0 E.	4 15 E.
NE. by N.	N. 39 15 E.	N. 35 30 E.	3 45 E.
NE.	N. 38 15 E.	N. 35 30 E.	2 45 E.
NE. by E.	S. 62 0 W.	S. 60 15 W.	1 45 E.
E. NE.	S. 63 0 W.	S. 62 0 W.	1 0 E.
E. by N.	S. 24 45 E.	S. 25 0 E.	0 15 E.
E.	S. 25 30 E.	S. 25 15 E.	0 15 W.
E. by S.	S. 27 0 E.	S. 25 30 E.	1 30 W.
E. SE.	S. 28 15 E.	S. 26 0 E.	2 15 W.
SE. by E.	N. 40 0 W.	N. 37 30 W.	2 30 W.
SE.	S. 31 30 E.	S. 28 0 E.	3 30 W.
SE. by S.	S. 61 45 W.	S. 66 0 W.	4 15 W.
S. SE.	S. 60 15 W.	S. 65 15 W.	5 0 W.
S. by E.	S. 58 45 W.	S. 64 0 W.	5 15 W.
S.	S. 57 15 W.	S. 62 30 W.	5 15 W.

FIG. 490

EXPERIMENT 16: STEEL MAGNET *R*.

Vessel heeled 20° to starboard and swung through eastern semicircle, resting two minutes on each point.

Magnet, a short steel tube which had been made a permanent magnet by an electric current several months previously: placed entirely above the compass, axis coincident with prolongation of compass-pivot; south pole nearest compass, and 6 inches from it—Fig. 490.

The difference in the deviations on corresponding points in the two quadrants, Table 68, is probably due to a want of exact coincidence of the axis of the magnet with the axis of the compass. The curve of Fig. 491 is enlarged twice.

It will be observed that the needle is drawn to the low side in both quadrants: were the magnet below the compass—south pole uppermost—the needle would be drawn to the high side in both quadrants; and the magnet in this case would represent the action of the hard iron of a vessel whose after body was pervaded by blue polarity and forward body by red polarity—the characteristic features of a ship built head north magnetic.

280. Experimental compensation of the deviations.— The *principle* underlying compensation of the deviations is very simple and will be illustrated by the following three cases: in each, the deviation was first produced by a disturbing magnet—then the SCORESBY was swung for a Table of Deviations—and finally the compensating materials were applied. There are different methods of performing the latter operation which will be described in Part Fifth, but the principle is the same in all, and it is *that* which is sought to be made clear.

EXPERIMENT 17: THE SEMICIRCULAR DEVIATION PRODUCED AND COMPENSATED.

The compass used in this experiment was a liquid four-needle type, No. 6211—in every particular like No. 7860, described in Fig. 402, Plate *P*.

All iron, magnets, and other disturbing materials were first removed from proximity of the SCORESBY, and the vessel swung until the pointer on the bow successively indicated equal (true) arcs of the brass circle on the floor, col. (1), Table 69. The corresponding heading of the vessel by compass was observed and recorded, col. (2). A steel magnet, 9½ inches long and 1¼ inches in diameter, was then placed horizontally in the plane of the compass needles, in a line bearing 45° on starboard bow, south pole directed toward compass, and 20 inches from its pivot—Fig. 492. The vessel was again swung through 360°, resting two minutes on each division of 20° of the brass circle on the floor, and the heading of the vessel by compass was observed, col. (3), Table 69. A comparison of cols. (2) and (3) gave the deviations, col. (4). Sixteen thin bar magnets, eight 11¼ inches long, and eight 8½ inches long, were now placed below the compass, and 29 inches from the plane of the needles, their length being in a vertical plane making an angle of 45° with the vertical plane through the keel, north poles all turned toward the "disturbing magnet"—Fig. 492. The vessel was then swung through 360°, resting two minutes on each division of 20° of the brass circle, and the heading of the SCORESBY noted and recorded, col. (5), Table 69.

The residual deviations are given in col. (6).

The system of "compensating-magnets" was then raised 1½ inches, without turning them in azimuth, the vessel swung as before, col. (7), and a final series of residuals

obtained, col. (8); the maximum attaining only 1°, the compass was deemed compensated.

The curves of the different deviations—without com-

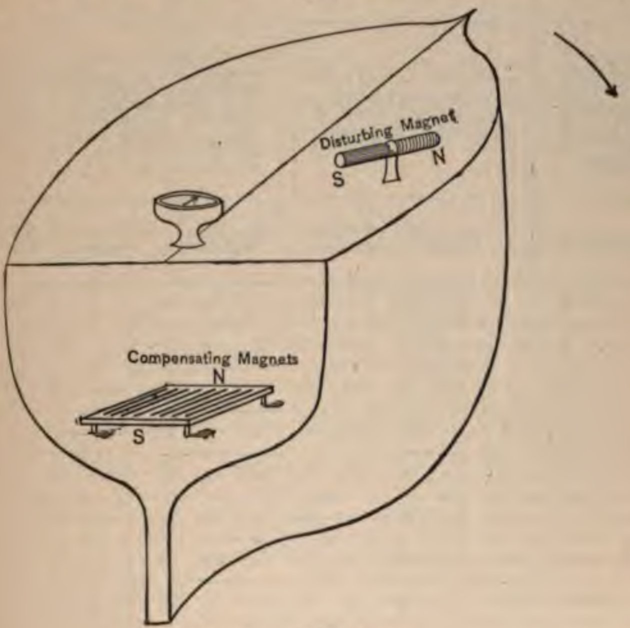


FIG. 492.—Experiment 17.

pensation at all (black); partially compensated (green); and finally compensated (red)—are given in Fig. 493.

Before placing the "*compensating* magnets"—in the northern semicircle, from east by way of north—the compass card was very steady and came quickly to rest, because the south pole of the *disturbing magnet* was attracting the north pole of the compass system; also, for equal true arcs of the circle, col. (1), Table 69, the corresponding compass arcs are very much less, besides being variable—see column headed "Differences" in col. (3); in the southern semicircle, the compass card was unsteady and required some time to come to rest, because the south pole of the

Table 69.

EXPERIMENT 17.

(1)	(2)		(3)		(4)
Heading of SCORESBY by brass circle on floor, *without* disturbing or compensating magnets on vessel.	Corresponding reading of compass No. 6211; *no* magnets on SCORESBY.		Corresponding reading of compass No. 6211, *with* disturbing magnet in "starboard angle" of 45°, south pole nearest compass, magnet horizontal and axis in plane of needles. Fig. 492.		Deviations produced by disturbing magnet = Difference of columns (2) and (3).
Headings, True.	Headings, Magnetic.	Differences.	Headings by Compass.	Differences.	Deviations.
N. 0° W.	N. 5° E.	20°	N. 11° W.	14°	16° E.
N. 20 W.	N. 15 W.	21	N. 25 W.	14	10 E.
N. 40 W.	N. 36 W.	21	N. 39 W.	14	3 E.
N. 60 W.	N. 57 W.	20	N. 53 W.	14	4 W.
N. 80 W.	N. 77 W.	20	N. 67 W.	14	10 W.
N. 100 W.	S. 83 W.	20	N. 81 W.	15	16 W.
N. 120 W.	S. 63 W.	19	S. 84 W.	16	21 W.
N. 140 W.	S. 44 W.	20	S. 68 W.	17	24 W.
N. 160 W.	S. 24 W.	19	S. 51 W.	20	27 W.
N. 180 W.	S. 5 W.	20	S. 31 W.	25	26 W.
N. 200 W.	S. 15 E.	19	S. 6 W.	31	21 W.
N. 220 W.	S. 34 E.	20	S. 25 E.	36	9 W.
N. 240 W.	S. 54 E.	20	S. 61 E.	33	7 E.
N. 260 W.	S. 74 E.	20	N. 86 E.	27	20 E.
N. 280 W.	N. 86 E.	20	N. 59 E.	21	27 E.
N. 300 W.	N. 66 E.	20	N. 38 E.	18	28 E.
N. 320 W.	N. 46 E.	20	N. 20 E.	16	26 E.
N. 340 W.	N. 26 E.	21	N. 4 E.	15	22 E.
N. 0 W.	N. 5 E.		N. 11 W.		16 E.

TABLE 69—*Continued.*

EXPERIMENT 17.

(5)		(6)	(7)		(8)
Reading of compass No. 6211, with compensating magnets placed below the plane of compass. Fig. 492.		Residual deviations after compensation in column (5).	Reading of compass No. 6211, after raising the system of compensating magnets of column (5) 1½ inches, vertically, *without* changing the direction of their starboard angle.		Final residual deviations after complete compensation = Difference of columns (2) and (7).
Headings by Compass.	Differences.	Deviations.	Headings by Compass.	Differences.	Deviations.
N. 4° E.		1° E.	N. 4° 30′ E.		0° 30′ E.
	18°			20° 30′	
N. 14 W.		1 W.	N. 16 0 W.		1 0 E.
	19			21 0	
N. 33 W.		3 W.	N. 37 0 W.		1 0 E.
	19			19 30	
N. 52 W.		5 W.	N. 57 30 W.		0 30 E.
	20			20 30	
N. 72 W.		5 W.	N. 78 0 W.		1 0 E.
	19			19 30	
S. 89 W.		6 W.	S. 82 30 W.		0 30 E.
	21			20 0	
S. 68 W.		5 W.	S. 62 30 W.		0 30 E.
	20			21 0	
S. 48 W.		4 W.	S. 43 30 W.		0 30 E.
	21			19 0	
S. 27 W.		3 W.	S. 24 0 W.		0 0
	22			19 0	
S. 5 W.		0	S. 5 0 W.		0 0
	21			19 30	
S. 16 E.		1 E.	S. 14 30 E.		0 30 W.
	22			19 0	
S. 38 E.		4 E.	S. 33 30 E.		0 30 W.
	21			19 30	
S. 59 E		5 E.	S. 53 0 E.		1 0 W.
	20			20 0	
S. 79 E.		5 E.	S. 73 0 E.		1 0 W.
	20			20 0	
N. 81 E.		5 E.	N. 87 0 E.		1 0 W.
	20			20 30	
N. 61 E.		5 E.	N. 66 30 E.		0 30 W.
	19			20 30	
N. 42 E.		4 E.	N. 46 0 E.		0 0
	19			20 30	
N. 23 E.		3 E.	N. 25 30 E.		0 30 W.
	19			21 0	
N. 4 E		1 E.	N. 4 30 E.		0 30 W.

... the south pole of the com-
... arcs of the circle, col. (1),
... are very much larger and
... "Differences" in col. (3).
... and compass arcs before
... compensation—column
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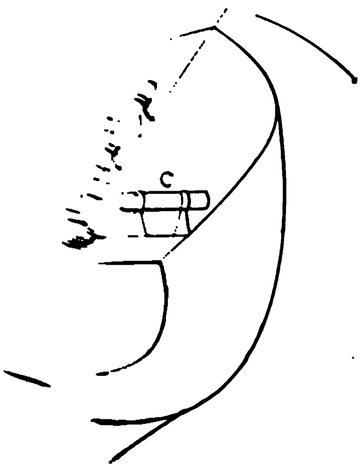


... Experiment 18.

... iron tubes—both disturbing (D)
... Table 70 gives details of the obser-
... their graphical representation.
... after compensation show that
... accurately done as it should be in
... for illustrative purposes, which
...

EXPERIMENT 19: THE HEELING ERROR PRODUCED AND COMPENSATED.

Compass No. 6211, which was in every respect like No. 7860, previously described, having been placed on the SCORESBY, the vessel, while upright, was swung until the pointer on the bow successively pointed to every 40° division of the brass circle on the floor, and the corresponding headings by the compass and circle were noted and recorded, cols. (1) and (2), Table 71.

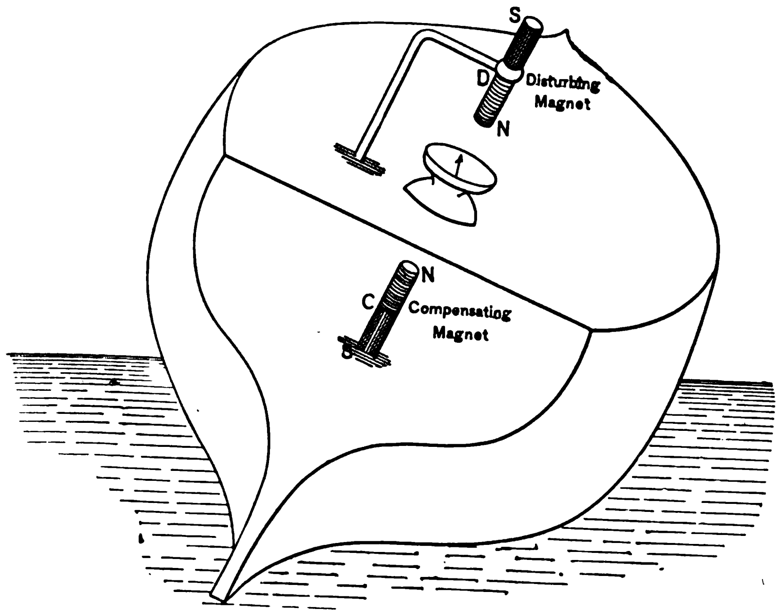


FIG. 496.—Experiment 19.

A solid steel cylindrical magnet (the disturbing magnet *D* of Fig. 496), 9½ inches long and 1½ inches in diameter, was then placed above the compass, its axis as nearly as possible in the prolongation of the compass-pivot, and

Table 70.

EXPERIMENT 18.

(1)	(2)		(3)	
Heading of Scoresby by brass circle on floor; no soft iron acting on compass.	Corresponding reading of compass No. 6211; no soft iron on Scoresby.		Corresponding reading of compass No. 6211, with soft-iron tube placed forward of compass, axis horizontal, in vertical plane through keel, and in plane of needles; nearest end ten inches from pivot.	
Headings, True.	Headings, Magnetic.	Differences.	Headings, by Compass.	Differences.
N. 0° W.	N. 5° 0′ E.	° 20.5	N. 4° 30′ E.	° 17.5
N. 20 W.	N. 15 30 W.	20.5	N. 13 0 W.	17.5
N. 40 W.	N. 36 0 W.	20.5	N. 30 30 W.	20.0
N. 60 W.	N. 56 30 W.	20.5	N. 50 30 W.	23.0
N. 80 W.	N. 77 0 W.	20.0	N. 73 30 W.	25.5
N. 100 W.	S. 83 0 W.	20.0	S. 81 0 W.	24.4
N. 120 W.	S. 63 0 W.	19.5	S. 57 0 W.	20.5
N. 140 W.	S. 43 30 W.	19.5	S. 36 30 W.	17.0
N. 160 W.	S. 24 0 W.	19.0	S. 19 30 W.	15.5
N. 180 W.	S. 5 0 W.	20.0	S. 4 0 W.	15.5
N. 200 W.	S. 15 0 E.	19.5	S. 11 30 E.	16.5
N. 220 W.	S. 34 30 E.	19.5	S. 28 0 E.	18.0
N. 240 W.	S. 54 0 E.	20.0	S. 46 0 E.	21.0
N. 260 W.	S. 74 0 E.	20.0	S. 67 0 E.	25.0
N. 280 W.	N. 86 0 E.	20.0	N. 88 0 E.	25.0
N. 300 W.	N. 66 0 E.	20.0	N. 63 0 E.	22.5
N. 320 W.	N. 46 0 E.	20.5	N. 40 30 E.	18.5
N. 340 W.	N. 25 30 E.	20.5	N. 22 0 E.	17.5
N. 360 W.	N. 5 0 E.		N. 4 30 E.	

TABLE 70—*Continued.*

EXPERIMENT 18.

(4)	(5)		(6)
Deviations produced by soft-iron tube = Difference of columns (2) and (3).	Reading of compass No. 6211 after deviations of column (4) were partly compensated by a soft-iron tube similar to the one used for causing deviations in transverse plane through pivot of compass on starboard side.		Residual deviations after compensation of the quadrantal deviation.
Deviations.	Headings, by Compass.	Differences.	Deviations.
0° 30′ E.	N. 5° 30′ E.	° 20.5	0° 30′ W.
2 30 W.	N. 15 0 W.	20.5	0 30 W.
5 30 W.	N. 35 30 W.	20.5	0 30 W.
6 0 W.	N. 56 0 W.	21.0	0 30 W.
3 30 W.	N. 77 0 W.	21.0	0 0
2 0 E.	S. 82 0 W.	20.5	1 0 E.
6 0 E.	S. 61 30 W.	19.5	1 30 E.
7 0 E.	S. 42 0 W.	18.5	1 30 E.
4 30 E.	S. 23 30 W.	19.0	0 30 E.
1 0 E.	S. 4 30 W.	19.0	0 30 E.
3 30 W.	S. 14 30 E.	19.0	0 30 W.
5 30 W.	S. 33 30 E.	19.0	1 0 W.
8 0 W.	S. 52 30 E.	19.5	1 30 W.
7 0 W.	S. 72 0 E.	20.0	2 0 W.
2 0 W.	N. 88 0 E.	20.0	2 0 W.
3 0 E.	N. 68 0 E.	21.0	2 0 W.
5 30 E.	N. 47 0 E.	19.5	1 0 W.
3 30 E.	N. 27 30 E.	20.5	2 0 W.
0 30 E.	N. 7 0 E.		2 0 W.

TABLE 71—*Continued.*

EXPERIMENT 19.

Heading by compass No. 6211; vessel heeled 20° to starboard, corresponding to headings by brass circle in column (1); disturbing magnet above compass, nearest end 11 inches from needles. See Fig. 496.	Heeling deviations produced by disturbing magnet placed as stated in column (6); vessel heeled 20° to starboard. Difference of columns (5) and (6).	Headings by compass No. 6211, *after* placing compensating magnet below compass, nearest end 12 inches from needles. See Fig. 496.	Residual heeling deviations after compensation; vessel heeled 20° to starboard. Difference of columns (5) and (8).	Heading by compass No. 6211 corresponding to column (1); vessel upright, disturbing magnet removed, and compensating magnet in place as in Fig. 496.	Deviations produced by compensating magnet, vessel upright. Difference of columns (2) and (10).
(6)	(7)	(8)	(9)	(10)	(11)
N. 29° 0′ E.	20° 15′ W.	N. 9° 0′ E.	0° 15′ W.	N. 4° 30′ E.	0° 45′ E.
N. 11 30 E.	23 0 W.	N. 10 45 W.	0 45 W.		
N. 8 30 W.	23 30 W.	N. 30 30 W.	1 30 W.	N. 35 15 W.	0 30 W.
N. 31 45 W.	21 0 W.	N. 51 0 W.	1 45 W.		
N. 62 30 W.	11 15 W.	N. 71 30 W.	2 15 W.	N. 75 30 W.	1 30 W.
S. 84 0 W.	1 45 E.	S. 87 45 W.	2 0 W.		
S. 52 0 W.	13 15 E.	S. 67 0 W.	1 45 W.	S. 65 0 W.	1 45 W.
S. 25 15 W.	20 0 E.	S. 46 30 W.	1 15 W.		
S. 2 45 W.	22 45 E.	S. 26 15 W.	0 45 W.	S. 25 0 W.	0 45 W.
S. 16 15 E.	22 0 E.	S. 6 0 W.	0 15 W.		
S. 33 0 E.	19 30 E.	S. 14 30 E.	1 0 E.	S. 14 30 E.	0 0
S. 48 15 E.	15 15 E.	S. 34 30 E.	1 30 E.		
S. 63 0 E.	11 0 E.	S. 54 0 E.	2 0 E.	S. 54 45 E.	0 45 E.
S. 77 15 E.	5 45 E.	S. 74 0 E.	2 30 E.		
N. 88 30 E.	0 30 E.	N. 87 0 E.	2 0 E.	N. 85 15 E.	1 15 E.
N. 75 45 E.	6 45 W.	N. 67 0 E.	2 0 E.		
N. 60 0 E.	11 0 W.	N. 47 30 E.	1 30 E.	N. 45 30 E.	0 30 E.
N. 45 0 E.	16 0 W.	N. 28 0 E.	1 0 E.		
N. 29 0 E.	20 15 W.	N. 8 45 E.	0 0	N. 4 45 E.	0 45 E.

the north (nearest) end 11 inches from the plane of the needles. The vessel, while upright, was again swung, col. (3), to ascertain any disturbing effect of the magnet *D* from not being in line with the pivot; the results are given in col. (4).

Without moving in the least the brass arm that held the magnet *D*, this magnet was withdrawn from the arm, and together with all other magnets and iron were removed to some distance from the SCORESBY.

The vessel was now heeled 20° to starboard and swung, col. (5).

The magnet *D* was then carefully returned to its brass arm holder, Fig. 496, and the vessel swung, col. (6): the heeling deviations to the high side thus produced by *D* are recorded in col. (7), and are represented by the green curve of Fig. 497.

A compensating magnet *C* in every respect similar to *D* was then placed symmetrically to *D*, below the compass, its north (nearest) end 12 inches from the plane of the needles, and the vessel swung, col. (8). A comparison of cols. (5) and (8) gives the residual deviations, col. (9), after compensation; they are represented by the red curve. Finally, to ascertain the effect of the compensating magnet, due to want of verticality, the vessel was brought to upright, the magnet *D* removed, and the vessel swung, col. (10), a comparison of which with col. (2) shows the results in col. (11).

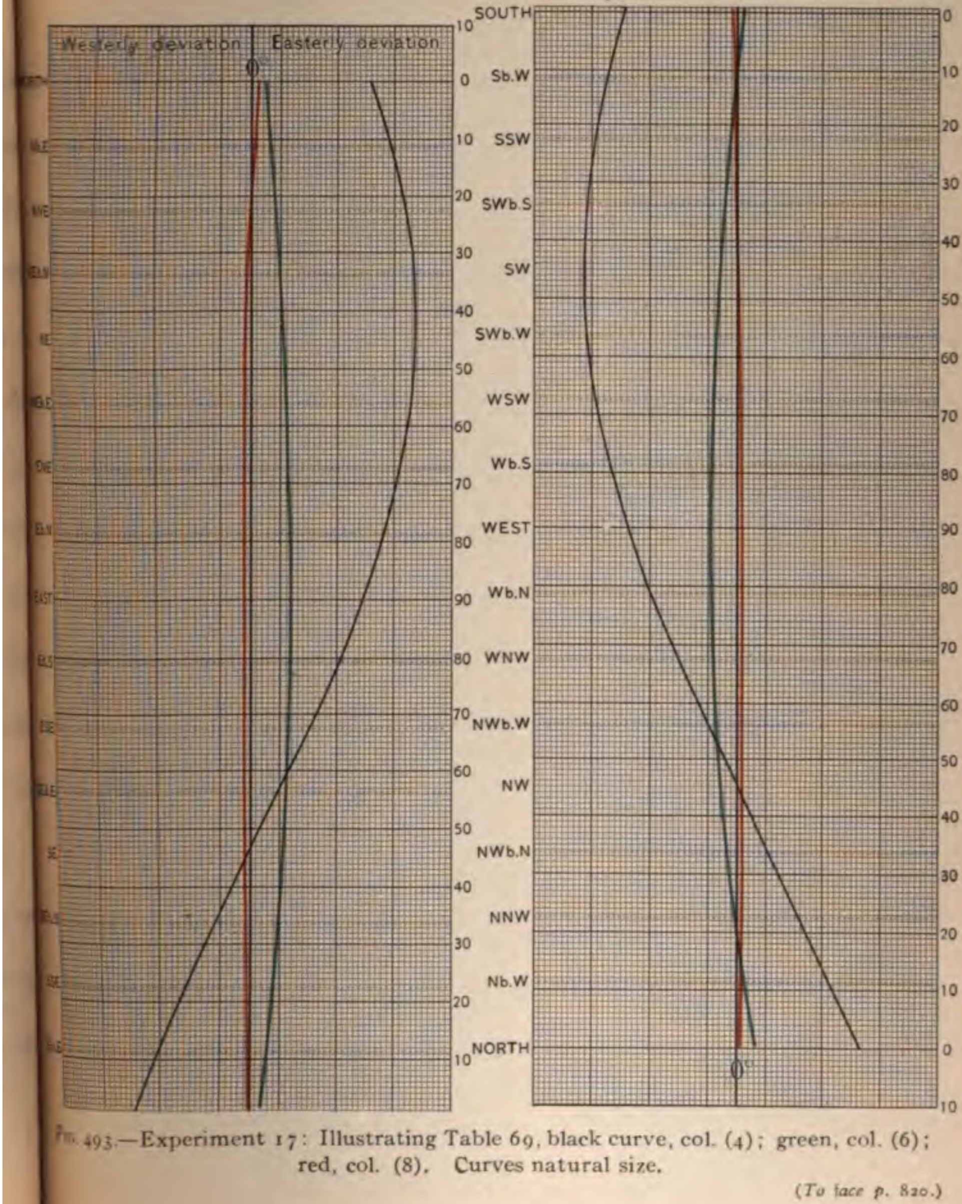
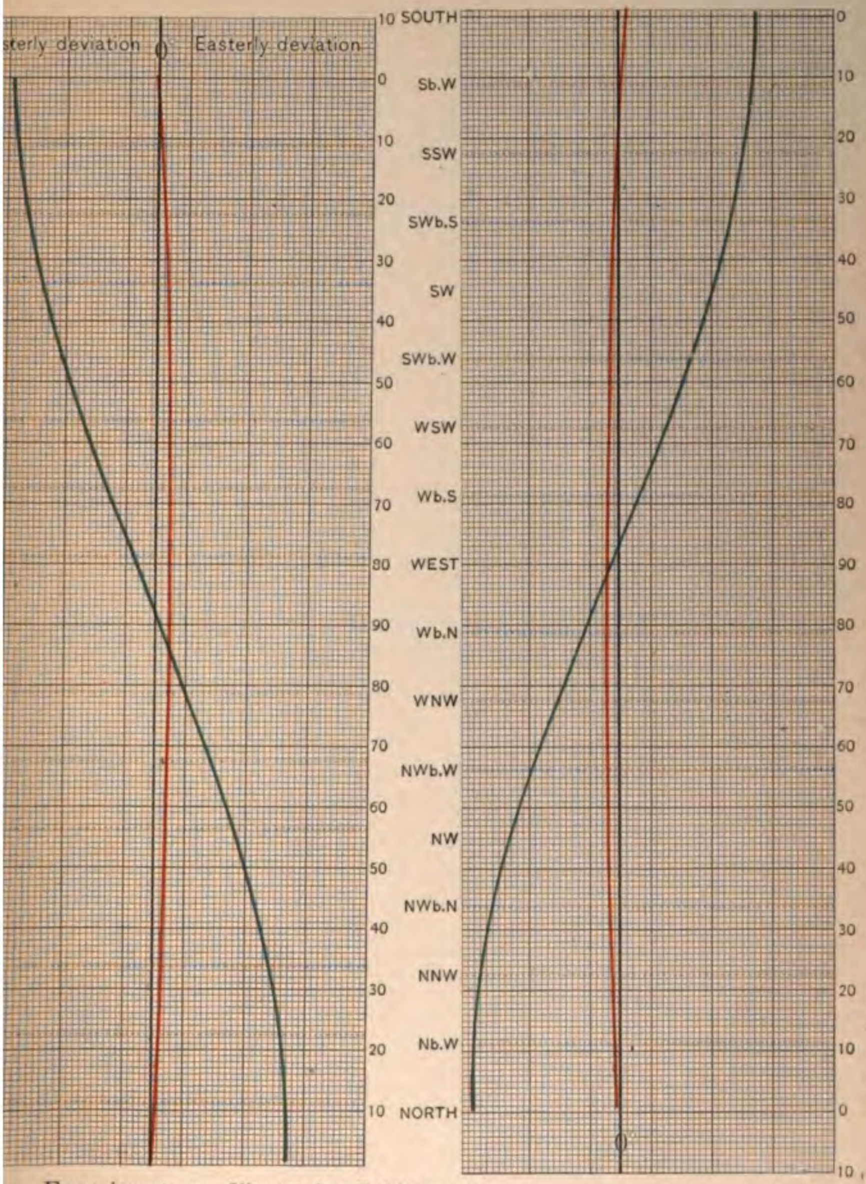


FIG. 493.—Experiment 17: Illustrating Table 69, black curve, col. (4); green, col. (6); red, col. (8). Curves natural size.

(To face p. 820.)



.—Experiment 19: Illustrating Table 71, the green curve, col. (7); and the red, col. (9). Curves natural size.

(To face back of Fig. 495.)

PART FOURTH.

THE MATHEMATICAL THEORY OF THE DEVIATIONS.

CHAPTER XXI.

HISTORICAL.

281. Present conditions producing deviations.—In Part Third are stated the varied sources of deviation—iron of different susceptibility to magnetism, and its retentiveness: some—the hull itself—like a permanent magnet whose power endures with little change; and more—the wrought and cast iron which is like a sluice to water—chiefly a conduit for magnetic flux, without retentive power.

If the flux be full and strong, such iron will become powerfully magnetic—if thin and weak, feebly so: if the iron lies horizontally, the deviations from it will have identical character and value in all parts of the world; but if vertical, they will differ in kind from the preceding, and be one thing in northern latitudes, and another in southern.

And when the ship heels, iron of every kind produces new disturbances, additional to those on an even keel.

Such is the present state of the case, but it was not so at first: the varied sources of deviation arose gradually as iron was introduced into ships; and the unfolding of the laws of these deviations was like that of all other phenomena—slow and tedious, yielding to enquiry only after much groping and some wandering from the right path, even by investigators of the greatest ability.

282. Earliest observations of the deviations.—Prior to 1800, it had been observed that the needle on ships deviated from the magnetic meridian. In 1666, Guillaume Denys, a hydrographer of Dieppe, France, noticed that two

compasses placed in different parts of a ship did not agree—that something in her structure affected them. In 1691–93, William Dampier, R.N., in an account of his voyage, states that he was puzzled at certain anomalies in the amount of the variation of the compass on passing the Cape of Good Hope. During the voyage of Captain James Cook, R.N. (1776–80), the astronomer of the expedition, William Wales, notes that "variations observed with the ship's head in different positions, and even in different parts of her, will materially differ from one another, and much more will observations observed on board different ships." And in 1794, Mr. Downie, R.N., master (navigator) of H.M.S. GLORY, in a report on a newly invented compass, observes that "the quantity and vicinity of iron in most ships has an effect in attracting the needle; for it is found by experience that the needle will not always point in the same direction when placed in different parts of the ship; also, it is rarely found that two ships steering the same course by their respective compasses will go exactly parallel to each other; yet these compasses, when compared on board the same ship, will agree exactly."

These were the first glimmerings of a condition that to-day has reached the largest and most important proportions; they were isolated observations, however, of a mere fact, without pursuit of its ramifications or enquiry into its meaning.

283. Discovery of the semicircular deviation, and the neutral line of ships.—Between 1801 and 1803, Captain Matthew Flinders, R.N., made a cruise in H.M.S. INVESTIGATOR to Australia. He knew of the effect of iron upon the compass, and set about ascertaining its laws. While in northern latitudes, he found that the deflection was greatest on easterly and westerly courses, gradually decreased as the ship swung toward either north or south, and disappeared on those points—in fact, a deviation of

the semicircular type; on reaching the Magnetic Equator, the deviation vanished on every course, reappeared in southern latitudes, but in reverse order; so that, whereas in north latitude, upon the ship swinging from N. toward E., the first maximum occurred at east and the second at west, now the first was at west and the second at east. He inferred—and rightly—that the phenomenon was a function of the Dip: that the iron (in all probability vertical wrought-iron stanchions) became magnets through induction of the Earth's vertical component, and so acquired an upper pole of one name in northern latitudes—lost this *end* polarity on the Magnetic Equator (where there is no vertical component)—and regained end polarity of opposite name in southern latitudes. The stanchions, no doubt, were symmetrically located with respect to a central vertical section, and thus their magnetic pull was fore-and-aft.

In Captain Flinders' ship—a wooden sailing vessel—the maximum deviation was only a few degrees: he proposed to correct it by means similar to the cause—vertical iron; and this is the origin of the Flinders bar used at the present day.

By trial with a compass in various parts of the ship, he also found that she had a *neutral line*, in which it were best to fix the compass, and thus avoid both the ailment and its remedy—advice which it is even more important to follow now.

284. The quadrantal deviation predicted. — The next advance in knowledge of the deviations was prompted by observations in Arctic whaling grounds.

William Scoresby was born in England in 1789 and died in 1857: his father commanded a whaler, and the son accompanied him to sea at the age of eleven, going on this voyage to the highest latitude then reached, 81° 12′ north. For many years the son cruised in Arctic regions

as a subordinate and in command of ships; then he left the sea and entered the ministry, where he acquired the distinction of doctor of divinity; but it was through his magnetical investigations, relating mostly to ships (which he continued even in his new calling) that he became so widely known.

Dr. Scoresby was struck by the large deviations he regularly experienced in high latitudes: other Arctic navigators—Ross, Parry, and Sabine (1818-24)—observed the same thing, but it does not appear that they divined the cause; that was done by Dr. Thomas Young, the founder of the Undulatory Theory of Light, who stated that they were due, partly to the diminished (horizontal) directive force of the Earth in Polar regions, and partly to the increased induction by the vertical component.

Furthermore, he treated of the effect of horizontal iron and predicted quadrantal errors arising therefrom; and this is the first mention of this type of deviation: in view of the use of iron as deck beams, anchor cables, screw shafts, engines and boilers, then successively coming into vogue, it was a most valuable prognosis.

About 1820, Prof. Peter Barlow investigated the deviations both experimentally and mathematically, and proposed a method of compensation that was followed for a time, but proving inadequate, especially in the new conditions of ship construction, it fell into disuse. His, however, was a painstaking effort which added much that was enlightening and accurate to the general knowledge of the subject; and if his method of compensation failed, it was not without benefit, for it spurred others on in the enquiry.

285. **Poisson's Theory of the Deviations.**—The eminent French mathematician and philosopher Poisson next attacked the subject in all its generality. Between 1824 and 1839, he communicated to the French Academy of Science several memoirs containing both his mathematical

theory of transiently induced magnetism (based on the physical theory of Coulomb) and his Theory of the Deviations of the Compass produced by the iron of Ships.

"After developing the theory of magnetic attractions and repulsions, and thereon establishing general equations, containing the laws alike of the distribution of magnetism in the interior of bodies magnetized by induction from other bodies, and of the attractions and repulsions which such bodies exert on points in given positions, he then discussed those equations for the particular case of a *sphere*, solid or hollow, and by supposing the Earth to be an inductive body, he deduced the deviations of a compass-needle for different positions in the vicinity of such a sphere, thus magnetized by the Earth. . . . He finally considered the simultaneous magnetic actions on a horizontal needle, of the Earth directly, and of any number of spherical masses rendered magnetic by the inductive action of the Earth, but supposed uninfluenced by each other's magnetic action; whence he passed by an easy inference to those general formulas for the disturbing action on a magnetic needle *of any system of bodies magnetized by induction from the Earth, whatever their respective forms or relative positions, and while supposed to be influenced by their own mutual magnetic actions*, which have generally furnished the point of departure for succeeding investigations on the Theory of Compass Deviations. . . . In Poisson's investigation, though recognizing the possible existence of a certain degree of coercive force, and therefore of *polar magnetism* in the iron of a ship, and indicating the steps to be taken for introducing this element among the other data of the problem, he yet omitted its actual consideration on grounds of expediency, it would appear, partly in view of its being then presumed very small in amount, and partly from the desire to avoid any unnecessary complication of the solution. This restriction, not inappli-

cable to the state of naval construction at that time, especially in France, could not, of course, continue to be admissible after the era of the use of iron in ship building, or even in the heavy machinery of steam vessels with wooden hulls. . . . Poisson further restricts his enquiry by the supposition ordinarily made by subsequent writers, that the distances of the masses of iron from the compass are sufficiently great, relatively to the LENGTH of the needle for the influence of the latter to be neglected.

"It may, however, be observed, that he specially investigated the necessary correction of the compass deviation so influenced, in his first memoir of 1824.

"He considered in the same memoir the reaction of a disturbing mass on the needle consequent upon the reflex action of the latter, which he also neglects, in the present investigation, as being practically insignificant.

"Both of the suppositions of this paragraph are involved in the *general formulæ for the disturbing action on a magnetic needle* in the memoir of 1824, previously referred to. . . .

"The investigations on compass deviations since the date of Poisson's last memoir (1838), commencing with Prof. Airy's first paper in 1839, have steadily advanced toward establishing a comprehensive and available theory in all that relates to this subject. Starting with the recognized necessity, in the actual condition of the problem, of appreciating and at once allowing for the dominating influence of *polar magnetism*, as compared with the magnetism induced in the soft iron, they were compelled to deal with the very element which Poisson, at his standpoint, was justified in disregarding. This immediately changed the whole aspect of the question. For, with the complex conditions thus introduced, and the more exacting requirements of experience in their practical treatment, came the necessity for constantly aiming at the com-

plete analysis of the magnetic phenomena of ships." (Prof. B. F. Greene, U. S. Navy.)

"M. Poisson gives, to express the action of the soft iron on the compass, formulæ involving coefficients to be determined by observation; and he has adapted the formulæ to observations made on shipboard sufficient in number to determine the coefficients in the particular case of the soft iron being symmetrically placed. The present writer afterwards modified Poisson's formulæ so as to adapt them to the form in which the data generally present them selves, viz., a vessel having hard as well as soft iron—both unsymmetrically distributed—and observations being made in a certain number of equidistant points." (Mr. Archibald Smith.)

286. An account of the author who modified Poisson's theory.—By a natural step, we now come to the man who amplified and perfected Poisson's theory as it practically exists to-day, and it would seem that those who daily direct their course by the results of his labor should know something more of him than as the author of the Admiralty Manual: I therefore present a few facts of his life, based on the obituary written, at the time of his death, by Lord Kelvin.

Mr. Archibald Smith was born in Glasgow, Scotland, August 10, 1813, and died December 26, 1872. His father was connected with the University of Glasgow, and there the son was educated after preliminary training; later, he completed a course at Cambridge and took the highest honors in mathematics, being what is technically known as Senior Wrangler.

His tastes were decidedly scientific and mathematical, though he was also an accomplished classical scholar; but he followed neither bent—he chose the legal profession. He passed much time, however, on intricate problems of mathematics, particularly on the Theory of

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swinging and intensity experiments, and they "determined that the *interior* of an iron vessel acted upon the compass as a permanent magnet." (Phil. Trans., 1839.)

The outcome of his experiments was an elaborate mathematical treatment of the subject and the practical compensation of the deviations: the former seems to have been soon lost to view in the general acceptance of the theory of Poisson and Smith; but the method of compensation became very popular and was extensively used in the mercantile marine.

Prof. Airy brought the *permanent* magnetism of ships into bold view; indeed, he deemed it so largely the source of semicircular deviation, as to disregard (as practically unimportant) the part due to vertical soft iron: it is not quite as he stated, that "the correction of the semicircular deviation made by fixed magnets in one latitude will be perfectly correct in every other latitude." (Results of exp., etc.; Weale, 1840.)

But about this time, "permanent" and "sub-permanent" magnetism were hazy conceptions in the scientific mind; also, the entire absence of connection—as to their respective effects—between horizontal and vertical soft iron was not fully appreciated; and indeed the whole subject was so thick with indefinite notions that the surprise is, not that men made mistakes, but that they saw so clearly. While it is one thing—as critics have done in this and *other* matters—to take a *retrospective* view of the whole field and separate the true from the faulty, it is quite another to labor successfully in that field when all was confusion and pitfalls.

Iron was rapidly replacing wood in the hulls of ships, and when it was seen that the compass erred more and more in its new abode, those who had to go to sea in such ships naturally lost heart; it was therefore a most signal service that Prof. Airy rendered by his method of com-

pensation: this consisted in neutralizing the semicircular deviation by two steel magnets fastened to the deck at distances found by trial—one athwartships and the other fore-and-aft—near the compass; and overcoming the quadrantal deviation by boxes of chain, nails, or other soft iron, suitably placed: it is *in essence* the method of compensation at the present day.

It was Lord Kelvin, I believe, who replaced the bars by bundles of wires and the chain by spheres: his other valuable appliances for practically dealing with the deviations as well as his theoretical contributions to magnetic science are too well known to require further mention.

Prof. Airy also urged the magnetic examination of every new iron ship, not only as a precaution for her safety but also to afford material for investigation of the subject.

288. The feature of sub-permanent magnetism found. But iron ships even with compensated compasses continued to come to grief, and enquiry arose as to the cause: Dr. Scoresby led the way in ascribing it to a *change* in the original magnetism of the ship—that it was partly shaken out by buffeting of the waves on her first trip, and that then the compensating magnets had an opposite and unknown effect.

This position was fully warranted by his early sea experience coupled with the trials he had made of plates of iron to determine the manner in which they would acquire and retain the magnetic condition.

Prof. Airy refuted Dr. Scoresby's contention in the measure it was asserted, and an earnest controversy arose between them, which lasted many months: then it subsided, "until attention was once more called to it by a fearful calamity. A new iron ship, the TAYLEUR, of 2,000 tons burden, sailed from Liverpool with emigrants in the year 1854. Before departure, she had been swung and

disclosed very large deviations — the steering compass, 60° max.

"Like all very large deviations, this undoubtedly was for the most part due to the inherent (and, as was supposed, the permanent) magnetism of the ship.

"A correction by magnets was applied in the usual way, and the compass made to give tolerably correct indications: in this trim she sailed. In going down the Channel, she experienced severe weather, and within two days after leaving port, was wrecked, with great loss of life, on the coast of Ireland. An enquiry into the disaster seemed to point to a grave compass error as the cause, and Dr. Scoresby called attention to the fact that the disaster might be accounted for on the hypothesis that a large portion of the magnetism had been shaken out of her during those two days. If this had happened, it was shown that the compensating magnets must have over-corrected the compass, and produced an error of precisely such a kind as would have led her to the point where she struck, when she was believed to have abundance of sea room. If Dr. Scoresby's hypothesis was correct (and the evidence in support of it was very cogent) an entirely new and very alarming quality of iron ships had been revealed: that the magnetism could be shaken out to some extent was generally admitted, but it was a startling novelty to be told that two days' straining in a heavy sea could cause such change as to be equal to two points of deviation, as appeared to be the case with the TAYLEUR.

"Dr. Scoresby's explanation was received with surprise by all, and with incredulity by many. Mr. Airy did not hesitate to pronounce it impossible that so rapid a change could have occurred, and perhaps it must be regarded as doubtful whether it did. . . .

"Dr. Scoresby took advantage of the sailing of another new iron vessel—the ROYAL CHARTER—to make a voyage

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of many were kept up during voyages to southern latitudes.

All this was supplemented by observations on ships of the British Navy by Captain F. J. Evans, R.N. From all the facts deduced, the following points were considered established: 1st, that the magnetism of iron ships is distributed according to precise and well-determined laws; 2d, that a definite magnetic character is impressed on every iron ship while building, which is never afterwards entirely lost; 3d, that a considerable reduction takes place in the magnetism of an iron ship just after completion, but that subsequently any change in its direction or amount is slow and gradual; 4th, that the original magnetism of an iron ship is constantly subject to small fluctuations from change of latitude arising from new magnetic inductions.

Subsequent experience—both varied and abundant—has confirmed these conclusions.

The credit of discovering that the distinctive magnetic character of an iron ship depends upon her compass heading on the stocks is due jointly to Captain Edward Johnson, R.N., and Dr. Scoresby: the former, in 1835, in his report of the first iron ship ever swung, says: "As, in the construction of iron vessels, hammering the numerous rivets might elicit magnetic influences, it would be well to note by compass the direction of their heads and sterns when building, with a view of ascertaining whether any distinct magnetic properties indicated by those parts are due to the line of direction of the vessel with respect to the magnetic meridian"; and Dr. Scoresby, long previous to the formation of the Liverpool Committee, had made many experiments and written much regarding the same point. The committee's enquiry only confirmed his theory.

The fact that a ship loses much of her magnetism dur-

ing the first voyage must also be credited to the observation of Dr. Scoresby.

290. Various European contributions to knowledge of the deviations.—The foregoing were the principal investigators of the Deviations; but others added to the general knowledge, without whose contributions the Mathematical Theory would not be what it is—the embodiment of the conditions under which deviations arise, and a practical instrument for analyzing those conditions. I will therefore briefly mention those of whom I have knowledge as having contributed to the subject.

Captain F. J. Evans, R.N., for many years Superintendent of Compasses, had extensive experience with the deviations of ships of the British Navy; he and Mr. Archibald Smith were joint authors of the Admiralty Manual of Deviations—to name which is alone sufficient testimony of Captain Evans' contributions.

Captain A. Collet, of the French Navy, was connected with the Marine of France in a capacity similar to the preceding: he incorporated some of the Admiralty Manual in a French Treatise, but surrounded the bare mathematical structure with such ample and original explanation as to render it attractive and easily intelligible.

Another French writer—M. Gaussin, de l'Ecole du Genie Maritime—added an important fact to the general fund: it is that the horizontal soft iron of the ship not being really as susceptible to magnetism as theory contemplates, does not, during the swinging, especially when rapid, acquire the degree of magnetism proper to the actual heading of the ship, but that of some previous instant; in other words, there is a retard of induction.

Mr. Archibald Smith adapted certain geometrical principles to a graphic method, to which he gave the name of Dygogram; but it is to Captain Colongue, of the Imperial Russian Navy, that the simplification of the Dygogram is

due, which renders it most useful to navigators. Mr. Eugen Gelcich, of Austria, is the author of a mathematical Treatise on the subject: it contains original views.

No doubt other European writers have contributed to the general knowledge of the deviations, but with their works I am not acquainted.

291. American contributions to knowledge of the Deviations.—In this country, Rear-Admiral John A. Howell, U.S.N., has bridged some wide gaps in the mathematical formulas, and thus made them passable to those not so well equipped for long striding in this field as the original authors of the Theory were.

The Office of Superintendent of Compasses for our Navy was established by Prof. B. F. Greene, U.S.N., who by his writings disseminated much valuable information among the officers of the Service regarding the compass and its deviations on iron ships. Upon his retirement at the age limit, the Office practically lapsed into abeyance for some time: in 1881, the writer of this Treatise revived it and performed the duties for four years; then, in the natural order, other line-officers of the Navy filled the place, each contributing to the advancement of the subject.

Conspicuous among them, however, has been Commander S. W. B. Diehl, U.S.N.—who at first was associated with the writer in the Office, and who subsequently was Superintendent for many years: during his time, the ships of our Navy rapidly changed from a wooden fleet under sail and steam to complicated Battleships of huge dimensions and armament; iron and steel became the sole environment of the compass, and many questions arose regarding the suitable location and proper compensation of the compass. Commander Diehl met them successfully—subjecting everything to test on the SCORESBY, so that it bore the stamp of prudent and careful experiment.

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[2.] By reference to Plates *S* and *T*, it will be seen that in Figs. 435 and 439, at (2) and (17) respectively, there can be positions of horizontal soft iron in which it has a bad effect on the compass: it may not itself produce perceptible deviation, but its poles act upon *like* poles of the needle—lessen the directive force—and thereby give freer rein to other deviating causes.

[3.] Theory considers the horizontal soft iron perfectly permeable to magnetism; but the fact is, that it has various degrees of hardness which prevents instant induction—there is a lag, so that a ship swinging with the starboard helm will have deviations different from those with the port helm, and this the greater in amount, the more rapid the swing to one side than the other.

[4.] Ships moored at a dock, riding to a constant wind, steering the same course—in fact, any situation in which they continue in the same general direction, acquire a temporary charge of magnetism that is due entirely to this circumstance: it dissipates soon after change of heading, but constitutes a veritable disturbance while it lasts. The longer the ship is in the same position, the greater will be the charge. I am reliably informed that iron vessels lying at the wharves of New York find a deviation of about 7° from this cause, which disappears soon after going to sea. The converse of this should be a note of warning to those approaching this port: coming from the southward or eastward, the ship is days on the same general course; magnetic induction is active, with the shock of waves and vibration of the screw to assist it; and deviations which the ship did not have on sailing, will be found at the end of the passage. Errors from this source are not indicated by the

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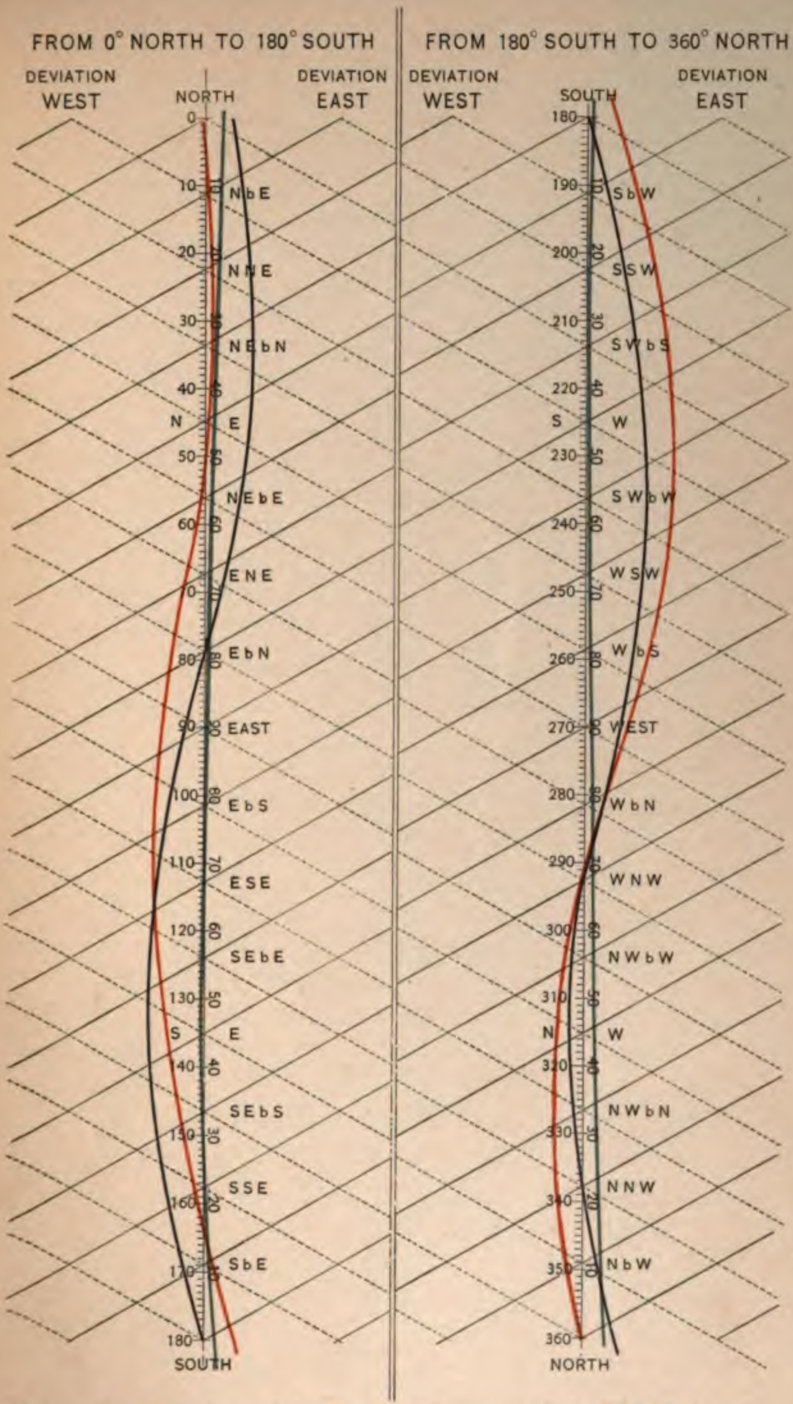

Fig 498.—Deviations of the Standard Compass of the U. S. S. Atlanta. Curves natural size. Red = *before* firing the battery and before compensation; Black = *after* firing the battery and before compensation; Green = after compensation.

(To face p. 840.)

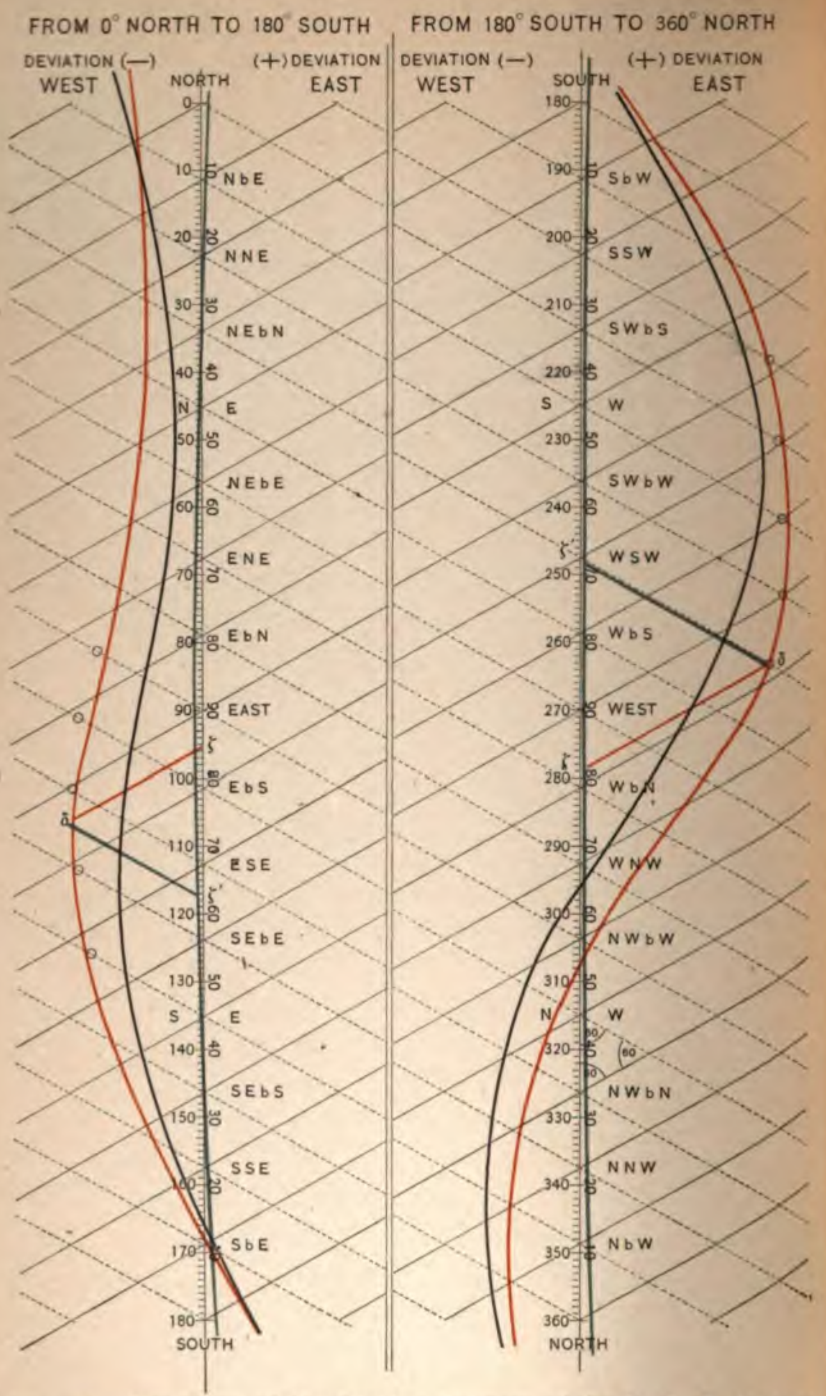

FIG. 499.—Deviations of the Steering Compass of the U. S. S. ATLANTA. Curves natural size. RED = *before* firing battery and before compensation; BLACK = *after* firing battery and before compensation; GREEN = after compensation.

(*To face back of Fig.* 498.)

urant peut produire, sur un compas placé a neuf mètres, e deviation de 7° 30′. Comme cette erreur se produit ılement pendant l'éclairage et que les observations qui rmittent de régler et de compenser les compas ont lieu dinairement le jour, on voit qu'il peut se passer un cern temps avant que le capitaine puisse être prévenu et méfier d'une erreur aussi considerable et aussi dangeıse puisqu'elle n'affectera la route que pendant queles heures de nuit. Il faudra donc, toutes les fois qu'on t usage à bord d'une machine à courant continu, avoir in de la munir de deux fils, placés l'un près de l'autre. faudra de plus s'assurer, de temps à autre, que l'isoleent de ces deux fils est parfait, parce qu'un défaut d'isoleent insuffisant pour produire une diminution sensible lumière pourrait produire cependent une erreur comrable a celle provenant de l'emploi d'un seul fil. Quand emploie au contraire une machine a courants alternatifs, ème à un seul fil, il n'y a rien a craindre pour le coms." (Capitaine A. Collet, de la Marine Française.)

"The field of a dynamo disturbed a compass on board armor-plated ship at a distance of 37 feet, although in intervening space there was an iron bulkhead, a steel ck two to three inches thick, and a second deck, partly steel ⅜ inch thick.

"The system of double wires should be employed, to oid the disturbing magnetic effects of single wires on compass." (Admiralty Manual.)

] Lightning is a source of error in both ship and compass by deranging the magnetism of the former d reversing that of the latter. It is of record that "dur- the years 1892–93, both the steamship CAPELLA and M.S. RALEIGH were struck by lightning, causing conlerable change in the magnetism of those ships." (Capt. J. Evans, R.N.)

"M. Arago cite un vaisseau génois qui croyant marcher

vers le nord, vint se briser sur la côte, pres d'Alger, les pôles de sa boussole ayant été renversés par un coup de foudre." (Daguin, Physique.)

[9.] In Art. 87 will be found some information relative to ridges of magnetic ore and submerged rocks, and a few well-authenticated instances of the latter having greatly affected the compass: indeed it is entirely reasonable that such should be the case.

[10.] Whether a telescopic smoke-stack be up or down; a heavy gun run in or out, or variously trained; stands of rifles in place or in use; wire pendants dangling loosely or stopped to the side; large cranes swung out for lowering boats or not:—all this periodic movement of such masses of iron *in vicinity of the compass* constitutes sources of error of different degrees, which can be determined only by experiment with the articles in the varied positions they may occupy. An instance is recorded of a *difference* of 16° in the deviations with the smoke-stack first up and then down.

[11.] It should occur to every one whose duties bring him near the compass that all articles of iron and steel on his person are magnetic and therefore liable to produce error: the bayonet of a sentry; the cutlass of a quartermaster; the spring in the helmsman's cap; the steel-rimmed eye-glasses of an officer, or his galvanized trumpet:—may be brought near enough to the card to disturb it temporarily. Rubbing the glass cover of the bowl with a silk or woolen cloth will excite electric currents in the glass which deflect the needle: they may be dissipated by moistening the glass.

[12.] Many of the foregoing errors cannot be avoided or compensated; they can only be determined by observation and corrected from tabulation: ceaseless vigilance by daily azimuths on the few courses likely to be run each day is the only safeguard against danger.

293. Waves of air that produce musical sounds illustrate waves of ether that cause deviations.—Motion of a magnetic body will be found at the bottom of nearly every compass disturbance, and this motion causes a movement in the ether—generally a wave—so that in the last analysis we have to deal with the representative of a wave, either graphically as a curve, or analytically by mathematical symbols.

In Articles 30 to 38, the superposition of waves is described, but its further illustration may be done here with advantage. A piano-wire vibrates not only as a whole, but also in parts—the half, third, fourth, and other equal subdivisions, thus giving the fundamental note and its harmonics: when, therefore, the keys respond to a musician's touch, there is a mingling of waves that may be represented by a curve of varied sinuosities; yet a resonator—a vibrating instrument of specific frequency, that responds only to waves of the same period just as a swing does to suitably timed impulses—will pick out each note so that it may be experimentally determined which ones form the composite sound.

Similarly, a ship has the fundamental semicircular deviation, upon which is superposed the quadrantal, sextantal, octantal, and minor deviations, together with many accidental errors—arising from waves in the ether caused by the varied movement of divers magnetic bodies—all portrayed by a compound curve, often of very odd and irregular contour: but mathematical analysis will separate the parts and determine the form and value of each.

Again, if a musical instrument sends out long waves, the sound is grave; if short waves in quick succession, the note is shrill; if it abounds in harmonics, the effect is most agreeable: and according to the degree of these qualities in each instrument, its musical character is determined.

The human voice is but the sound of a reed instrument with a fundamental note and its harmonics; and all know how it varies from one person to another—the melody of the prima donna at one extreme, and the harshness of the cabman at the other; yet both result from waves of air. So with the ship: each has her distinctive character portrayed by deviations that arise from waves peculiar to movements of her own magnetic composition.

294. Periodic and harmonic motion.—The deviations of the compass belong to a large body of phenomena that have the same characteristic—periodicity.

The ebb and flow of the tide; the waxing and waning of the magnetic elements in secular, annual, and diurnal periods of time; the rise and fall of temperature with the day and season; the growth and subsidence of electric flow in both the alternating and continuous currents:—these are all periodic, and typical of very much that takes place in nature. The oscillation of a pendulum or movement of a balance-wheel is the simplest illustration of periodicity: however long either motion continues, it but repeats itself—starts from rest, acquires a certain velocity, and sinks to rest again. So with the Deviations: let the ship circle round and round as often as she will in the same locality, and the values of the first circle but recur.

This general law finds its mathematical expression in Fourier's Series.

Consider Fig. 500: a point P moves with *uniform* velocity round a circle: as it does so, its successive projections H upon the diameter $\overline{AC}$ will move with *variable* velocity—start from rest at A, gradually increase its speed until O is reached, while P revolves from A to B, then as gradually decrease speed in going toward C, while P proceeds from B to C; at C, the point H begins a retrograde movement toward A exactly like its outward motion, while P continues on through the semicircle CDA: this

motion of H along $\overline{AC}$, as indicated by the arrows E and E', is both periodic and harmonic.

To represent it graphically, *suppose* perpendiculars

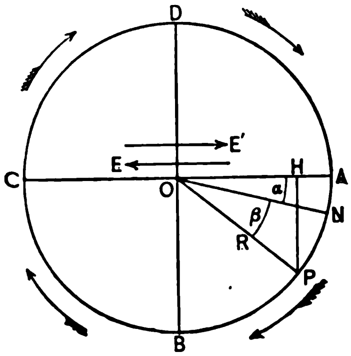

FIG. 500.

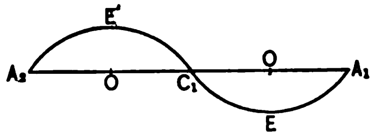

FIG. 501.—Semicircular.

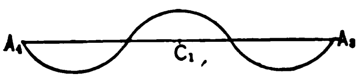

FIG. 502.

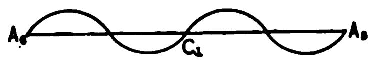

FIG. 503.—Quadrantal.

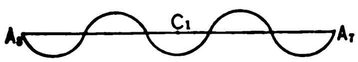

FIG. 504.

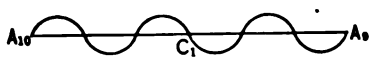

FIG. 505.—Sextantal.

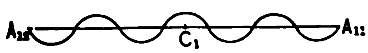

FIG. 506.

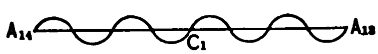

FIG. 507.—Octantal.

Harmonic Motion Illustrated.

erected to the line $A_1C_1A_2$ (Fig. 501)—below, on the right of C_1; and above, on the left—each equal in length to the velocity of the point H at the corresponding part of the

diameter AOC: draw a curve through the ends of all the perpendiculars, and it is found to be of the wave form.

The following names have been given to certain features of this harmonic motion: the time in which the point P revolves once round the circle is called the *Period*—denote it by T; it is also the time that H moves from A to C and back again, or from A_1 to A_2, that is, the time to describe the curve on that line; the distance of H from O, Fig. 500, during any stage of its motion, either in linear measure as $\overline{OH}$ or angular measure as POH, is called the *Displacement*—it is equivalent to *Phase*, which term is also used; the maximum displacement $\overline{OA}$, or when H is at either end of the diameter $\overline{AC}$, is called the *Amplitude;* this distance is identical with $\overline{BD}$, so that when H is also on this latter diameter, at O, it has the maximum velocity, and thus, in Fig. 501, both $\overline{OE}$ and $\overline{OE'}$ represent the amplitude of the wave form.

The point P may start from A, or from N the extremity of any radius making an angle α with $\overline{AO}$: in the latter case, the constant α, equal to the portion of the period required to describe $\overline{AN}$, will enter into all the calculations.

Let $\overline{OA}=\overline{OP}=R$; $\pi=$a semicircle$=180°$; $A=$acceleration of H; $v=$linear velocity of H; $\omega=$angular velocity of $\overline{OP}$, as P revolves round the circle; $x=\overline{OH}$, the variable distance of H from the center; and $y=\overline{HP}$, the variable distance of P from the diameter $\overline{AC}$: x and y are then the coördinates of P. In Fig. 500,

$$\cos POH=\frac{\overline{OH}}{\overline{OP}};\quad \therefore x=R.\cos(\beta+\alpha). \qquad (1)$$

If P starts from N, the angle β will be equal to the velocity (ω) multiplied by the time (t) in which the radius moves from $\overline{ON}$ to $\overline{OP}$, that is, $\beta=\omega.t$; whence (1) becomes

$$x=R.\cos(\omega.t+\alpha). \qquad (2)$$

If $\alpha = 0$, or the point starts from the extremity A of the diameter, then

$$x = R.\cos \omega.t. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Differentiating this,

$$dx = -R.\omega.\sin \omega.t.dt. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Since velocity (v) equals distance (x) divided by time (t), we have from eq. (4),

$$\frac{dx}{dt} = v = -R.\omega.\sin \omega.t. \quad . \quad . \quad . \quad . \quad (5)$$

And since acceleration (A) is a *change* of velocity, we find by differentiating (5),

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = A = -R.\omega^2.\cos \omega.t. \quad . \quad . \quad . \quad (6)$$

The time (T)—the Period—that P requires to make one cycle is equal to the circumference ($ABCD = 2\pi$) divided by the angular velocity (ω) of $\overline{OP}$, that is,

$$T = \frac{2\pi}{\omega}; \quad \therefore \omega = \frac{2\pi}{T}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Substituting this in (3), (5), and (6), they become

$$\text{Displacement} = x = R.\cos \frac{2\pi}{T}.t. \quad . \quad . \quad . \quad (8)$$

$$\text{Velocity} = v = \frac{dx}{dt} = -R.\frac{2\pi}{T}.\sin \frac{2\pi}{T}.t. \quad . \quad . \quad (9)$$

$$\text{Acceleration} = A = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -R.\frac{4.\pi^2}{T^2}.\cos \frac{2\pi}{T}.t. \quad (10)$$

As the functions *sine* and *cosine* always return to the same values after each cycle of 360° ($=2\pi$), it is seen that equations (8), (9), and (10) express periodic and harmonic motion.

The line $\overline{HP}$ ($=y$) obviously undergoes periodic change which is expressed by

$$y = R.\sin \omega.t. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

This is similar to eq. (3), and it depends upon the location of the points of maxima and minima which equation is to be employed: if A be the origin of motion of P, the maxima of harmonic *velocity* occur at O on the vertical diameter, and its minima at A and C, and eq. (11) will express these conditions; if, however, *displacement* of H is considered, its maxima occur at A and C, and minima at O on the vertical diameter, and eq. (3) expresses these facts.

Recurring to Fig. 501, it is seen that the particular harmonic motion there delineated, has two maxima (at E and E') and two minima (at A_1 and C_1) within the period T: but it is easily conceivable that there can be phenomena having a multiplicity and variety of maxima and minima within their cycles—that is to say, a primary Period associated with several subordinate Periods of successively decreasing length. A piano-wire, for example, often vibrates as a whole at the same time that its half, third, fourth, and other serial subdivisions are vibrating—it is the case of a fundamental note and its harmonics: the form of the curve representing the composite sound will depend on the number and amplitude of the harmonics and their coincidence or difference of phase.

An assemblage of terms similar to eq. (3), representing all the harmonics of any phenomenon, constitutes Fourier's Series; and if the phenomenon is deficient in any particular

harmonic, its term is absent from the series. It is thus seen that every variety of undulation in water, air, or ether—symmetrical or irregular (for the latter results only from superposition of the former)—can be expressed by Fourier's Series.

Figs. 501 to 507 represent the series $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, the individual terms of which have 2, 3, 4, 5, 6, 7, and 8 points of maxima and minima, respectively—that is, sub-periods of these values—associated with a primary.

If the even-numbered terms *only* ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$) of this series be retained, we have, as it were, the magnetic harmonics of a ship—the semicircular, quadrantal, sextantal, and octantal deviations, to which may be added, indefinitely, other terms ($\frac{1}{10}$, $\frac{1}{12}$, etc.), that is, the decantal, dodecantal, etc., deviations. The constant deviation is provided for in Fig. 500 by reckoning from N as the initial point, and in eq. (2) by the angle α. A crude idea of the relative importance of the several components of the total deviation is afforded by the size of the sinuosities in Figs. 501, 503, 505, and 507.

295. How the deviations arise.—The general action of one magnet upon another has been described in Art. 176, and in Art. 225 in connection with Figs. 373 to 376; the action of the Ship upon the Compass is only a particular case of this general phenomenon.

Let Fig. 508 represent a ship steaming in a circle for the purpose of determining a Table of Deviations: suppose her forward body pervaded by blue magnetism and the after body by red; and let the compass be placed within the influence of the blue region.

The direction of the needle—its balance between this deviating power of the Ship and the directive force of the Earth—is indicated at C on board and by the arrows $A, B, \ldots, H$ within the circle of swing.

Beginning with the ship heading north at (1), her mag-

netic influence is in line with the needle and causes no deviation; as she changes course, the force gradually acquires leverage, causing the needle to deflect toward the east—position (2) and arrow B; both leverage and deviation attain a maximum at (3), and arrow A, when the ship heads east; the swing continuing, the hold o: the ship's magnetism gradually relaxes and the needl returns toward the meridian—coinciding with it as tl ship heads south, position (5), where the disturbing i fluence is again parallel to the compass: the action in southern semicircle is similar, only that here the ne is deflected to the westward—increasingly so in the quarter—attaining a maximum at position (7)—and g ually returning to the meridian during the swing ir S.W. quarter.

It will be perceived that throughout the western circle the poles of the ship and needle nearest each are of opposite name, and hence the directive f increased; in the eastern semicircle, on the contr converse of this is the case, and the directive force i ished: on northerly courses, therefore, the com be abnormally steady, and on southerly, very sl the maximum of both conditions occurring when head is north and south respectively, and taperin a mean value as she heads either east or west.

These are the conditions for this particular vary with the location of the regions of polar one.

The deviations are graphically represented by drawing perpendiculars to the vertical li the right between 0° and 180°, and to the to 360°—each proportional to the deviat heading of the ship, and tracing a curve t we find it to be the familiar wave form

Harmonic motion was illustr

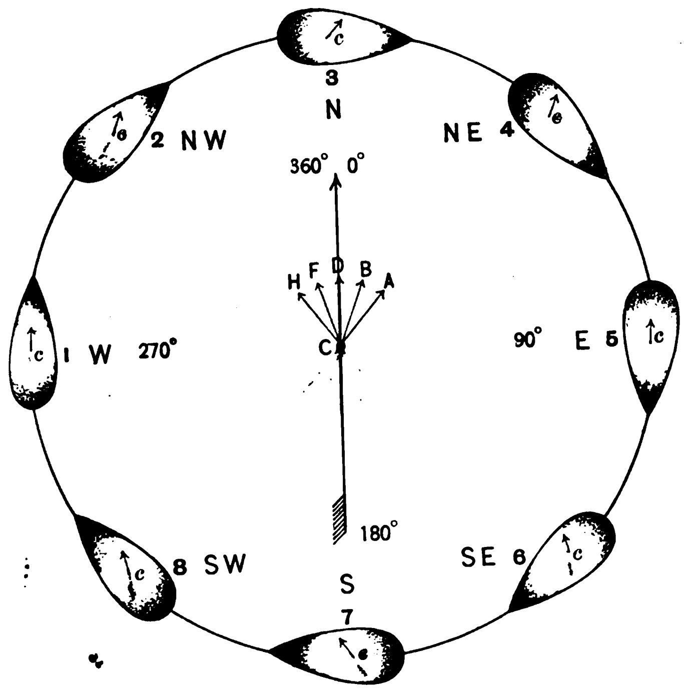

FIG 508.

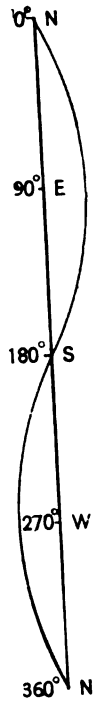

FIG. 509.

(*To face p.* 850.)

equally so by Fig. 508: comparing both, the point P revolving about a center is identical with the ship steaming in a circle; the to-and-fro motion of the point H in Fig. 500 is analogous to the alternate deflection east and west of the needle in Fig. 508; portions of the Period T correspond to different Azimuths of the ship's head; and the representation of the results is the same—a wave form.

Section Two: Definition of Terms and Meaning of Symbols.

Throughout the rest of this Treatise many terms and symbols are used, which will now be defined; and the meaning thus attached to them will be applicable hereafter.

296. Trigonometrical functions.

[1.] A ship steaming in a circle, describes angles of every value from 0° to 360°, and the deviation is wholly dependent on the angle or azimuth of her head—the course: it is not, however, with the angle directly that the relationship is generally carried on, but with certain of its properties—its Trigonometrical functions.

In Fig. 510, describe a circle and let x be one of the acute angles of two similar right-angled triangles, ABC and TDC: the sides of each of these triangles will be of different lengths according to the value of x, so that in the same triangle, the *ratio* of any two of its sides will represent the value of x; according to the sides taken, this ratio has received certain names, *sine*, *cosine*, *tangent*, etc. Thus:

$$\sin x = \frac{\overline{AB}}{\overline{CA}} = \overline{AB} \text{ (when } \overline{CA} = 1); \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\tan x = \frac{\overline{AB}}{\overline{CB}} = \frac{\overline{TD}}{\overline{CD}} = \frac{\overline{TD}}{\overline{CA}} = \overline{TD} \text{ (when } \overline{CA} = 1); \quad . \quad (13)$$

$$\sec x = \frac{\overline{CA}}{\overline{CB}} = \frac{\overline{CT}}{\overline{CD}} = \frac{\overline{CT}}{\overline{CA}} = \overline{CT} \text{ (when } CA = 1). \quad . \quad (14)$$

The *cosine*, *cotangent*, and *cosecant* of x are respectively the sine, tangent, and secant of its complement, CAB, $(=ACF=y)$; that is,

$$\cos x = \sin y = \frac{\overline{AF}}{\overline{CA}} = \frac{\overline{CB}}{\overline{CA}} = \overline{CB} \text{ (when } \overline{CA} = 1); \quad . \quad (15)$$

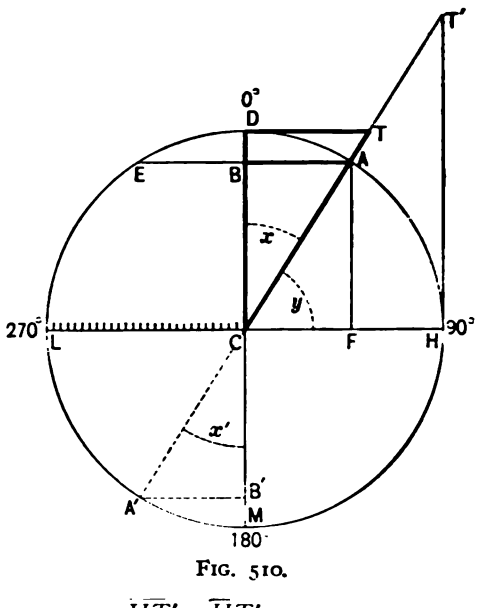

FIG. 510.

$$\cot x = \tan y = \frac{\overline{HT'}}{\overline{CH}} = \frac{\overline{HT'}}{\overline{CA}} = \overline{HT'} \text{ (when } \overline{CA} = 1); \quad (16)$$

$$\operatorname{cosec} x = \sec y = \frac{\overline{CT'}}{\overline{CH}} = \frac{\overline{CT'}}{\overline{CA}} = \overline{CT'} \text{ (when } CA = 1). \quad (17)$$

When the radius $\overline{CA}$ of the circle is unity, then $\overline{AB}$, $\overline{CB}$, $\overline{TD}$, etc., in eqs. (12) to (17) represent the *ratios* sine,

nt, etc., of x; and it is these *lines* that are in mind when we think of the Trigonometrical nt the conception is correct only when the unity. If this radius be divided into any numal parts, say one thousand, the whole length $\overline{CL}$ ty, then applying this as a standard of measure B, $\overline{TD}$, etc., it will be found that each line is equal fic number of these parts for a definite value of x: nce, if $x = 33° 45'$, the line $\overline{AB}$ will have .555 such $\overline{CB} = .831$; and $\overline{TD} = .668$; and these numbers are tural sine, cosine, and tangent respectively of 33° 45', milarly for all other angles.

nt, as a rule, it is with the logarithms of these numhat calculations are made, whence, in addition to s of natural sines, etc., we have Tables of log. sines, that is,

log of (natural sine) .555 = log sin 33° 45′ = 9.7448;
log of (natural cos) .831 = log cos 33° 45′ = 9.9199;
log of (natural tan) .668 = log tan 33° 45′ = 9.8249.

ı.] The natural sines of the angles corresponding to the points of the compass are frequently used in calulations of the deviations; they are denoted by S_1, S_2, S_3, etc.: thus,

$$S_1 = \text{nat. sin. } 11° 15' \text{ (N. by E.)} = .1951.$$

In Fig. 511, the angular values of the points, reckoned rom North, are given in the second circle from outward, nd the symbols appropriate to them, S_1, S_2, S_3, etc., in he third circle: in the quarters of the central space, the lgebraic signs of the sine and cosine in each quadrant re given.

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minutes (′),
string might be
and meas-
that the lines AB,
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tentative process
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times its
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. . . . . (18)

$= 206265''$. . (19)

following are

. . . (20)

. . . (21)

. . (22)

arc of 1°.
radius equal
converse, or
of such por-

tion of the circumference as is equal in length to the radius.

Whatever the length of the radius, R, the relation of

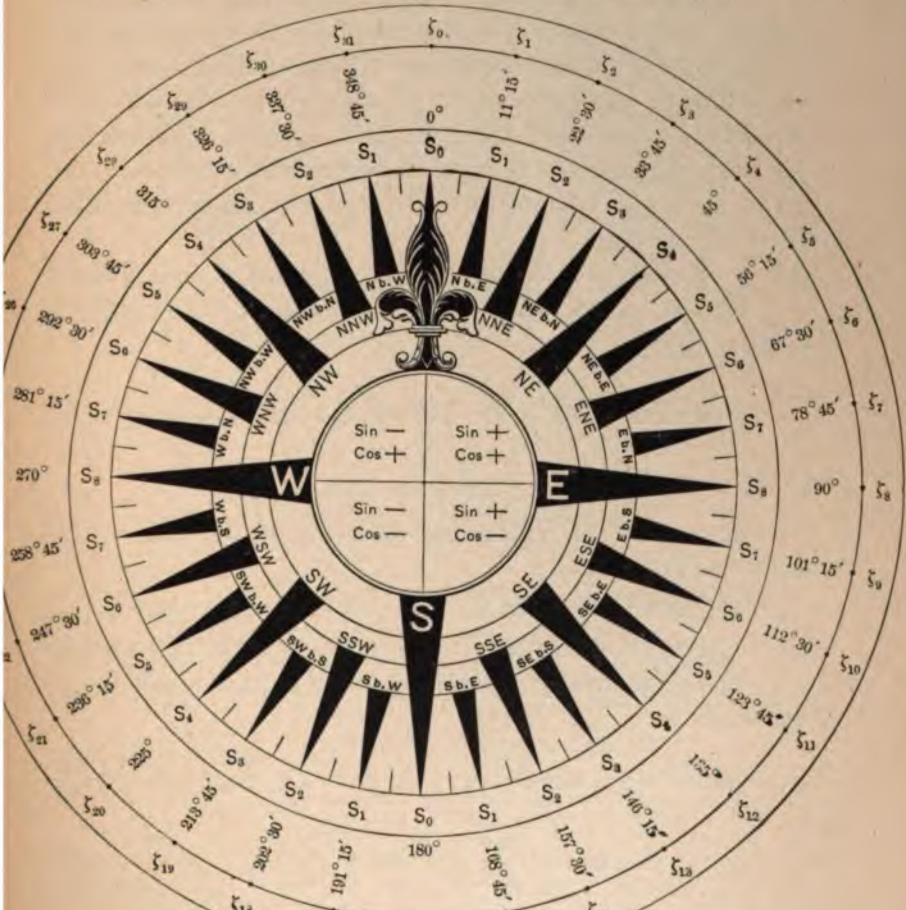

FIG. 511.

eq. (19) is invariable. There are thus two methods of expressing arcs: 1st, as a portion of a graduated circumference, as $x = 22° \, 15' \, 30''$, and 2d, in decimal parts of a

radius taken as unity, and this is analogous to representing angles by their natural sines, cosines, etc.; the 2d method is often employed.

If the arc is expressed in degrees, minutes, and seconds, and we wish to express it in parts of the radius, it must be multiplied by the last members of eqs. (20), (21), or (22), according as the given arc is in degrees, in minutes, or in seconds: on the other hand, if given in parts of radius and we wish to pass to degrees, or minutes, or seconds, the given arc must be multiplied by the corresponding values of R in eq. (19).

[4.] In Fig. 510, the *versed-sine*, or *versin* as it is usually written, of the arc $\overline{AD}$ corresponding to the angle x, is $\overline{BD}$, that is,

$$\text{versin } x = \overline{BD} = \overline{CD} - \overline{CB} = \overline{CA} - \overline{CB} = 1 - \cos x, \quad (23)$$

the last member being deduced from eq. (15).

Let $2x = \text{arc } ADE$, then $\overline{AE}$ is its chord, and $\overline{AE} = 2\overline{AB}$, whence by eq. (12)

$$\text{chord } 2x = 2 \sin x; \quad \text{or,} \quad \text{chord } x = 2 \sin \tfrac{1}{2}x. \quad . \ (24)$$

[5.] For convenience of reference a few of the formulas of Trigonometry most frequently required will be inserted here.

$$\sin 0° = 0; \quad \cos 0° = 1; \quad \sin 90° = 1; \quad \cos 90° = 0. \quad (25)$$

$$\tan x = \frac{\sin x}{\cos x}; \quad \cot x = \frac{\cos x}{\sin x}. \quad . \ . \ . \quad (26)$$

$$\sin^2 x + \cos^2 x = 1. \quad . \ . \ . \ . \ . \ . \quad (27)$$

$$2 \cos^2 \tfrac{1}{2}x = 1 + \cos x. \ . \ . \ . \ . \ . \ . \quad (28)$$

$$2 \sin^2 \tfrac{1}{2}x = 1 - \cos x. \ . \ . \ . \ . \ . \ . \quad (29)$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y. \quad . \quad . \quad (30)$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y. \quad . \quad . \quad (31)$$

$$\sin(-y) = -\sin y; \quad \cos(-y) = \cos y. \quad . \quad . \quad (32)$$

$$\sin(y - 90^\circ) = -\sin(90^\circ - y) = -\cos y. \quad . \quad (33)$$

$$\cos(y - 90^\circ) = \cos(90^\circ - y) = \sin y. \quad . \quad . \quad . \quad (34)$$

$$\sin(x+y) + \sin(x-y) = 2 \sin x \cos y. \quad . \quad . \quad (35)$$

$$\sin(x+y) - \sin(x-y) = 2 \cos x \sin y. \quad . \quad . \quad (36)$$

$$\cos(x+y) + \cos(x-y) = 2 \cos x \cos y. \quad . \quad (37)$$

$$\cos(x+y) - \cos(x-y) = -2 \sin x \sin y. \quad . \quad (38)$$

$$\sin 2x = 2 \sin x \cos x. \quad . \quad . \quad . \quad . \quad . \quad (39)$$

$$\cos 2x = \cos^2 x - \sin^2 x. \quad . \quad . \quad . \quad . \quad (40)$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

In eq. (38), let $x = mz$, and let $y = z$, then (38) becomes

$$\cos(m+1)z - \cos(m-1)z = -2 \sin mz \sin z; \quad . \quad (42)$$

or
$$\cos(m-1)z - \cos(m+1)z = 2 \sin mz \sin z. \quad . \quad (43)$$

This last formula will be found useful hereafter.

[6.] Recurring to Fig. 511, let the points of the compass, beginning with North, be denoted by ζ_0, ζ_1, ζ_2, . . . , ζ_{31}: then *any two* courses, diametrically opposite, will have *sines* and *cosines* of the same numerical value but of opposite algebraic sign, and hence their sum is zero. For example,

$$\sin \zeta_5 = -\sin \zeta_{21}; \quad \therefore \sin \zeta_5 + \sin \zeta_{21} = 0. \quad . \quad (44)$$

$$-\cos \zeta_{14} = \cos \zeta_{30}; \quad \therefore \cos \zeta_{30} + \cos \zeta_{14} = 0. \quad . \quad (45)$$

That the *sines* and *cosines* of courses differing by 180° are equal, may be seen by Fig. 510; for $x = x'$; $\overline{AB}$,(sin) $= -\overline{A'B'}$,(sin); and $\overline{CB}$,(cos) $= -\overline{CB'}$,(cos).

Let Σ indicate summation of quantities: then if Σ_{32} denote the sum of like quantities for each of the 32 points of the compass, by analogy with eqs. (44) and (45), we have:

$$\Sigma_{32} \sin \zeta = 0; \quad \Sigma_{32} \cos \zeta = 0. \quad . \quad . \quad . \quad (46)$$

$$\Sigma_{32} \sin 2\zeta = 0; \quad \Sigma_{32} \cos 2\zeta = 0. \quad . \quad . \quad (47)$$

$$\Sigma_{32} \sin \zeta \cos 2\zeta = 0; \quad \Sigma_{32} \sin \zeta \sin 2\zeta = 0. \quad . \quad (48)$$

$$\Sigma_{32} \sin \zeta \cos 2\zeta = 0; \quad \Sigma_{32} \cos \zeta \sin 2\zeta = 0. \quad . \quad (49)$$

$$\Sigma_{32} \cos \zeta \cos 2\zeta = 0; \quad \Sigma_{32} \sin 2\zeta \cos 2\zeta = 0. \quad (50)$$

$$\sin^2 \zeta_0 = \sin^2 \zeta_{16} = 0; \quad \sin^2 \zeta_8 = \sin^2 \zeta_{24} = 1. \quad . \quad (51)$$

$$\sin^2 \zeta_4 = \sin^2 \zeta_{12} = \sin^2 \zeta_{20} = \sin^2 \zeta_{28} = \tfrac{1}{2}. \quad . \quad . \quad (52)$$

$$\sin^2 \zeta_5 = \cos^2 \zeta_3; \quad \sin^2 \zeta_6 = \cos^2 \zeta_2; \quad \sin^2 \zeta_7 = \cos^2 \zeta_1. \quad (53)$$

$$\Sigma_{32} \sin^2 \zeta = 16; \ \Sigma_{32} \cos^2 \zeta = 16; \quad \Sigma_{32} \sin^2 2\zeta = 16;$$
$$\Sigma_{32} \cos^2 2\zeta = 16. \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

Formulas (46) to (54) will be found of use later.

[7.] The transformation of coördinates will be explained, as it enters into the formulas of the heeling error.

Let $\overline{CX}$ and $\overline{CY}$, Fig. 512, be rectangular axes to which the point P is referred; turn them through the angle α, and draw $\overline{PD}$ and $\overline{BF}$ parallel to Y, and $\overline{EB}$ to X. Then the angle $EPB = \alpha$; $x = \overline{CD}$; $y = \overline{PD}$; $x' = \overline{CB}$; $y' = \overline{PB}$.

$$x = \overline{CD} = \overline{CF} - \overline{DF} = \overline{CF} - \overline{EB}. \quad . \quad . \quad . \quad (55)$$

$$\overline{CF} = x'.\cos\alpha; \quad \overline{EB} = y'.\sin\alpha, \text{ whence} \quad . \quad . \quad (56)$$

$$x = x'.\cos\alpha - y'.\sin\alpha. \quad . \quad . \quad . \quad . \quad . \quad (57)$$

$$y = \overline{PD} = \overline{ED} + \overline{PE} = \overline{BF} + \overline{PE}. \quad . \quad . \quad . \quad (58)$$

$$\overline{BF} = x'.\sin\alpha; \quad \overline{PE} = y'.\cos\alpha, \text{ whence} \quad . \quad . \quad (59)$$

$$y = x'.\sin\alpha + y'\cos\alpha. \quad . \quad . \quad . \quad . \quad (60)$$

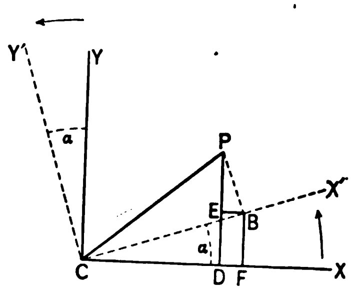

FIG. 512.

Multiply (57) by cos α, and (60) by sin α; add the results, and by means of (27) we find

$$x' = x.\cos\alpha + y.\sin\alpha. \quad . \quad . \quad . \quad . \quad (61)$$

Then multiply (57) by sin α, and (60) by cos α; subtract the results, and by means of (27) we have

$$y' = -x.\sin\alpha + y.\cos\alpha. \quad . \quad . \quad . \quad . \quad (62)$$

Equations (57), (60), (61), and (62) give the old coördinates in terms of the new, and conversely.

To pass from spherical to right-line coördinates, consider Fig. 513: P is a point on the surface of a sphere, and E its projection on the horizontal plane through the equator; the arc FP ($=$angle α) and the angle $HCL = HFL = \beta$

are the spherical coördinates, and $\overline{CD}=x$; $\overline{ED}=y$; $\overline{EP}=\overline{CM}=z$, the rectilinear: then from the figure,

$$x=\overline{CE}.\cos ECD, \quad \text{and} \quad \overline{CE}=\overline{MP}=r.\sin\alpha, \text{ whence} \quad (63)$$

$$x=r.\sin\alpha.\cos\beta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

$$y=\overline{CE}.\sin ECD, \text{ whence } y=r.\sin\alpha.\sin\beta. \quad . \quad . \quad (65)$$

$$z=\overline{CP}.\cos MCP=r.\cos\alpha. \quad . \quad . \quad . \quad . \quad (66)$$

Equations (64), (65), and (66) give the rectilinear coördinates in terms of the spherical.

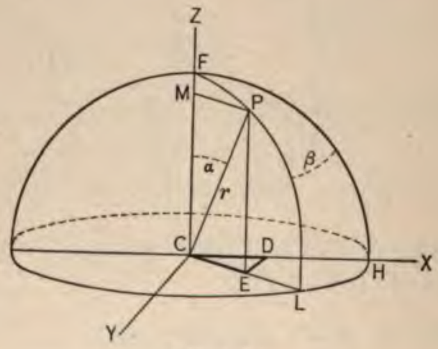

Fig. 513.

297. Magnetic Elements of the Earth and Ship, separate and combined.

[1.] The magnetic elements of the Earth have been so fully treated in Chapter VII and Arts. 196 and 253, that only a diagram for reference, is needed here—Fig. 514.

θ = Dip: This has been designated by D, but in view of this letter being used soon for one of the magnetic coefficients, the Dip will hereafter be represented by θ.

T = Total Intensity: It has been, and will continue to be, designated by T; this letter also stands for the

Period of harmonic motion and for the time in which a magnetic needle makes one oscillation; it will continue to represent all three quantities, as the context in each case will show which is meant.

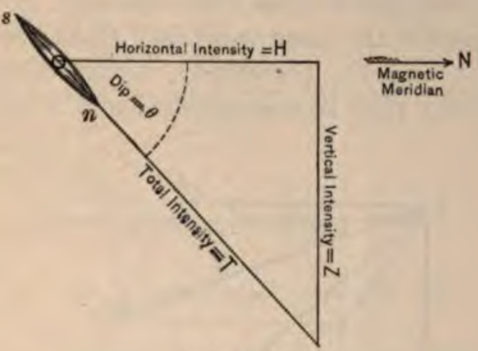

FIG. 514.

DYNE: This is defined in Art. 180.

$2l$: This means the EXACT LENGTH of a magnetic needle—not that between the estimated location of poles.

M represents the MAGNETIC MOMENT of the needle.

m is the POLE-STRENGTH of a needle.

[2.] The composition and resolution of forces enter fundamentally into the investigation of the Deviations, and the principles of the process will be now briefly explained. In Fig. 515, let $\overline{CX}$, $\overline{CY}$, $\overline{CZ}$ be axes at right angles to each other, and let $\overline{CM}$ represent a force f: within the solid angle formed by planes through the axes, f may take any one of an infinity of directions making different angles with the axes; the length of the projections of f on the axes will accordingly vary, and upon these projections a rectangular form, as in Fig. 515, may be constructed.

The projection of $\overline{CM}$ *directly* upon the axes may be made by multiplying it by the *cosine* of the angle it makes

with each; but for the purpose of this article, it will first be projected on the horizontal plane as the trace $\overline{CF}$, and then this on X and Y as $\overline{CS}$ and $\overline{CA}$ respectively: $\overline{CM}$ is projected directly on Z, as $\overline{CB}=AN=FM$.

By this method we obtain the COMPONENTS f_1, f_2, f_3, of f in the coördinate axes: it is evident, however, that f_1 and f_2 have not the same value as if f had been projected *directly* on X and Y. Conversely, if three forces f_1, f_2, f_3, are given, they may be compounded into a ***resultant*** f.

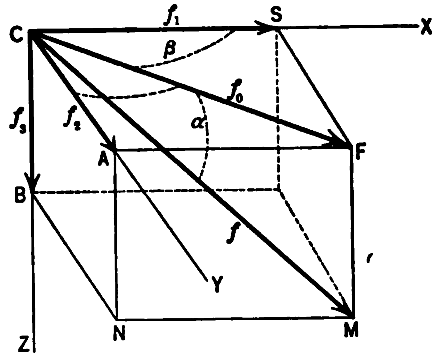

FIG. 515.

Analytically, the components of f are thus deduced: $\overline{CF}=f_0$ is the trace of f upon the horizontal plane; it makes the angle α with f, and β with f_1; BCM and ACF are the complements of α and β respectively; and CSF, CAF, and CAN are each 90°, the figure being rectangular. Then

$$f_0=f.\cos\alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

$$f_1=\overline{CS}=f_0.\cos\beta=f.\cos\alpha.\cos\beta. \quad . \quad . \quad . \quad (68)$$

$$f_2=\overline{CA}=\overline{SF}=f_0.\sin\beta=f.\cos\alpha.\sin\beta. \quad . \quad . \quad (69)$$

$$f_3=\overline{CB}=\overline{FM}=f.\sin\alpha. \quad . \quad . \quad . \quad . \quad (70)$$

[3] Transferring Fig. 515 to Fig. 516, $\overline{CM}$ in the latter represents the direction and strength of the Ship's

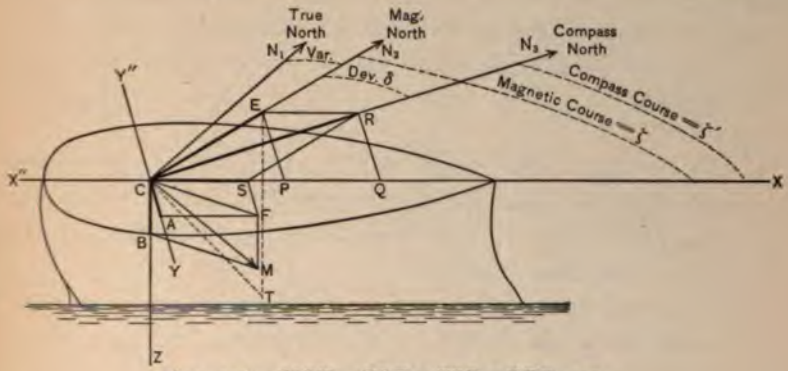

FIG. 516: $YCX = CPE = CQR = CET = 90$.

magnetic force acting upon the compass; and $\overline{CS}$, $\overline{CA}$, $\overline{CB}$ are its components—fore-and-aft, athwartships, and vertically toward the keel.

We have thus *begun* with the *resultant* force of the ship, $\overline{CM}$, and resolved it into three principal components; each of these may be separated into two parts—one representing the effect of hard iron, the other of soft; again, each of the latter may be divided into three classes, according to the direction of the iron; and finally, if we proceed to the ultimate masses contributing to the total magnetic disturbance, we obtain individualities whose number is legion: each mass may have its effect expressed by equations similar to (68), (69), and (70).

When these equations embody the conditions peculiar to each kind of iron, they naturally fall into distinct categories: summing up the latter, we have equations expressive of the magnetic force of the ship, and this is the framework of the mathematical theory of the deviations.

Thus it is seen that throughout this process it is forces of definite direction and strength that are dealt with—

compounded and resolved as occasion requires—and that these forces may have any other name, physical as well as magnetic, so far as the mathematical treatment is concerned: that considers force in the abstract only.

Fig. 516 illustrates many of the symbols that will now be defined:

V = VARIATION: the angle N_1CN_2 formed by meridians through the True and the Magnetic North, respectively.

ζ = MAGNETIC COURSE: heading of ship reckoned eastward from magnetic North.

ζ' = COMPASS COURSE: heading of ship reckoned eastward from North indicated by the compass.

δ = DEVIATION, the angle $N_2CN_3 = \zeta - \zeta'$.

$X, Y, Z = \overline{CX}, \overline{CY}, \overline{CZ}$ respectively: rectangular axes *fixed in ship* and revolving with it (the two first in the horizontal plane) round the third; C is the compass pivot; $\overline{CX}$ is in the central vertical plane through keel; $\overline{CY}$, athwartships; and $\overline{CZ}$, vertical; these directions are positive, and their opposites, CX'', CY'', CZ'' are negative.

$T = \overline{CT}$: EARTH'S TOTAL MAGNETIC FORCE.

θ = angle ECT = DIP.

$Z = \overline{ET}$ = VERTICAL COMPONENT of Earth's total force T.

$H = \overline{CE}$ = HORIZONTAL COMPONENT of Earth's total force T.

$X = \overline{CP}$ = Component of H in axis of X.

$Y = \overline{EP}$ = Component of H in axis of Y (here negative).

$\overline{CM}$ = Total magnetic force of *ship* alone.

$\overline{CF}$ = Trace of $\overline{CM}$ on deck, ship on even keel

$\overline{CS}$ = Component of $\overline{CF}$ in axis of X.

$\overline{CA} = \overline{SF}$ = Component of $\overline{CF}$ in axis of Y.

$\overline{FM} = \overline{CB}$ = Component of $\overline{CM}$ in axis of Z.

$H' = \overline{CR}$ = Resultant of $\overline{CE}$ and $\overline{CS}$; that is, resultant of horizontal forces of Earth and Ship, which resultant is always in the direction of the compass needle: $\overline{N_1C}$, $\overline{N_2C}$, and $\overline{N_3C}$ are all horizontal.

$X' = \overline{CQ}$ = Component of H' in axis of X; that is, the combined magnetic force of Earth and Ship in direction of ship's head.

$Y' = \overline{QR}$ = Component of H' in axis of Y; that is, the combined magnetic force of Earth and Ship toward side (in this case to port, and hence negative).

$Z' = \overline{ET} \pm \overline{FM}$ = Combined magnetic force of Earth and Ship in axis of Z, or downward.

λ: In Fig. 516, $\overline{CR}(=H')$ represents the combined horizontal force of Earth and Ship upon the compass for one particular heading; but it varies with every heading, as explained in Art. 295; it is determined by oscillation experiments with a horizontal needle while the ship is held successively on 32, 16, 8, 4, or 2 equidistant points and thus we obtain values H_1', H_2', ... , H_n'; a value of H, the Earth's horizontal intensity, is obtained by similar observations ashore in the same locality with the same needle; then

$$\lambda = \frac{\frac{1}{n}(H_1' + H_2' + H_3' \ldots H_n') \cos \delta}{H}. \quad . \quad . \quad (71)$$

That is, λ is the ratio of the *mean* value of the combined force of Earth and Ship (projected on the magnetic meridian) to the Earth's horizontal intensity: briefly, it is called the "*mean force to magnetic north*," and does not vary with change of geographical position.

μ: Heretofore used to denote magnetic permeability, it will now be employed, in addition, to represent another quantity: the context in every case will indicate the meaning attached to it.

Referring to Fig. 516, it is seen that $Z'(=\overline{ET} \pm \overline{FM})$ is the combined vertical force of Earth and Ship for a particular heading; and it is evident that if this heading be in the direction of the compass-needle, the downward pull

compounded and res…
these forces may hav…
as magnetic, so far as…
cerned: that consider…

Fig. 516 illustrate…
be defined:

V = VARIATION: the…
through the True…
ζ = MAGNETIC COURSE…
from magnetic N…
ζ' = COMPASS COURSE…
from North indi…
δ = DEVIATION, the…
$X, Y, Z = \overline{CX}, \overline{CY}$, …
in ship and re…
zontal plane) roun…
$\overline{CX}$ is in the cen…
athwartships; an…
tive, and their op…
$T = \overline{CT}$: EARTH'S…
θ = angle ECT = …
$Z = \overline{ET}$ = VERTIC…
$H = \overline{CE}$ = HORIZ…
$X = \overline{CP}$ = Comp…
$Y = \overline{EP}$ = Comp…
$\overline{CM}$ = Total ma…
$\overline{CF}$ = Trace of …
$\overline{CS}$ = Compone…
$\overline{CA} = \overline{SF}$ = Con…
$\overline{FM} = \overline{CB}$ = Co…
$H' = \overline{CR}$ = Res…
horizonta…
is always in
X_2C, and X_3C…

The actual iron of the ship is neither absolutely hard entirely soft, but of many degrees of both; and this tion is reflected in the fluctuating values of the mag- coefficients: if it is all quite hard, the coefficients be nearly constant under every vicissitude; but if ly soft, they will vary extremely with geographical nge.

Another hypothesis of the theory is, that the length the compass-needle is very small compared with the ance of the nearest iron.

The magnetism of a ship is called permanent, sub- manent, and transient.

The *transient* is particularly that induced in soft iron; also such as finds temporary lodgement from steering same course for some time: it is induced by the Earth both hard and soft iron. The *sub-permanent* is the *sur-* rge that enters the hard iron while the ship is building: generally wastes away during the first voyage. The *manent* is what remains in the hard iron after all loose gnetism has been shaken out: it gives the ship her mag- ic character.

As magnetic forces—however varied in direction—can resolved into three axes, so the material in which they ide—the hard and soft iron—may be classed in three nes, whatever be the real inclination of the iron in the p.

In Fig. 516, the axes X-Y-Z, fixed in the ship, are the ections in which the components of all the magnetic ces are resolved.

Q, R: these represent the effect of *hard* iron: P, fore- and-aft; Q, athwartships; and R, vertically; they resent the *permanent* magnetism, though in truth this ofttimes weakened by slight loss or strengthened by nsient gain—all which appears in the fluctuating values the coefficients.

will be greatest or least according to which end of needle is proximately acted upon, whereas at right ar to the meridian the pull will have a mean value: the therefore varies with the heading of the ship.

Values of Z' are determined by oscillation exper with a small dip circle turned until the needle is in each case, and then oscillated while the ship successively on 2, 4, or more equidistant points: obtain Z_1', Z_2', . . ., Z_n'. The value of Z, th vertical intensity alone is obtained by similar o on shore in the same locality with the same ne

$$\mu = \frac{\frac{1}{n}(Z_1' + Z_2' + \ldots Z_n')}{Z} \quad . \quad .$$

That is, μ is the ratio of the mean value of Ship's combined vertical force to that of t and as Z varies in different parts of the as seen by eq. (72).

J = Coefficient of the Heeling Error, which in the chapter on that subject.

Some Greek letters are occasionally tise, and it will be convenient to have t reference: α (alpha); β (beta); δ (((theta); λ (lambda); μ (mu); π (pi): ω (omega).

298. The mathematical representat iron.—In Part Third, the theory o illustrated *physically* by steel bars an the effect of these can be represent will now be explained.

The mathematical theory consi ship divided into two classes—the to magnetize, but retentive of wh *soft*, which both easily acquires. netism, upon removal of the i

being then diffused along the sides of the iron, equally at each end, and therefore ineffective.

This is not the customary explanation of the action of transient magnetism in horizontal soft iron: the one generally made is, that the Earth's horizontal component is resolved in the direction of the iron, and that it is this resolved part—varying with the *cosine* of the azimuth—that produces the induced effect: but if we consider a *cube* of soft iron, and turn it in azimuth, the side facing the north will always exhibit red polarity, whether the *same* side faces north or east; that is, the induced magnetic condition remains unchanged in absolute direction while the body in which it exists, turns. Indeed the experiment has been tried with a sphere of soft iron, and however rapidly it was spun round, the segment toward the lower end of the dipping-needle had constantly one kind of magnetism and the diametrically opposite segment the other kind, showing that the induced magnetic condition had a fixed direction in space. (See Fig. 551.)

If now the cube of soft iron be lengthened into a bar of small cross-section, or a long rod or tube of small diameter, there is every reason to expect that the induced magnetic condition will maintain a fixed direction in space, while the bar, rod, or tube which embodies it, is turned, thus giving opposite poles at the ends when its length coincides with the magnetic meridian, but along the sides when across the meridian.

The matter is illustrated by Fig. 551, where the distribution of transient magnetism in a long soft-iron tube is shown with it pointing north, northeast, and east: the neutral-line dividing both polarities is fixed in space—always at right angles to the dipping-needle.

The field of the *permanent* magnetism of the ship herself will either assist or oppose the Earth's influence; so that in reality the inducing force is, $H \pm$ ship and $Z \pm$ ship,

and a, b, $c \ldots k$ are the ratios of the induced charges to these quantities, which ratios are thus dependent for their absolute values on the strength of the inducing force and the amount, arrangement, and capacity for induction of the soft iron of the ship.

It is almost needless to state that the ship only strengthens or weakens the induced charge of the Earth, without altering the nature of its action as a FORCE in different azimuths or with geographical change: that remains as already stated.

299. The magnetic coefficients—Exact and Approximate. The deviations are expressed by Fourier's series, and while this is theoretically infinite, still only a few terms are necessary for the degree of accuracy requisite in these calculations: five generally suffice.

A: The first term (A) is a summation of *possible* errors.

$B.\sin \zeta' + C.\cos \zeta'$. . (73): This embraces the second and third terms of the series, each made up of two factors—the coefficients B and C (constant for the same time, place, and conditions), and the variable ζ'; the effect of the latter is to produce two maxima and two minima in the deviations during the circuit of swing.

The *Period* of these terms—explained in Art. 294—is a circle with values of opposite sign in each half, and hence it is called the *Semicircular* deviation: it represents the joint action of permanent magnetism in hard iron and transient magnetism in vertical soft iron.

$D.\sin 2\zeta' + E.\cos 2\zeta'$. . (74): This embraces the fourth and fifth terms, which, with all subsequent pairs, is (like the preceding) made up of two parts each composed of two factors—a coefficient and a variable: the former is constant for the same time, place, and conditions, while the latter introduces a different period into each pair of terms. In this, the coefficients are D and E, and the period is a semicircle, with maxima and minima values of

the deviation of opposite sign in each half, that is, in each quadrant, and hence this is called the *Quadrantal* deviation: it represents the action of transient magnetism in horizontal soft iron.

$F.\sin 3\zeta'+G.\cos 3\zeta'$. . (75): This embraces the sixth and seventh terms: coefficients F and G, and period one third of a circle, with maxima and minima values of opposite sign in each half of this, that is, in every sixth of the circle, and hence it is called the *Sextantal* deviation.

These terms are only occasionally used, while all subsequent ones are rarely employed.

The greatest variety of values that any quantity can have, is expressed by the tangent of an angle; for while the angle increases from 0° to 90°, its tangent varies from zero to infinity: therefore, denoting the radius of a circle by R, the following relations are always true:

$$R.\sin\alpha=C, \quad \text{and} \quad R.\cos\alpha=B. \quad . \quad . \quad (76)$$

Whence, by means of eq. (26),

$$\tan\alpha=\frac{C}{B}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

The angle α is called the *Starboard angle*, and indicates the direction of the resultant force producing semicircular deviation. Squaring (76) and adding, we have, by means of eq. (27),

$$R_2\,(\sin^2\alpha+\cos^2\alpha)=B^2+C^2; \quad \therefore R=\sqrt{B^2+C^2}. \quad (78)$$

Substituting the values of (76) in (73), this becomes by means of eqs. (30) and (78),

$$B.\sin\zeta'+C.\cos\zeta'=R.\cos\alpha\sin\zeta'+R.\sin\alpha\cos\zeta'; \; . \quad (79)$$

$$=R\,(\sin\zeta'\cos\alpha+\cos\zeta'\sin\alpha); \quad . \quad (80)$$

$$=\sqrt{B^2+C^2}[\sin(\zeta'+\alpha)]. \quad . \quad . \quad . \quad . \quad (81)$$

The maximum value of this will occur when $(\zeta'+\alpha)=90°$, for then $\sin(\zeta'+\alpha)=1$; hence the maximum value of the semicircular deviation is

$$B.\sin\zeta'+C.\cos\zeta'=\sqrt{B^2+C^2}. \quad . \quad . \quad . \quad (82)$$

Similarly, if we make

$$\tan\beta=\frac{E}{D} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

we shall find in the same way from eq. (74)

$$D.\sin 2\zeta'+E\cos 2\zeta'=\sqrt{D^2+E^2}[\sin(2\zeta'+\beta)]; \quad (84)$$

$$=\sqrt{D^2+E^2}. \quad . \quad . \quad . \quad . \quad . \quad (85)$$

And this last, for a like reason, is the maximum value of the quadrantal deviation.

The coefficients are expressed either in *degrees* and *minutes*, and then they are designated by the *Italic* capitals *A*, *B*, *C*, *D*, *E*, known as the *Approximate Coefficients;* or they are expressed in *parts of the radius* as explained in Art. 296, par. [3], and then they are represented by the old English capitals $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$. These are known as the *Exact Coefficients;* and it will be seen later, when their values are deduced, that they are nearly the *sines* of the arcs denoted by the approximate coefficients: when, therefore, the arc and its sine—both expressed in the same measure—do not differ by an amount greater than is admissible in the deviations, either set of coefficients may be used. Limits of inaccuracy are shown in Table 72: the degrees of line (*b*) are converted into parts

TABLE 72.

(*a*)	(1)	(2)	(3)	(4)	(5)
(*b*)	Arc in degrees	5°	10°	15°	20°
(*c*)	Arc in parts of radius = 1	.0873	.1745	.2618	.3491
(*d*)	Sine in parts of radius = 1	.0872	.1736	.2588	.3420
(*e*)	Difference in parts of radius	.0001	.0009	.0030	.0071
(*f*)	Difference in minutes of arc	.34′	3.09′	10.3′	24.4′

of radius by multiplying each by eq. (20); the results are given in line (*c*); the quantities in line (*d*) are taken from a table of natural sines; the differences in line (*e*) are in parts of radius, and to convert them into minutes, each must be multiplied by $R = 3437'.7$ from eq. (19). If then we wish to restrict the inaccuracy to 25′, we may use either set of coefficients for deviations up to 20°.

Section Three: The Primary Equations and Their Adaptation for Use.

300. Poisson's equations.—However the ship's head may change in azimuth, the compass-needle always takes the direction of the resultant of all the forces acting on it; that is, the resultant of the disturbing force of the Ship and the directive force of the Earth. These several forces —resolved in three directions—have been explained in Arts. 297 and 298, and illustrated by Fig. 516, so that it only remains to sum them up each in its own axis, and express their balance, thus:

$$X' = X + a.X + b.Y + c.Z + P. \quad . \quad . \quad . \qquad (86)$$

$$Y' = Y + d.X + e.Y + f.Z + Q. \quad . \quad . \quad . \qquad (87)$$

$$Z' = Z + g.X + h.Y + k.Z + R. \quad . \quad . \quad . \qquad (88)$$

These are Poisson's fundamental equations: in them X', Y', Z' represent, with reference to the ship (Fig. 516), the longitudinal, transverse, and vertical COMPONENTS of the *resultant* of all the forces acting on the compass; while the quantities on the right of the sign of equality in each equation represent the *parts* of these COMPONENTS—that is, X, Y, Z, the force of the Earth, and the others, various groups of disturbing forces of the Ship.

Of the latter P, Q, R represent the effect of permanent

magnetism. The transient magnetism of vertical soft iron being induced by the Earth's vertical component, is a determinate part of that component, which part is denoted, for the several groups of this kind of iron, by c, f, k, or $c.Z$, $f.Z$, $k.Z$: as Z does not vary with the azimuth, neither will these; but as it does with geographical change, so will these. Similarly, the transient magnetism of horizontal soft iron is a specific part of the Earth's horizontal component, which, for the several groups of this kind of iron is represented by a, b, d, e, g, h, or $a.H$, $b.H$, $d.H$, $e.H$, $g.H$, $h.H$: but the FORCE exerted by this magnetism varies with the azimuth, and this fact is expressed by connecting the above quantities with the axes of X and Y, which, being fixed in the Ship and revolving with it, therefore represent every possible value of the *effective* force as $a.X$, $d.X$, $g.X$, $b.Y$, $e.Y$, $h.Y$: when either X or Y coincides with the meridian, they become equivalent to H; but otherwise, in their revolution as the ship swings, they represent the various values of H in the successive azimuths. It has been explained elsewhere why these do not vary with geographical change.

When quantities proper to the problem are introduced into Poisson's equations, and they are adapted for use, they afford methods for determining the magnetic character of a ship by means of certain coefficients; also methods for computing a table of deviations from these coefficients; and for predicting the change that will occur in the deviations upon change of geographical position; finally—in connection with oscillation experiments—for calculating the heeling error.

That is to say, the mathematical theory greatly lessens the labors of the navigator besides supplying a scientific analysis of the conditions involved; and that it accurately takes into account these conditions and provides for them within the limits of error admissible in the deviations,

has been abundantly shown by comparison of results with experiment.

301. Quantities proper to the problem introduced into Poisson's equations.—All the symbols now to be used are explained in Art. 297. Referring to Fig. 516, we have the following:

$$\overline{CP}=\overline{CE}.\cos ECP, \quad \text{or} \quad X=H.\cos\zeta. \quad . \quad . \quad (89)$$

$$\overline{EP}=-\overline{CE}.\sin ECP, \quad \text{or} \quad Y=-H.\sin\zeta. \quad . \quad (90)$$

$$\overline{ET}=\overline{CE}.\tan ECT, \quad \text{or} \quad Z=H.\tan\theta. \quad . \quad . \quad (91)$$

$$\overline{CQ}=\overline{CR}.\cos RCQ, \quad \text{or} \quad X'=H'.\cos\zeta'. \quad . \quad (92)$$

$$\overline{QR}=-\overline{CR}.\sin RCQ, \quad \text{or} \quad Y'=-H'.\sin\zeta'. \quad . \quad (93)$$

The sign is negative in (90) and (93) because the angles are reckoned in the direction Y'' in the axis of Y.

Substituting the values of (89) to (93) in (86), (87), (88), and dividing (86) and (87) by H, and (88) by Z, we have:

$$\frac{H'}{H}.\cos\zeta'=(1+a)\cos\zeta-b.\sin\zeta+c.\tan\theta+\frac{P}{H}. \quad (94)$$

$$-\frac{H'}{H}.\sin\zeta'=d.\cos\zeta-(1+e)\sin\zeta+f.\tan\theta+\frac{Q}{H}. \quad (95)$$

$$\frac{Z'}{Z}=\frac{g}{\tan\theta}.\cos\zeta-\frac{h}{\tan\theta}.\sin\zeta+1+k+\frac{R}{Z}. \quad . \quad (96)$$

These are Poisson's equations with the quantities proper to the problem introduced, and their meaning is this: the denominator in each denotes the unit of measure, so that (96) expresses the total downward pull on the needle in terms of the Earth's vertical force when the ship is on an even keel; similarly, (94) and (95) express the total

horizontal pull in terms of the Earth's horizontal force—the first toward the ship's head and the second toward the starboard side (in this case to port, as indicated by the negative sign). To resolve a force in any direction is to multiply it by the *cosine* of the angle included by the old and new directions; the component at right angles to the new direction is given by multiplying by the *sine* of the same angle: in Fig. 516, $\overline{CR}(=H')$ is the total horizontal pull, so that to multiply this by $\cos\zeta'$ and $\sin\zeta'$ respectively, as in eqs. (94) and (95), gives the components to bow and side.

As these components vary with every heading of the ship, it is desirable to have their values in two fixed directions—that is, in the magnetic meridian and in an east and west line: by observing Fig. 516, it will be seen that to multiply eq. (94) by $\cos\zeta$, and (95) by $\sin\zeta$, resolves both into the meridian, and then taking their difference, we get their value in this direction: performing the operations, we have

$$\frac{H'}{H}.\cos\zeta'\cos\zeta=(1+a)\cos^2\zeta-b.\sin\zeta\cos\zeta$$
$$+c.\tan\theta.\cos\zeta+\frac{P}{H}.\cos\zeta. \quad (97)$$

$$-\frac{H'}{H}.\sin\zeta'\sin\zeta=d.\cos\zeta\sin\zeta-(1+e)\sin^2\zeta$$
$$+f.\tan\theta\sin\zeta+\frac{Q}{H}\sin\zeta. \quad (98)$$

Subtracting (98) from (97), we have

$$\left.\begin{aligned}\frac{H'}{H}(\cos\zeta'\cos\zeta+\sin\zeta'\sin\zeta)&=-(d+b)\cos\zeta\sin\zeta\\ &+(1+a)\cos^2\zeta+(1+e)\sin^2\zeta\\ &+\left(c.\tan\theta+\frac{P}{H}\right)\cos\zeta-\left(f.\tan\theta+\frac{Q}{H}\right)\sin\zeta.\end{aligned}\right\}\quad(99)$$

By means of eq. (31), Art. 296,

$$\cos\zeta\cos\zeta'+\sin\zeta\sin\zeta'=\cos(\zeta-\zeta')=\cos\delta. \quad . \quad (100)$$

By analogy with eq. (28), Art. 296, $\cos^2\zeta=\frac{1+\cos 2\zeta}{2}$. (101)

By analogy with eq. (29), Art. 296, $\sin^2\zeta=\frac{1-\cos 2\zeta}{2}$. (102)

By means of eq. (39), Art. 296, $\cos\zeta\sin\zeta=\frac{\sin 2\zeta}{2}$. . (103)

Substituting the values of eqs. (100) to (103) in (99), it becomes

$$\left.\begin{aligned}\frac{H'}{H}.\cos\delta=&-(d+b)\frac{\sin 2\zeta}{2}+(1+a)\left(\frac{1+\cos 2\zeta}{2}\right)\\&+(1+e)\left(\frac{1-\cos 2\zeta}{2}\right)+\left(c\tan\theta+\frac{P}{H}\right)\cos\zeta\\&-\left(f.\tan\theta+\frac{Q}{H}\right)\sin\zeta,\end{aligned}\right\} \quad (104)$$

which may be put in this form:

$$\left.\begin{aligned}\frac{H'}{H}\cos\delta=&-\left(\frac{d+b}{2}\right)\sin 2\zeta+\frac{1}{2}+\frac{a}{2}+\frac{\cos 2\zeta}{2}\\&+\frac{a\cos 2\zeta}{2}+\frac{1}{2}+\frac{e}{2}-\frac{\cos 2\zeta}{2}-\frac{e.\cos 2\zeta}{2}\\&+\left(c\tan\theta+\frac{P}{H}\right)\cos\zeta-\left(f\tan\theta+\frac{Q}{H}\right)\sin\zeta.\end{aligned}\right\} \quad (105)$$

Whence

$$\left.\begin{aligned}\frac{H'}{H}\cos\delta=&\left(1+\frac{a+e}{2}\right)+\left(c\tan\theta+\frac{P}{H}\right)\cos\zeta\\&-\left(f.\tan\theta+\frac{Q}{H}\right)\sin\zeta+\left(\frac{a-e}{2}\right)\cos 2\zeta\\&-\left(\frac{d+b}{2}\right)\sin 2\zeta.\end{aligned}\right\} \quad (106)$$

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This is an accurate formula for calculating the deviation (δ) on any compass course (ζ'); but δ enters into three terms of the second member and thus renders it unavailable for general use: certain changes, however, can be made in it—introducing errors, it is true, but admissible ones—and then the rigorous formula becomes an approximate one of great utility.

In eq. (124) the deviation is found by means of its *sine* ($\sin\delta$) and the coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$ are expressed in parts of radius, as explained in Art. 296, par. [3]: in the changed formula, the arc itself (δ) in degrees and minutes will be used in the first member, and δ will be omitted entirely from the second member; to indicate this approximate condition of (124), the coefficients will be denoted by *Italic* capitals, and expressed in degrees and minutes; that is,

$$\delta = A + B.\sin\zeta' + C.\cos\zeta' + D.\sin 2\zeta' + E.\cos 2\zeta'. \quad (125)$$

In order to form an idea of the inaccuracy that may arise from the use of (125), it will be seen by Table 72 that for a maximum deviation of 20°, to use δ for $\sin\delta$, introduces an error of 25′ in the calculated deviation. In eq. (124), of the coefficients $\mathfrak{A}$, $\mathfrak{D}$, $\mathfrak{E}$, affected by functions of δ, the first and last are small quantities at best; the two principal terms remain unchanged in both (124) and (125).

As the sine increases, and the cosine decreases, with the angle, multiplying $\mathfrak{A}$ by $\cos\delta$ will decrease it; therefore to omit $\cos\delta$ in first term of (124) is to increase its value, so that A is larger than $\mathfrak{A}$; to omit δ in fourth term of (124) decreases its value, so that $D.\sin 2\zeta'$ is less than $\mathfrak{D}.\sin(2\zeta'+\delta)$; and to omit δ from the last term of (124) increases its value, so that $E.\cos 2\zeta'$ is larger than $\mathfrak{E}.\cos(2\zeta'+\delta)$. Thus the *tendency* of the inaccuracies in these terms is to counterbalance each other; and it is safe to state that up to 20° deviation, formula (125)

may be used without introducing a greater error than 25′ into the resulting calculated deviations.

303. Expansion of the Deviation Formula into Fourier's Series.—Mathematical quantities are said to be of different orders when they bear a certain numerical relation to each other and have a definite initial value; both relation and value may be arbitrarily set, depending, however, on the nature of the phenomenon under investigation and the degree of refinement suitable for its treatment: what, with perfect propriety, could be called "small quantities" in compass deviations, would be gross and inappropriate in delicate astronomical calculations.

Consider the expansion of the *sine* and *cosine* of an arc (δ) in terms of the arc itself by Maclaurin's series—a theorem of calculus by which a single variable is developed by successive differentiation into an infinite number of terms:

$$\sin\delta=\delta-\frac{\delta^3}{2.3}+\frac{\delta^5}{2.3.4.5}-\frac{\delta^7}{2.3.4.5.6.7}+\text{etc.} \quad (126)$$

$$\cos\delta=1-\frac{\delta^2}{2}+\frac{\delta^4}{2.3.4}-\frac{\delta^6}{2.3.4.5.6}+\text{etc.} \quad . \quad . \quad (127)$$

In the second member of each of these equations, the first term is the primary value and we may call it of the first order; then the second term of (127) is of the second order: each term grows rapidly less in numerical value, so that by stopping at any particular one as the last to enter the computations, we thereby determine the degree of accuracy of the result.

Reproducing eq. (124)—*after* expanding sin $(2\zeta'+\delta)$ and cos $(2\zeta'+\delta)$ by means of eqs. (30) and (31), Art. 296—we have

$$\left.\begin{aligned}\sin\delta=\mathfrak{A}.\cos\delta+\mathfrak{B}.\sin\zeta'+\mathfrak{C}.\cos\zeta'&\\+\mathfrak{D}.\sin 2\zeta'.\cos\delta+\mathfrak{D}.\cos 2\zeta'.\sin\delta&\\+\mathfrak{E}.\cos 2\zeta'\cos\delta-\mathfrak{E}.\sin 2\zeta'.\sin\delta.&\end{aligned}\right\}\quad(128)$$

In this, *all the coefficients* are expressed in parts of radius, as explained in Art. 296, par. [3]; and if we consider δ, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, small quantities of the first order, then $\mathfrak{A}$ and $\mathfrak{E}$ may be considered small quantities of the second order, or of the same order as squares or products of the first order; quantities of the third order will be the same as cubes of the first order; and so on.

Let the THIRD ORDER be the last to enter our calculations, and substitute in (128) the values of sin δ and cos δ from (126) and (127): when this is done, and all terms containing quantities of the fourth, fifth, and subsequent orders are rejected, equation (128) becomes

$$\left.\begin{aligned}\delta-\frac{\delta^3}{6}=\mathfrak{A}+\mathfrak{B}.\sin\zeta'+\mathfrak{C}.\cos\zeta'+\mathfrak{D}.\sin 2\zeta'&\\ -\mathfrak{D}.\frac{\delta^2}{2}\sin 2\zeta'+\mathfrak{D}.\delta.\cos 2\zeta'&\\ +\mathfrak{E}.\cos 2\zeta'-\mathfrak{E}.\delta.\sin 2\zeta'.&\end{aligned}\right\}\quad(129)$$

Transposing, and collecting in first member all terms having δ for a factor, we have

$$\left.\begin{aligned}\delta\,(1-\mathfrak{D}.\cos 2\zeta'+\mathfrak{E}.\sin 2\zeta')=\mathfrak{A}+\mathfrak{B}.\sin\zeta'+\mathfrak{C}.\cos\zeta'&\\ +\mathfrak{D}.\sin 2\zeta'+\mathfrak{E}.\cos 2\zeta'-\frac{\mathfrak{D}}{2}.\delta^2.\sin 2\zeta'+\frac{\delta^3}{6}.&\end{aligned}\right\}\quad(130)$$

Remembering that *products* of quantities of the first order become quantities of the second order, if, in (130), we reject all terms containing quantities of the second, third, and succeeding orders, and retain only those of the *first*, it becomes

$$\delta=\mathfrak{B}.\sin\zeta'+\mathfrak{C}.\cos\zeta'+\mathfrak{D}.\sin 2\zeta'.\quad . \quad . \quad (131)$$

Now find the square and the cube of both members of (131), and substitute in (130) the values of δ^2 and δ^3 thus found: after similar terms of opposite sign are cancelled, and other like terms are reduced according to their numeri-

cal coefficients, eq. (130) becomes (132), which is correct to the third order inclusive.

$$\left.\begin{aligned}
\delta\,(1-\mathfrak{D}.\cos 2\zeta'+\mathfrak{E}.\sin 2\zeta') &= \mathfrak{A}+\mathfrak{B}.\sin\zeta' \\
&+\mathfrak{C}.\cos\zeta'+\mathfrak{D}.\sin 2\zeta'+\mathfrak{E}.\cos 2\zeta' \\
&+\tfrac{1}{6}.\mathfrak{B}^3.\sin^3\zeta'+\tfrac{1}{2}.\mathfrak{B}^2.\mathfrak{C}.\sin^2\zeta'.\cos\zeta' \\
&+\tfrac{1}{2}.\mathfrak{B}.\mathfrak{C}^2.\sin\zeta'\cos^2\zeta'-\tfrac{1}{2}.\mathfrak{B}.\mathfrak{D}^2.\sin^2 2\zeta'\sin\zeta' \\
&+\mathfrak{C}^3.\cos^3\zeta'-\tfrac{1}{2}.\mathfrak{C}.\mathfrak{D}^2\cos\zeta'.\sin^2 2\zeta' \\
&-\tfrac{1}{3}.\mathfrak{D}^3.\sin^3 2\zeta'.
\end{aligned}\right\}\qquad(132)$$

To get the value of δ from this, divide both members by $(1-\mathfrak{D}.\cos\ 2\zeta'+\mathfrak{E}.\sin\ 2\zeta')$. Squares of the second order, as $\mathfrak{E}^2$, are of the fourth order; squares of the first multiplied by one of the second, as $\mathfrak{D}^2.\mathfrak{E}$, are of the fourth; products of the second, as $\mathfrak{A}.\mathfrak{E}$, are of the fourth; the product of two of the first and one of the second, as $\mathfrak{C}.\mathfrak{D}.\mathfrak{E}$, is of the fourth; fourth powers of the first order and higher powers of other orders are of the fourth and beyond it: all these will occur in the quotient found by dividing the second member of (132) by the factor of δ in the first member; retaining in this quotient only quantities to the *third* order included, and arranging the retained terms according to the coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, we have:

$$\left.\begin{aligned}
\delta &= \mathfrak{A}+\mathfrak{A}.\mathfrak{D}.\cos 2\zeta'+\mathfrak{B}.\sin\zeta'+\mathfrak{B}.\mathfrak{D}.\sin\zeta'.\cos 2\zeta' \\
&+\mathfrak{B}.\mathfrak{D}^2.\sin\zeta'.\cos^2 2\zeta'-\mathfrak{B}.\mathfrak{E}.\sin\zeta'.\sin 2\zeta' \\
&+\tfrac{1}{6}.\mathfrak{B}^3.\sin^3\zeta'+\tfrac{1}{2}\mathfrak{B}^2.\mathfrak{C}.\sin^2\zeta'.\cos\zeta' \\
&+\tfrac{1}{2}.\mathfrak{B}.\mathfrak{C}^2.\sin\zeta'.\cos^2\zeta'-\tfrac{1}{2}.\mathfrak{B}.\mathfrak{D}^2.\sin\zeta'.\sin^2 2\zeta' \\
&+\mathfrak{C}.\cos\zeta'+\mathfrak{C}.\mathfrak{D}.\cos\zeta'.\cos 2\zeta' \\
&+\mathfrak{C}.\mathfrak{D}^2\cos\zeta'.\cos^2 2\zeta'-\mathfrak{C}.\mathfrak{E}.\cos\zeta'.\sin 2\zeta' \\
&+\tfrac{1}{6}\mathfrak{C}^3.\cos^3\zeta'-\tfrac{1}{2}.\mathfrak{C}.\mathfrak{D}^2\cos\zeta'.\sin^2 2\zeta'+\mathfrak{D}.\sin 2\zeta' \\
&+\mathfrak{D}^2.\sin 2\zeta'.\cos 2\zeta'+\mathfrak{D}^3.\sin 2\zeta'.\cos^2 2\zeta' \\
&-\mathfrak{D}.\mathfrak{E}.\sin^2 2\zeta'-\tfrac{1}{3}.\mathfrak{D}^3.\sin^3 2\zeta' \\
&+\mathfrak{E}.\cos 2\zeta'+\mathfrak{E}.\mathfrak{D}.\cos^2 2\zeta'.
\end{aligned}\right\}\qquad(133)$$

By means of (28) and (29), Art. 296, and the formulas relative to multiple angles, the powers and products of

angular functions in (133) can be converted into individual multiples of the angle; this will be done as follows, grouping together such of the terms as have the same coefficient for a factor:

$$\mathfrak{B}.\mathfrak{D}.(\sin\zeta'\cos 2\zeta')=\tfrac{1}{2}.\mathfrak{B}.\mathfrak{D}.\sin 3\zeta'-\tfrac{1}{2}\mathfrak{B}.\mathfrak{D}.\sin\zeta'. \qquad (134)$$

$$\left.\begin{aligned}&\mathfrak{B}.\mathfrak{D}^2(\sin\zeta'\cos^2 2\zeta'-\tfrac{1}{2}.\sin\zeta'\sin^2 2\zeta')\\ &\qquad=\mathfrak{B}.\mathfrak{D}^2(\tfrac{1}{2}\sin\zeta'+\tfrac{1}{2}\sin\zeta'.\cos 4\zeta'-\tfrac{1}{4}\sin\zeta'\\ &\qquad+\tfrac{1}{4}.\sin\zeta'\cos 4\zeta')=\tfrac{1}{4}.\mathfrak{B}.\mathfrak{D}^2.\sin\zeta'\\ &\qquad-\tfrac{3}{8}.\mathfrak{B}.\mathfrak{D}^2\sin 3\zeta'+\tfrac{3}{8}.\mathfrak{B}.\mathfrak{D}^2\sin 5\zeta'.\end{aligned}\right\} \qquad (135)$$

$$-\mathfrak{B}.\mathfrak{E}.(\sin\zeta'.\sin 2\zeta')=\tfrac{1}{2}.\mathfrak{B}.\mathfrak{E}.\cos 3\zeta'-\tfrac{1}{2}.\mathfrak{B}.\mathfrak{E}.\cos\zeta'. \qquad (136)$$

$$\left.\begin{aligned}\tfrac{1}{6}.\mathfrak{B}^3.(\sin^3\zeta')&=\tfrac{1}{6}\mathfrak{B}^3[\sin\zeta'(\sin^2\zeta')]\\ &=\tfrac{1}{6}\mathfrak{B}^3[\sin\zeta'(\tfrac{1}{2}-\tfrac{1}{2}\cos 2\zeta')]\\ &=\tfrac{1}{6}\mathfrak{B}^3(\tfrac{1}{2}\sin\zeta'-\tfrac{1}{2}\cos 2\zeta'\sin\zeta')\\ &=\tfrac{1}{8}.\mathfrak{B}^3.\sin\zeta'-\tfrac{1}{24}.\mathfrak{B}^3.\sin 3\zeta'.\end{aligned}\right\} \qquad (137)$$

$$\left.\begin{aligned}\tfrac{1}{2}.\mathfrak{B}^2.\mathfrak{C}(\sin^2\zeta'.\cos\zeta')&=\tfrac{1}{2}\mathfrak{B}^2.\mathfrak{C}[\tfrac{1}{2}-\tfrac{1}{2}\cos 2\zeta']\cos\zeta'\\ &=\tfrac{1}{8}\mathfrak{B}^2.\mathfrak{C}.\cos\zeta'-\tfrac{1}{8}\mathfrak{B}^2.\mathfrak{C}\cos 3\zeta'.\end{aligned}\right\} \qquad (138)$$

$$\left.\begin{aligned}\tfrac{1}{2}\mathfrak{B}.\mathfrak{C}^2(\sin\zeta'\cos^2\zeta')&=\tfrac{1}{2}\mathfrak{B}.\mathfrak{C}^2[\tfrac{1}{2}+\tfrac{1}{2}\cos 2\zeta']\sin\zeta'\\ &=\tfrac{1}{8}\mathfrak{B}.\mathfrak{C}^2\sin\zeta'+\tfrac{1}{8}\mathfrak{B}.\mathfrak{C}^2\sin 3\zeta'.\end{aligned}\right\} \qquad (139)$$

$$\mathfrak{C}.\mathfrak{D}(\cos\zeta'\cos 2\zeta')=\tfrac{1}{2}\mathfrak{C}.\mathfrak{D}\cos\zeta'+\tfrac{1}{2}\mathfrak{C}.\mathfrak{D}\cos 3\zeta'. \quad . \qquad (140)$$

$$\left.\begin{aligned}&\mathfrak{C}.\mathfrak{D}^2(\cos\zeta'\cos^2 2\zeta')=\mathfrak{C}.\mathfrak{D}^2[\tfrac{1}{2}+\tfrac{1}{2}\cos 4\zeta']\cos\zeta'\\ &\quad=\tfrac{1}{2}\mathfrak{C}.\mathfrak{D}^2\cos\zeta'+\tfrac{1}{4}\mathfrak{C}.\mathfrak{D}^2\cos 3\zeta'+\tfrac{1}{4}\mathfrak{C}.\mathfrak{D}^2\cos 5\zeta'.\end{aligned}\right\} \qquad (141)$$

$$-\mathfrak{C}.\mathfrak{E}(\cos\zeta'\sin 2\zeta')=-\tfrac{1}{2}\mathfrak{C}.\mathfrak{E}\sin\zeta'-\tfrac{1}{2}\mathfrak{C}.\mathfrak{E}\sin 3\zeta'. \qquad (142)$$

$$\left.\begin{aligned}\tfrac{1}{6}\mathfrak{C}^3(\cos^3\zeta')&=\tfrac{1}{6}\mathfrak{C}^3[\cos\zeta'(\cos^2\zeta')]\\ &=\tfrac{1}{6}\mathfrak{C}^3[\cos\zeta'(\tfrac{1}{2}+\tfrac{1}{2}\cos 2\zeta')]\\ &=\tfrac{1}{8}\mathfrak{C}^3\cos\zeta'+\tfrac{1}{24}\mathfrak{C}^3\cos 3\zeta'.\end{aligned}\right\} \qquad (143)$$

$$\left.\begin{aligned}&-\tfrac{1}{2}\mathfrak{C}.\mathfrak{D}^2(\cos\zeta'\sin^2 2\zeta')=-\tfrac{1}{2}\mathfrak{C}.\mathfrak{D}^2[\tfrac{1}{2}-\tfrac{1}{2}\cos 4\zeta']\cos\zeta'\\ &=-\tfrac{1}{4}\mathfrak{C}.\mathfrak{D}^2\cos\zeta'+\tfrac{1}{8}\mathfrak{C}.\mathfrak{D}^2\cos 3\zeta'+\tfrac{1}{8}\mathfrak{C}.\mathfrak{D}^2\cos 5\zeta'.\end{aligned}\right\} \qquad (144)$$

$$\mathfrak{D}^2.(\sin 2\zeta'\cos 2\zeta')=\tfrac{1}{2}\mathfrak{D}^2.\sin 4\zeta'. \quad . \quad . \qquad (145)$$

$$\left.\begin{aligned}&\mathfrak{D}^3(\sin 2\zeta'\cos^2 2\zeta'-\tfrac{1}{3}\sin^3 2\zeta')\\&\quad=\mathfrak{D}^3\sin 2\zeta'\,(\tfrac{1}{2}+\tfrac{1}{2}\cos 4\zeta')\\&\quad-\tfrac{1}{3}\mathfrak{D}^3\sin 2\zeta'\,(\tfrac{1}{2}-\tfrac{1}{2}\cos 4\zeta')\\&\quad=\tfrac{2}{6}\mathfrak{D}^3\sin 2\zeta'+\tfrac{4}{6}\,.\,\mathfrak{D}^3\cos 4\zeta'\sin 2\zeta'\\&\quad=2\mathfrak{D}^3\cos\zeta'\sin\zeta'\,(\tfrac{1}{3}+\tfrac{2}{3}\cos 4\zeta')\\&\quad=2\mathfrak{D}^3\cos\zeta'\,[\tfrac{1}{3}\sin\zeta'+\tfrac{2}{3}(\tfrac{1}{2}\sin 5\zeta'-\tfrac{1}{2}\sin 3\zeta')]\\&\quad=\tfrac{2}{6}\mathfrak{D}^3\sin 2\zeta'+\tfrac{4}{12}\mathfrak{D}^3\sin 6\zeta'+\tfrac{4}{12}\mathfrak{D}^3\sin 4\zeta'\\&\quad-\tfrac{4}{12}\mathfrak{D}^3\sin 4\zeta'-\tfrac{4}{12}\mathfrak{D}^3\sin 2\zeta'=\tfrac{1}{3}\mathfrak{D}^3\sin 6\zeta'.\end{aligned}\right\}\quad(146)$$

$$\left.\begin{aligned}\mathfrak{E}\,.\,\mathfrak{D}\,(\cos^2 2\zeta'-\sin^2 2\zeta')&=\mathfrak{E}\,.\,\mathfrak{D}\,[(\tfrac{1}{2}+\tfrac{1}{2}\cos 4\zeta')\\&\quad-(\tfrac{1}{2}-\tfrac{1}{2}\cos 4\zeta')]=\mathfrak{E}\,.\,\mathfrak{D}\cos 4\zeta'.\end{aligned}\right\}\quad(147)$$

Substituting the last members of equations (134) to (147), both inclusive, for their equivalents in eq. (133), and grouping the terms according to the same multiple of the angle, eq. (133) becomes

$$\left.\begin{aligned}\delta=\mathfrak{A}&+[\mathfrak{B}+\tfrac{1}{8}\mathfrak{B}\,.\,\mathfrak{C}^2+\tfrac{1}{8}\mathfrak{B}^3+\tfrac{1}{4}\mathfrak{B}\,.\,\mathfrak{D}^2-\tfrac{1}{2}\mathfrak{B}\,.\,\mathfrak{D}-\tfrac{1}{2}\mathfrak{C}\,.\,\mathfrak{E}]\sin\zeta'\\&+[\mathfrak{C}-\tfrac{1}{2}\mathfrak{B}\,.\,\mathfrak{E}+\tfrac{1}{8}\mathfrak{B}^2\,.\,\mathfrak{C}+\tfrac{1}{2}\mathfrak{C}\,.\,\mathfrak{D}+\tfrac{1}{4}\mathfrak{C}\,.\,\mathfrak{D}^2+\tfrac{1}{8}\mathfrak{C}^3]\cos\zeta'\\&+[\mathfrak{D}]\sin 2\zeta'+[\mathfrak{A}\,.\,\mathfrak{D}+\mathfrak{E}]\cos 2\zeta'\\&+[\tfrac{1}{2}\mathfrak{B}\,.\,\mathfrak{D}-\tfrac{3}{8}\mathfrak{B}\,.\,\mathfrak{D}^2-\tfrac{1}{24}\mathfrak{B}^3+\tfrac{1}{8}\mathfrak{B}\,.\,\mathfrak{C}^2-\tfrac{1}{2}\mathfrak{C}\,.\,\mathfrak{E}]\sin 3\zeta'\\&+[\tfrac{1}{2}\mathfrak{B}\,.\,\mathfrak{E}-\tfrac{1}{8}\mathfrak{B}^2\,.\,\mathfrak{C}+\tfrac{1}{2}\mathfrak{C}\,.\,\mathfrak{D}+\tfrac{3}{8}\mathfrak{C}\,.\,\mathfrak{D}^2+\tfrac{1}{24}\mathfrak{C}^3]\cos 3\zeta'\\&+[\tfrac{1}{2}\mathfrak{D}^2]\sin 4\zeta'+[\mathfrak{E}\,.\,\mathfrak{D}]\cos 4\zeta'+[\tfrac{3}{8}\mathfrak{B}\,.\,\mathfrak{D}^2]\sin 5\zeta'\\&+[\tfrac{3}{8}\mathfrak{C}\,.\,\mathfrak{D}\,]\cos 5\zeta'+[\tfrac{1}{3}\mathfrak{D}^3]\sin 6\zeta'.\end{aligned}\right\}\quad(148)$$

The variables of this equation are δ and ζ': the first term of the second member and the quantities within brackets of all the other terms are constants; represent them by *Italic* capitals, and then eq. (148) is

$$\left.\begin{aligned}\delta=A&+B\,.\sin\zeta'+C\,.\cos\zeta'+D\,.\sin 2\zeta'+E\,.\cos 2\zeta'\\&+F\,.\sin 3\zeta'+G\,.\cos 3\zeta'+H\,.\sin 4\zeta'+K\,.\cos 4\zeta'\\&+L\,.\sin 5\zeta'+M\,.\cos 5\zeta'+N\,.\sin 6\zeta'.\end{aligned}\right\}\quad(149)$$

This is FOURIER'S SERIES: only twelve terms, however, appear here, because quantities to only the third order in-

cluded, were admitted; this condition was made when eqs. (126) and (127) were introduced into the series of equations of which (149) is the final result. If the fourth, fifth, or any other order of quantities had been used, a correspondingly greater number of terms would appear in (149); and as (126) and (127) are infinite, so would (149) be if (126) and (127) had been introduced in their entirety.

In (149) the azimuth of the ship's head by compass (ζ') is expressed in degrees and minutes; but all the other quantities (δ, A, B, C, etc.) can be expressed either in degrees and minutes or in parts of radius, as explained in Art. (296).

As eq. (149) is the basic *working* formula of the deviations, it will be useful to take a retrospective view of the manner in which it has been deduced: in PART THIRD the forces in action to produce deviation were represented physically; in Chapter XXI a résumé of the history of the deviations was given; *Section one* of the present chapter summarizes the sources of deviations, besides exemplifying their mode of occurrence; *Section two* defines the terms and symbols used in the investigation, and prepares the way for the general mathematical interpretation in eqs. (86), (87), and (88) of the physical facts stated in Part Third; the quantities proper to the problem were next introduced and the equations eventually became (94), (95), and (96); by varied transformation of these we arrived at expressions for the combined and separate force of the Ship and Earth, and also eq. (124)—a rigorous formula for calculating the deviation on any compass course; lastly, this was variously transformed throughout Art. 303, until we obtained Fourier's Series. In this series it will be perceived that the function *sine* and *cosine* of every possible multiple of the variable ζ' enters as a factor of the terms: this means that phenomena of every possible "*Period*" has here its mathematical expression; or, as Period and Wave are mutually

convertible terms—as explained in Vol. I—it means that symmetrical waves of all sizes are represented by Fourier's Series; it is only their varied superposition that brings out the odd, irregular curves so often seen in their resultant—the deviations.

CHAPTER XXIII.

THE MAGNETIC COEFFICIENTS.

Section One: The Method of Least Squares.

304. Necessity for adjustment of the observation-equations.—Eq. (125), for calculating the deviations, contains five unknown quantities—A, B, C, D, E—and if observations be made on *only five* points, we should then have but five equations similar to (125), and the unknown quantities could be deduced directly by any process of algebra; but we may observe on 8, 16, or 32 points and have a corresponding number of equations, or on each half point and have 64—or even on every degree of the circle and have 360 equations: but in all such cases we have only five unknown quantities to be determined. When their values are ascertained from observations on only five points, such values satisfy the equations; but when determined from *any* five of a larger number of equations, then the values obtained from the five selected equations will not satisfy the others from among which they were chosen.

It is evident, however, that the larger the number of observations to determine a certain quantity, the more accurate its resulting value; so that from 32 equations based on careful observations we should get better values of A, B, C, D, E than from five.

As it is thus desirable to use a large number of observations, they must be adjusted so that the most probable

values of the coefficients shall be obtained; and this is done by the Method of Least Squares, now to be briefly explained —chiefly according to the mode of treatment by Prof. Mansfield Merriman.

305. The Principles of Probability.—In mathematics, the certainty of an event is denoted by unity—its impossibility by zero; and as all its phases take place between these extremes, the *probability* of any one or several of these phases occurring is represented by a fraction: the numerator denotes the number of ways in which the event may happen or fail, and the denominator the total number of possible ways.

Thus, in tossing a coin, there are only *two* possible ways in which it can turn up—either head or tail; and the one is as likely to occur as the other: the probability of throwing a head is therefore $\frac{1}{2}$, and of a tail, $\frac{1}{2}$; their sum is unity, or it is certain that the coin will turn up head or tail.

If a bag contain three red balls and five blue ones, and another bag contain four red balls and seven blue ones, the probability of drawing a red ball from the first bag is $\frac{3}{8}$ and from the second $\frac{4}{11}$. Each bag represents the vicissitudes of certain independent events—the drawing of balls of different colors from them; but now consider them jointly—to draw a ball with each hand at the same time out of both bags: it is evident that a certain one ball of the first bag may be drawn out with each ball of the second bag and thus form a different pair every time; the same is true of a certain other ball of the first bag and all of the second; and also of a certain third ball of the first bag and all of the second; and so on until all of the first have been successively united to all of the second: this is equivalent to taking the product of the number of balls in both bags, that is $8 \times 11 = 88$, which is the total number of different possible ways of drawing out two balls from both bags.

Reasoning in the same way, since there are three red balls in the first bag and four red ones in the second, the

probability of drawing two red ones from both bags at the same time is $\frac{3}{8}\times\frac{4}{11}$; for, the total number of different possible combinations of all the balls in both bags being 88, as shown above, one of the three red balls of the first bag may be successively united to each of the four red ones in the second bag and form a different pair; then the second red ball of the first bag with each of the four red ones of the second bag; and finally, the third red ball of the first bag with each of the red ones of the second bag, which is equivalent to multiplying together their number in both bags, or $\frac{3\times 4}{8\times 11}=\frac{3}{8}\times\frac{4}{11}$: similarly, the probability of drawing two blue balls together is $\frac{5\times 7}{8\times 11}=\frac{5}{8}\times\frac{7}{11}$. That is to say, the probabilities of the primary events being separately $\frac{3}{8}$ and $\frac{4}{11}$ for the respective bags, the probability of the compound event for both bags is the product of these individual probabilities.

To consider generally a compound event made up of any number n of simple events, let p be the probability of an event happening in one trial and f its failing, so that $p+f=1$: from what has just been stated, the probability that all will *happen* is p^n, since that of each is p, and n their number; the probability that $(n-1)$ events will happen and one fail, in n ways, is $n.p^{(n-1)}.f$; similarly, for $(n-2)$ events happening and two failing, we have, according to the algebraic theory of combination, $\frac{n(n-1)}{2}.p^{(n-2)}.f^2$. If, then, $(p+f)^n$ be expanded by the binomial formula, it becomes

$$(p+f)^n=p^n+n.p^{(n-1)}f+\frac{n(n-1)}{1.2}.p^{(n-2)}f^2$$
$$+\frac{n(n-1)(n-2)}{1.2.3}.p^{(n-3)}.f^3+\text{etc.} \quad (150)$$

In this, the first term is the probability that all will happen; the second term, that $(n-1)$ will happen and one fail; the third term, that $(n-2)$ will happen and two fail; and so on. If $p=f=\frac{1}{2}$, it corresponds to the case of throwing n coins—say 6; substituting these quantities in (150), it gives the following series as the numerical values of the several terms:

$$\tfrac{1}{64}+\tfrac{6}{64}+\tfrac{15}{64}+\tfrac{20}{64}+\tfrac{15}{64}+\tfrac{6}{64}+\tfrac{1}{64} \quad . \quad . \quad . \quad . \quad (151)$$

That is, if six coins be thrown, the *probability* of the relative numbers of heads and tails appearing in a single throw is:

That all will be heads. . . . . $\frac{1}{64}$
That five will be heads and one tail. . . . . $\frac{6}{64}$
That four will be heads and two tails. . . . $\frac{15}{64}$
That three will be heads and three tails. . $\frac{20}{64}$
That two will be heads and four tails. . . . $\frac{15}{64}$
That one will be head and five tails. . . . . $\frac{6}{64}$
That all will be tails. . . . . $\frac{1}{64}$

And the sum of these seven probabilities is, of course, unity.

If at O, Fig. 517, we erect a perpendicular Op equal to 20 parts of a scale divided into 64ths, and then on each side of it, at equal distances (e_1, e_2, e_3), erect other perpendiculars of 15, 6, and 1 part respectively in length, and draw a curve through their ends, we shall have a graphic illustration of the probabilities given above numerically: it is called the *probability curve*, and is typical of all matters whose proba bilities are calculated: it is not of identical contour for all cases, but varies suitably for each, while preserving the characteristic form shown in Fig. 517.

The MOST PROBABLE *event among several is that which has the greatest mathematical probability;* in the above case, it is $\frac{20}{64}$, or that in throwing six coins they will turn up equally heads and tails.

If a skillful marksman fire a thousand rifle-shots at a target 300 yards distant, they will mostly hit the bull's-eye and its vicinity—equally all round: if a series of rings encircle the bull's-eye, but few stray shots will fall in the outer one, while they will rapidly increase in the rings toward the center. Plotting the number in each ring as

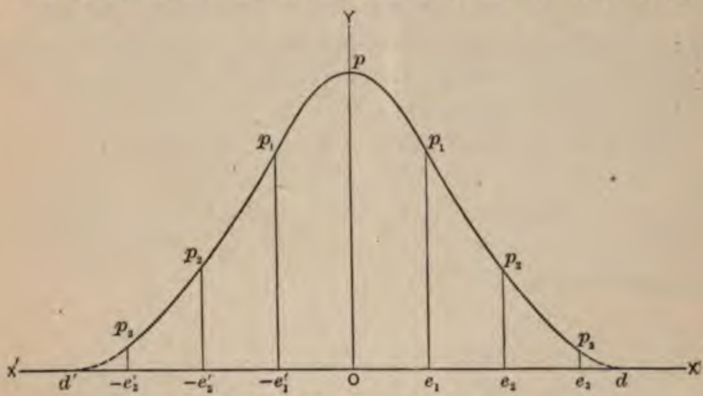

FIG. 517.

an ordinate, and the diameter of the ring as an abscissa, we should get a curve very much like that of Fig. 517; from the examination of a large number of cases like this, these axioms have been formulated: 1st, small errors are very much more frequent than large ones; 2d, positive and negative errors are equally frequent; 3d, very large errors do not occur.

Every shot that misses the bull's-eye is in error, and to the amount of its distance from it: there is the greatest probability that an expert marksman will hit the center or near it, and the least probability that his shots will fall near the limits of the target; and thus the probability is a function of the error.

Referring to Fig. 517, if errors (x), $\overline{Oe_1}$, $\overline{Oe_2}$, etc., be laid off as abscissæ, and probabilities (y), $\overline{Op}$, $\overline{e_1p_1}$, etc., as ordinates, then the equation of the curve is

$$y=f(x). \qquad \qquad (152)$$

The second member, when definitely determined, must express analytically the axioms stated above; that is, 1st, the maximum ordinate or greatest mathematical probability must correspond to a zero error; the curve must be symmetrical with regard to the axis of Y, since plus and minus errors of the same size are equally probable; and 3d, as x increases, or the errors become larger in size, y must decrease, or the probability of their occurrence grows less, until, when x is *very* large, y must be zero, i.e., very large errors do not occur.

306. The Law of Error.—Every quantity has a *true* value which it is the aim of observation to determine: frequently, however, it is not attained, but a difference occurs—an *error*, x; if z denote the true value, and m the observed one, then

$$z-m=x. \qquad (153)$$

If we calculate the *most probable* value, u, of a quantity, then the difference between *it* and the individual observation, m, is called a *residual*, v, that is,

$$u-m=v. \qquad (154)$$

In a large number of accurate observations the true and most probable values practically coincide, so that (153) and (154) then become identical.

The most probable value of a quantity which has been measured many times (m_1, m_2, $m_3 \ldots m_n$) with equal care is the arithmetical mean of the observations; thus,

$$u=\frac{m_1+m_2+m_3+\ldots+m_n}{n}, \qquad (155)$$

whence $\qquad n.u=m_1+m_2+m_3+\ldots+m_n. \qquad (156)$

But as n stands for the number of single observations, this may be transposed and written as follows:

$n.u-m_1-m_2-m_3-m_n=0$; or

$$(u-m_1)+(u-m_2)+\ldots+(u-m_n)=0; \qquad (157)$$

which, being interpreted, is, that the arithmetical mean requires that the algebraic sum of the residuals shall be zero; for, by (154), $(u-m_1)$, $(u-m_2)$, etc., are the residuals.

Now, referring to eq. (152) and its meaning, we may write several such:

$$y_1=f(x_1);\quad y_2=f(x_2);\quad y_n=f(x_n).\quad .\quad .\quad (158)$$

These express the probabilities of the occurrence of n separate systems of errors; and, by Art. 305, the probability P of the occurrence of all together will be their product: that is,

$$P=y_1 \cdot y_2 \cdot y_n=f(x_1)\,.\,f(x_2)\,.\,f(x_n).\quad .\quad .\quad (159)$$

Taking the logarithm of (159), we have

$$\log P=\log f(x_1)+\log f(x_2)+\log f(x_n).\quad .\quad (160)$$

It has been shown in Art. 305 that the most probable value of the unknown quantity z, to which the probabilities (y) and errors (x) of eq. (158) relate, will be that which renders the compound probability P a maximum; the first differential of (160) must therefore be equal to zero: the reason for this last statement will be found in any Calculus under the heading Maxima and Minima; besides, the matter is explained on page 633 of this Treatise.

Indicating the differentiation of (160), and dividing by dz, as the equation relates to that unknown quantity, we have

$$\frac{dP}{dz}=\frac{d\,.f(x_1)}{f(x_1)dz}+\frac{d\,.f(x_2)}{f(x_2)dz}+\frac{d\,.f(x_n)}{f(x_n)dz}=0.\quad .\quad .\quad (161)$$

When the differentiation of the numerator is carried out, it generally takes this *typical form:*

$$d\,.f(x)=f'(x)\,.f(x)\,.dx.\quad .\quad .\quad .\quad .\quad (162)$$

Adapting the numerator of each term of the second member of (161) to the form of the second member of (162),

and substituting the results in (161), it becomes, after reduction,

$$f'(x_1)\frac{dx_1}{dz}+f'(x_2)\frac{dx_2}{dz}+f'(x_n)\frac{dx_n}{dz}=0. \quad . \quad . \quad (163)$$

The errors $x_1, x_2 \ldots x_n$ of this are given by the general form, eq. (153): differentiating that for each value of x, and remembering that m is constant, being the numerical value of the unknown quantity found by observation, we have from (153)

$$dz=dx; \therefore \frac{dx}{dz}=1; \text{ and by analogy } \frac{dx_1}{dz}=\frac{dx_2}{dz}=\frac{dx_n}{dz}=1. \quad (164)$$

This simplifies eq. (163), so that it becomes

$$f'(x_1)+f'(x_2)+\ \ldots\ +f'(x_n)=0. \quad . \quad . \quad (165)$$

As already explained, in a large number of careful observations the residuals (v) and errors (x) become identical, and the sum of each equal to zero, that is,

$$v_1+v_2+v_3+\ \ldots\ +v_n=x_1+x_2+x_3+\ \ldots\ +x_n=0. \quad (166)$$

This can be the same as (165) *only* when the symbol f' means that each term of that equation is multiplied by a constant, or when

$$f'(x_1)+f'(x_2)+\ \ldots\ +f'(x_n)=c.x_1+c.x_2+\ \ldots\ +c.x_n. \quad (167)$$

Deducing the values of $f'(x_1)$, $f'(x_2)$, etc., from the general form (162), and substituting their equivalents in (167), this becomes

$$\frac{d.f(x_1)}{f(x_1).dx_1}+\frac{d.f(x_2)}{f(x_2)dx_2}+\ \ldots\ +\frac{d.f(x_n)}{f(x_n)dx_n}=c.x_1+c.x_2+\ \ldots\ +c.x_n. \quad (168)$$

Since this is true whatever be the number (n) of observations, the corresponding terms of the two members are equal: hence if x be any error, and $y=f(x)$, as predicated in eq. (152), we shall have

$$dy=d.f(x); \quad . \quad . \quad . \quad . \quad . \quad . \quad (169)$$

using this general value of x, instead of x_1 in the *first* term of each member of (168), and equating those terms, we have, after substituting values of (152) and (169):

$$\frac{d.f(x)}{f(x)dx}=c.x;\ \text{or}\ \frac{dy}{y.dx}=c.x;\ \text{or}\ \frac{dy}{y}=c.x.dx. \quad . \quad (170)$$

Integrating this and denoting the constant of integration by k', we have

$$\int\frac{dy}{y}=\log y,\quad \text{and}\quad \int c.x.dx=\frac{c.x^2}{2}+k';. \quad (171)$$

whence $$\log y=\tfrac{1}{2}c.x_2+k'. \quad . \quad . \quad . \quad . \quad (172)$$

As the second member is the logarithm of the first member, the quantity y will be equal to the base of the Naperian system (denoted by e) raised to a power indicated by the second member: and as the *sum* of logarithms represents the *product* of numbers for which they stand, so will the two numbers $e^{k'}$ and $e^{\frac{1}{2}cx^2}$ become a product, since their logarithms are connected by plus in (172); hence

$$y=e^{k'}.e^{\frac{1}{2}c.x^2}.. \quad . \quad . \quad . \quad . \quad . \quad (173)$$

According to the principles of probability established in Art. 305, the probability y must *de*crease as the error x *in*creases in size, and this can be effected only by making c negative, so that it is necessarily so: let $c=-2h^2$; $\therefore \frac{1}{2}c=-h^2$; and as $e^{k'}$ is a constant, represent it by k; whence (173) becomes

$$y=k.e^{-h^2.x^2}. \quad . \quad . \quad . \quad . \quad . \quad (174)$$

This is the equation of the Probability Curve, Fig. 517, and the second member is the definite form sought for the undetermined $f(x)$ of eq. (152). EQ. (174) EXPRESSES THE LAW OF PROBABILITY OF OBSERVATION ERRORS.

307. The Principle of Least Squares.—Let a quantity whose true value is z be the object of a series of observations, $m_1, m_2 \ldots m_n$; these will scarcely be identical with

the true value, but differ from it by small errors, $x_1, x_2 \ldots x_n$: then by means of eq. (153) we have

$$z - m_1 = x_1; \quad z - m_2 = x_2; \quad z - m_n = x_n. \quad . \quad (175)$$

The separate probabilities of the errors $x_1, x_2 \ldots x_n$ occurring, are by eq. (174):

$$y_1 = k.e^{-h^2x_1^2}; \quad y_2 = k.e^{-h^2x_2^2}; \; y_n = k.e^{-h^2x_n^2}. \quad . \quad (176)$$

The factor h represents the precision of each observation in the opinion of the observer, and since for the purpose of this enquiry all the observations are regarded equally trustworthy, h is the same in all the equations of (176).

By Art. 305, the probability P of committing all the errors $(x_1, x_2 \ldots x_n)$ together, is equal to the product of their individual probabilities as given by (176); that is,

$$P = y_1 . y_2 . y_n = k^n . e^{-h^2(x_1^2 + x_2^2 + x_n^2)}. \quad . \quad . \quad (177)$$

And, as explained in the same article, the most probable system of errors is that which will give P the greatest mathematical probability, that is, *make it a maximum;* also, the most probable system of errors *corresponds* to the most probable value of the unknown quantity z: in eq. (177), P will be a maximum when the exponent of the second member $(x_1^2 + x_2^2 + x_n^2)$ has the least value, that is, when

$$x_1^2 + x_2^2 + x_n^2 = \text{a minimum}. \quad . \quad . \quad . \quad (178)$$

As, however, in a large number of accurate observations, the errors and residuals are identical, we have

$$x_1^2 + x_2^2 + x_n^2 = v_1^2 + v_2^2 + v_n^2 = \text{a minimum}. \quad . \quad (179)$$

So that, from eqs. (177), (178), and (179) *the Principle of Least Squares is, that for observations of the same precision, the most probable value* (*P*) *of the unknown quantity* (*z*) *is that which renders the sum of the squares of the residuals* (v^2, *etc.*) *a minimum;* and it is from this fact that the method takes its name—the residuals are the *least* quan-

tities that enter the calculations, and it is their *squares* that are considered.

Only one unknown quantity, and that for direct observation, has been dealt with; but the method can be extended to any number and to indirect observations. The varying precision of the several observations and the means of estimating the probable error of results have not been treated, as they do not enter the Deviation calculations based on the principles of Least Squares: only the part essential to the work in hand has been deemed necessary to be set forth here; but this understood, the other parts, if desired, are readily learned from any treatise on Least Squares.

308. The observation, residual, and normal equations.—The case of three unknown quantities, each multiplied by a constant, with five observations of equal precision, will now be considered; and what is true of this definite number of quantities and observations admits of unlimited extension.

Let a, b, c, be the constants; z_1, z_2, z_3, the unknown quantities; and m_1, m_2, m_3, m_4, m_5, the observations upon them: let the equations be as follows:

$$\left.\begin{aligned} a_1z_1+b_1z_2+c_1z_3&=m_1,\\ a_2z_1+b_2z_2+c_2z_3&=m_2,\\ a_3z_1+b_3z_2+c_3z_3&=m_3,\\ a_4z_1+b_4z_2+c_4z_3&=m_4,\\ a_5z_1+b_5z_2+c_5z_3&=m_5.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (180)$$

These are the *observation equations*, each arising from one observation of the connected quantities: they must be adjusted, since, from the explanation made in Art. 304, their number exceeds that of the quantities to be determined. If, instead of regarding z_1, z_2, z_3 as true values, we consider them the most probable values, then by subtracting their observed values m_1, m_2 . . . m_5 from them—

that is, transposing $m_1, m_2 \ldots m_5$ to the left-hand member in each equation—we get a series of residuals as stated in connection with equation (154): performing this operation and denoting the residuals by v_1, v_2, v_3, v_4, v_5, we have this set of *residual equations:*

$$\left.\begin{aligned} a_1z_1+b_1z_2+c_1z_3-m_1&=v_1,\\ a_2z_1+b_2z_2+c_2z_3-m_2&=v_2,\\ a_3z_1+b_3z_2+c_3z_3-m_3&=v_3,\\ a_4z_1+b_4z_2+c_4z_3-m_4&=v_4,\\ a_5z_1+b_5z_2+c_5z_3-m_5&=v_5.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (181)$$

In these, let R_1 stand for all the terms in the first member of the first equation, except that containing z_1, that is, let $R_1=b_1z_2+c_1z_3-m_1$; and similarly, let $R_2 \ldots R_5$ represent the corresponding terms in the other equations of the group; then they become

$$\left.\begin{aligned} a_1z_1+R_1&=v_1,\\ a_2z_1+R_2&=v_2,\\ a_3z_1+R_3&=v_3,\\ a_4z_1+R_4&=v_4,\\ a_5z_1+R_5&=v_5.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (182)$$

Squaring each member of these, and adding together all those on the right hand and also all those on the left hand of the signs of equality, we have

$$(a_1z_1+R_1)^2+(a_2z_1+R_2)^2+(a_3z_1+R_3)^2+(a_4z_1+R_4)^2 \\ +(a_5z_1+R_5)^2=v_1^2+v_2^2+v_3^2+v_4^2+v_5^2. \quad (183)$$

According to the principle established in eq. (179), the most probable value of z_1 in (183) will be that which renders the second member a minimum, that is,

$$v_1^2+v_2^2+v_3^2+v_4^2+v_5^2=\text{a minimum.} \quad . \quad . \quad (184)$$

This, of course, requires that its equal—the first member of (183)—be also a minimum; and to effect this, its first differential must, by the principles of Maxima and Minima,

be placed equal to zero: differentiating the first member of (183), then, with respect to z_1, we have, after omitting the common factor $2.dz_1$ from all the terms:

$$a_1[a_1z_1+R_1]+a_2[a_2z_1+R_2]+a_3[a_3z_1+R_3]+a_4[a_4z_1+R_4]+a_5[a_5z_1+R_5]=0. \quad (185)$$

Again: in the first equation of group (181) let S_1 stand for all the terms in the first member, except that containing z_2, or let $S_1=a_1z_1+c_1z_3-m_1$; and similarly, let $S_2 \ldots S_5$ represent the corresponding terms in the other equations of the group, then (181) becomes:

$$\left.\begin{array}{l} b_1z_2+S_1=v_1, \\ b_2z_2+S_2=v_2, \\ b_3z_2+S_3=v_3, \\ b_4z_2+S_4=v_4, \\ b_5z_2+S_5=v_5. \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (186)$$

Proceeding with these in the same manner as with group (182), we obtain an equation similar to (183); and the most probable value of z_2 will be obtained from it in the same way that (185) was deduced from (183): we may therefore write the last equation of the process:

$$b_1[b_1z_2+S_1]+b_2[b_2z_2+S_2]+b_3[b_3z_2+S_3]+b_4[b_4z_2+S_4]+b_5[b_5z_2+S_5]=0. \quad (187)$$

Finally: in the first equation of group (181) let T_1 stand for all the terms in the first member, except that containing z_3, that is, let $T_1=a_1z_1+b_1z_2-m_1$; and similarly, let $T_2 \ldots T_5$ represent the corresponding terms in the other equations of group (181); then we should get for z_3, a group like (182) for z_1, and like (186) for z_2; that is,

$$\left.\begin{array}{l} c_1z_3+T_1=v_1 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ c_5z_3+T_5=v_5 \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (188)$$

And proceeding with group (188) in the same way as with the other two groups, we should eventually obtain eq. (189)

to express the most probable value of z_3 in the same manner that (185) and (187) do for z_1 and z_2 respectively:

$$c_1[c_1z_3+T_1]+c_2[c_2z_3+T_2]+c_3[c_3z_3+T_3]+c_4[c_4z_3+T_4]+c_5[c_5z_3+T_5]=0. \qquad (189)$$

Eqs. (185), (187), and (189) are the *normal equations;* and no more can be formed from the group of residuals (181): the five of that group—the same in number as the observations—have therefore been reduced to three normal equations; and from these the three unknown quantities (z_1, z_2, z_3) can be determined by usual algebraic processes, and the values so gotten are the most probable, and will satisfy the normal equations.

It only remains to point out the characteristic feature of the normals, and this is most important, because it affords the *principle* upon which any number of observation-equations of the deviations can be reduced to the same number of normals as the magnetic coefficients to be determined.

In each normal equation there are five *sets* of quantities—each *set* within brackets; the first *term* of each *set* in the *same* normal contains one particular unknown quantity, connected, however, with different coefficients; in all three normals—(185), (187), (189)—each quantity within brackets, that is, in reality, each residual equation of group (181), is multiplied (outside the brackets) by the coefficient of the unknown quantity in its own residual equation; or, more explicitly, the first normal (185) for z_1 arises from multiplying the first equation of group (181) by a_1, the second of the group by a_2, the third by a_3, the fourth by a_4, and the fifth by a_5, and adding the results; and each of these multipliers ($a_1, a_2 \ldots a_5$) is the coefficient of z_1 in the several equations of group (181). Similarly for (187), $b_1, b_2 \ldots b_5$ are the multipliers; and $c_1, c_2 \ldots c_5$ for (189). From this is deduced the *Rule* for forming the Normal Equations: *For each observation, write an equation of the connected quan-*

tities and thence deduce the residual equations; from these form the normal equations thus:—take each residual equation and multiply it by the coefficient of the first unknown quantity in that equation; add the results and the sum will be the normal equation for that unknown quantity. Similarly, take each residual equation and multiply it by the coefficient of the second unknown quantity in its own equation; do the same for each unknown quantity.

Section Two: The Magnetic Coefficients deduced by the Method of Least Squares.

309. Observation-equations that may form the Basis of the Operations.—The deviations may be calculated by eq. (149), but it contains twelve terms—many more than are essential for the customary accuracy, and needs at least twelve observations to determine the coefficients A, B, $C, \ldots\ldots M$, N. Five terms generally suffice, and eq. (124), which is exact, is often used as the starting-point: it has, however, the quantity sought (δ) connected with the known quantities, and this is troublesome.

Eq. (125) is very convenient; it affords only the approximate coefficients, however, but these once determined, the exact coefficients, to the number of twelve, can be deduced from them, and then accuracy to the third order of small quantities, when required, is attainable: (125) will therefore be used as the basic equation.

310. The Coefficients obtained by Method of Least Squares.—Reproducing (125), it is given number (190) here in the natural sequence,

$$\delta = A + B.\sin\ \zeta' + C.\cos\ \zeta' + D.\sin\ 2\zeta' + E.\cos\ 2\zeta'. \qquad (190)$$

The deviation δ is observed on the compass course ζ', so that δ and ζ' are known; the quantities to be determined

are A, B, C, D, E—and five equations like (190) are the least that will give them: any algebraic process will afford the solution in this case. In order to take advantage of whatever number of observations may be made, recourse must be had to the method of Least Squares.

Let the observations be made on the 32 compass-courses ζ_0', ζ_1', $\zeta_2' \ldots \zeta_{31}'$; and let the corresponding deviations be designated by δ_0, δ_1, $\delta_2 \ldots \delta_{31}$; then the 32 equations similar to (190) are:

$$\left.\begin{array}{l}
\delta_0 = A + B.\sin\zeta_0' + C.\cos\zeta_0' + D.\sin 2\zeta_0' + E.\cos\zeta_0', \\
\delta_1 = A + B.\sin\zeta_1' + C.\cos\zeta_1' + D.\sin 2\zeta_1' + E.\cos\zeta_1', \\
\delta_2 = A + B.\sin\zeta_2' + C.\cos\zeta_2' + D.\sin 2\zeta_2' + E.\cos\zeta_2', \\
\delta_3 = A + B.\sin\zeta_3' + C.\cos\zeta_3' + D.\sin 2\zeta_3' + E.\cos\zeta_3', \\
\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\
\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\
\delta_{31} = A + B.\sin\zeta_{31}' + C.\cos\zeta_{31}' + D.\sin 2\zeta_{31}' + E.\cos\zeta_{31}'.
\end{array}\right\} \quad (191)$$

In these, the *sines* and *cosines* may be replaced by their symbols S_0, S_1, $S_2 \ldots S_8$, shown in Fig. 511; but it must be remembered—*first*, that $\sin 2\zeta_1' = \sin\zeta_2'$, $\sin 2\zeta_2' = \sin\zeta_4'$, $\cos 2\zeta_1' = \cos\zeta_2'$, etc.; that is, for each multiple angle $(2\zeta')$ we must use the symbol that represents the sine or cosine of twice the single angle; *second*, that sines and cosines of angles whose sum is 90° are complementary; so that, although S_0, S_1, $S_2 \ldots S_8$ represent only sines, still they can be used for cosines by employing those proper to the complement of the angle; for instance, $\sin\zeta_2' = S_2$, but $\cos\zeta_2' = S_6$, because $\zeta_2' = 22° 30'$ and $\zeta_6' = 67° 30'$, and $\zeta_2' + \zeta_6' = 90°$; *third*, that certain values of sine and cosine reduce the terms in which they occur either to unity or to zero; this is shown in paragraphs [5] and [6] of Art. 296; when such is the case, the factor will still be inserted to preserve the symmetry of the equations: with these considerations in view, and regard for the proper algebraic signs of the functions, group (191) becomes

$$\left.\begin{array}{l}\delta_0=A+B.S_0+C.S_0+D.S_0+E.S_0,\\ \delta_1=A+B.S_1+C.S_7+D.S_2+E.S_6,\\ \delta_2=A+B.S_2+C.S_6+D.S_4+E.S_4,\\ \delta_3=A+B.S_3+C.S_5+D.S_6+E.S_2,\\ \quad\cdots\cdots\cdots\cdots\cdots\\ \quad\cdots\cdots\cdots\cdots\cdots\\ \quad\cdots\cdots\cdots\cdots\cdots\\ \delta_{31}=A-B.S_1+C.S_7-D.S_2+E.S_6.\end{array}\right\} \quad . \quad (192)$$

These are the observation equations. To form the normal equation for A, each of the 32 equations of this group must be multiplied by the factor of A, which being unity throughout, does not change the value of any term; then adding together all the members on each side of the sign of equality, we have

$$\delta_0+\delta_1+\delta_2+\delta_3+\ldots+\delta_{31}=32A+[\text{terms in } B, C, D, E]. \quad (193)$$

But by eqs. (46) and (47), page 858, all the terms within brackets in (193) reduce to zero, so that the normal for A is

$$32A=\delta_0+\delta_1+\delta_2+\delta_3+\ \ldots\ +\delta_{31};$$

whence
$$A=\frac{\delta_0+\delta_1+\delta_2+\delta_3+\ \ldots\ +\delta_{31}}{32} \quad . \quad . \quad . \quad (194)$$

To form the normal for B, each of the 32 equations of group (192) must be multiplied by the factor of B in its own equation, that is, the first one of the group by S_0, the second by S_1, the third by S_2, and so on; and then the members on each side of the sign of equality added together: by Fig. 511 it is seen that (numerically) the factor S_0 for the observation δ_0 at north is the same as that for δ_{16} at south, and that the factor S_1 for the observation δ_1 at N. by E. is the same as that for δ_{17} at S. by W., and so on; we may therefore group the left-hand members of the separate equations of (192) when added together, according to the factors S_0, S_1, $S_2 \ldots S_8$; with proper regard for the signs

of sine and cosine in the different quadrants, the normal equation for B, then, is (195).

NORMAL EQUATION FOR B.

Left-hand member. *Right-hand member.*

$$\left.\begin{array}{l}
\tfrac{1}{2}(\partial_0-\partial_{16})S_0\\
+\tfrac{1}{2}(\partial_1-\partial_{17})S_1\\
+\tfrac{1}{2}(\partial_2-\partial_{18})S_2\\
+\tfrac{1}{2}(\partial_3-\partial_{19})S_3\\
+\tfrac{1}{2}(\partial_4-\partial_{20})S_4\\
+\tfrac{1}{2}(\partial_5-\partial_{21})S_5\\
+\tfrac{1}{2}(\partial_6-\partial_{22})S_6\\
+\tfrac{1}{2}(\partial_7-\partial_{23})S_7\\
+\tfrac{1}{2}(\partial_8-\partial_{24})S_8\\
+\tfrac{1}{2}(\partial_9-\partial_{25})S_7\\
+\tfrac{1}{2}(\partial_{10}-\partial_{26})S_6\\
+\tfrac{1}{2}(\partial_{11}-\partial_{27})S_5\\
+\tfrac{1}{2}(\partial_{12}-\partial_{28})S_4\\
+\tfrac{1}{2}(\partial_{13}-\partial_{29})S_3\\
+\tfrac{1}{2}(\partial_{14}-\partial_{30})S_2\\
+\tfrac{1}{2}(\partial_{15}-\partial_{31})S_1
\end{array}\right\}
=\left\{\begin{array}{l}
\tfrac{1}{2}A\left\{\begin{array}{l} S_8-S_8+2\,(S_1-S_1+S_2\\ \quad -S_2+S_3-S_3+S_4\\ \quad -S_4+S_5-S_5+S_6\\ \quad -S_6+S_7-S_7)\end{array}\right\}\\
+\tfrac{1}{2}B\left\{\begin{array}{l} 2S_8^2+4(S_1^2+S_2^2+S_3^2\\ \quad +S_4^2+S_5^2+S_6^2+S_7^2)\end{array}\right\}\\
+\tfrac{1}{2}C\left\{\begin{array}{l} 2(S_4S_4-S_4S_4)+4(S_1S_7\\ \quad -S_1S_7+S_2S_6-S_2S_6\\ \quad +S_3S_5-S_3S_5)\end{array}\right\}\\
+\tfrac{1}{2}D\left\{\begin{array}{l} 2(S_1S_2-S_1S_2+S_2S_4\\ \quad -S_2S_4+S_3S_6-S_3S_6\\ \quad +S_4S_8-S_4S_8+S_5S_6\\ \quad -S_5S_6+S_6S_4-S_6S_4\\ \quad +S_7S_2-S_7S_2)\end{array}\right\}\\
+\tfrac{1}{2}E\left\{\begin{array}{l} S_8S_0-S_8S_0+2(S_1S_6\\ \quad -S_1S_6+S_2S_4-S_2S_4\\ \quad +S_3S_2-S_3S_2+S_5S_2\\ \quad -S_5S_2+S_6S_4-S_6S_4\\ \quad +S_7S_6-S_7S_6)\end{array}\right\}
\end{array}\right\} \qquad (195)$$

All the factors within brackets of the second member of (195), except that of B, have two identical terms of opposite sign; so that A, C, D, E reduce to zero. The value of the factor of B will now be found: remembering the signification of S_0, $S_1 \ldots S_8$, Art. 296, we have by Trig., $S_8=1$, $\therefore S_8^2=1$; the pairs S_1 and S_7, S_2 and S_6, S_3 and S_5, are complementary, so that $\sin^2+\cos^2$ of each pair is equal to unity, that is, $S_1^2+S_7^2=1$, $S_2^2+S_6^2=1$, and $S_3^2+S_5^2=1$;

also $S_4^2 = \frac{1}{2}$: substituting these in the factor of B, it becomes

$$\{2(1) + 4(1 + 1 + 1 + \tfrac{1}{2})\} = 16. \quad . \quad . \quad . \quad (196)$$

Denoting the left-hand member of (195) by

$$\Sigma[\tfrac{1}{2}(\delta_0 \ldots \delta_{31})(S_0 \ldots S_8)], \quad . \quad . \quad . \quad (197)$$

and substituting (196) and (197) in (195), the latter becomes

$$\Sigma[\tfrac{1}{2}(\delta_0 \ldots \delta_{31})(S_0 \ldots S_8)] = \tfrac{1}{2}B\{16\} = 8B; \quad . \quad (198)$$

whence
$$B = \frac{\Sigma[\tfrac{1}{2}(\delta_0 \ldots \delta_{31})(S_0 \ldots S_8)]}{8}. \quad . \quad . \quad (199)$$

To form the normal equation for C each of the 32 equations of (192) must be multiplied by the factor of C in its own equation; that is, the first by S_0, the second by S_7, the third by S_6, and so on; then the procedure is the same as that to get the normal equation for B, and indeed that for C is quite similar to the one for B: the identities and differences are these—the left-hand members of both are the same; of the terms of the right-hand members, those in A and D have identical factors within brackets; of the terms in B and C, the factors within brackets are alike, except that S_8^2 occurs in the normal for B and S_0^2 in that for C; also, these factors are interchanged—that of B in the normal for B is the factor of C in the normal for C; the factor of C in the normal for B is the factor of B in the normal for C; factor of E is the same in both normals, except that it has $(S_8S_0 - S_8S_0)$ in the normal for B, and $(S_0^2 - S_0^2)$ in that for C. The procedure with the normal for C is the same as that for B between equations (195) and (199), so that the value of C, which is similar to (199), may be written at once:

$$C = \frac{\Sigma[\tfrac{1}{2}(\delta_0 \ldots \delta_{31})\ (S_0 \ldots S_8)]}{8}. \quad . \quad . \quad (200)$$

To form the normal for D, multiply the first equation of group (192) by S_0, the second by S_2, the third by S_4, and so on—each equation by the factor of D in that equation;

and then proceed as with B between eqs. (194) and (195), whence eq. (201), which is the normal for D.

As with (195) for B, so with (201) for D, the observations of $\delta_0, \delta_1, \delta_2 \ldots \delta_{31}$ are grouped according to the factors $S_0, S_1, S_2 \ldots S_8$, due regard being had to algebraic signs; also, (195) was divided throughout by 2, as indicated by the factor $\frac{1}{2}$ connected with every term—similarly, (201) is divided throughout by 4, as indicated by the factors $(\frac{1}{2})(\frac{1}{2})$ in the first member, and $\frac{1}{4}$ in the second.

NORMAL EQUATION FOR D.

Left-hand member. *Right-hand member.*

$$\left.\begin{array}{l}
\frac{1}{2}\left(\frac{\delta_0+\delta_{16}}{2}\right)S_0-\frac{1}{2}\left(\frac{\delta_8+\delta_{24}}{2}\right)S_0 \\
+\frac{1}{2}\left(\frac{\delta_1+\delta_1}{2}\right)S_2-\frac{1}{2}\left(\frac{\delta_9+\delta_{25}}{2}\right)S_2 \\
+\frac{1}{2}\left(\frac{\delta_2+\delta_{18}}{2}\right)S_4-\frac{1}{2}\left(\frac{\delta_{10}+\delta_{26}}{2}\right)S_4 \\
+\frac{1}{2}\left(\frac{\delta_3+\delta_{19}}{2}\right)S_6-\frac{1}{2}\left(\frac{\delta_{11}+\delta_{27}}{2}\right)S_6 \\
+\frac{1}{2}\left(\frac{\delta_4+\delta_{20}}{2}\right)S_8-\frac{1}{2}\left(\frac{\delta_{12}+\delta_{28}}{2}\right)S_8 \\
+\frac{1}{2}\left(\frac{\delta_5+\delta_{21}}{2}\right)S_6-\frac{1}{2}\left(\frac{\delta_{13}+\delta_{29}}{2}\right)S_6 \\
+\frac{1}{2}\left(\frac{\delta_6+\delta_{22}}{2}\right)S_4-\frac{1}{2}\left(\frac{\delta_{14}+\delta_{30}}{2}\right)S_4 \\
+\frac{1}{2}\left(\frac{\delta_7+\delta_{23}}{2}\right)S_2-\frac{1}{2}\left(\frac{\delta_{15}+\delta_{31}}{2}\right)S_2
\end{array}\right\}=\left\{\begin{array}{l}
\frac{1}{4}A[2(S_8-S_8) \\
\quad +4(S_2-S_2+S_4 \\
\quad -S_4+S_6-S_6)] \\
+\frac{1}{4}B[2(S_1S_2 \\
\quad -S_1S_2+S_2S_4 \\
\quad -S_2S_4+S_3S_6 \\
\quad -S_3S_6+S_8S_4 \\
\quad -S_8S_4+S_6S_5 \\
\quad -S_6S_5+S_4S_6 \\
\quad -S_4S_6+S_2S_7 \\
\quad -S_2S_7)] \\
+\frac{1}{4}C[2(S_1S_2 \\
\quad -S_1S_2+S_2S_4 \\
\quad -S_2S_4+S_3S_6 \\
\quad -S_3S_6+S_8S_4 \\
\quad -S_8S_4+S_6S_5 \\
\quad -S_6S_5+S_4S_6 \\
\quad -S_4S_6+S_2S_7 \\
\quad -S_2S_7)] \\
+\frac{1}{4}D[4S_8^2+8(S_2^2 \\
\quad +S_4^2+S_6^2)] \\
+\frac{1}{4}E[4(S_4S_4 \\
\quad -S_4S_4) \\
\quad +8(S_2S_6-S_2S_6)]
\end{array}\right\} \quad (201)$$

In (201), the factors within brackets that are connected with A, B, C, and E contain two identical quantities of opposite sign, so that those terms reduce to zero, leaving only that containing D; the value of this is found in a manner similar to that of B in (195), that is: $S_8^2=1$; $S_2^2+S_6^2=1$; $S_4^2=\frac{1}{2}$; substituting these in the factor of D, it is $[4(1)+8(1+\frac{1}{2})]=16$; hence the second member of (201) is

$$\tfrac{1}{4}D[16]=4D. \quad . \quad . \quad . \quad . \quad . \quad (202)$$

Substituting this and the equivalent of the first member, represented by a quantity analogous to (197) and designated by Σ_1, in (201), this becomes

$$\Sigma_1[\tfrac{1}{2}(\delta_0 \ldots \delta_{31})(S_0 \ldots S_8)]=4D; \quad . \quad . \quad (203)$$

whence

$$D=\frac{\Sigma_1[\frac{1}{2}(\delta_0 \ldots \delta_{31})(S_0 \ldots S_8)]}{4}. \quad . \quad . \quad (204)$$

To form the normal equation for E, each equation of group (192) is multiplied by the factor of E in it, that is, the first by S_0, the second by S_8, the third by S_4, and so on; the procedure then is the same as that for obtaining the normal for B: the left-hand member of E is identical with that for D, so that (203) may be used at once to represent it; the normal for E, then, is

$$\Sigma_1[\tfrac{1}{2}(\delta_0 \ldots \delta_{31})(S_0 \ldots S_8)]=$$
$$\left\{\begin{array}{l} \tfrac{1}{4}A[2(S_0-S_0)+4(S_2-S_2+S_4-S_4+S_6-S_6)] \\ +\tfrac{1}{4}B[S_0S_8-S_0S_8+2(S_2S_3-S_2S_3+S_4S_2-S_4S_2 \\ \quad +S_6S_1-S_6S_1+S_2S_5-S_2S_5+S_4S_6-S_4S_6 \\ \quad +S_6S_7-S_6S_7)] \\ +\tfrac{1}{4}C[S_0^2-S_0^2+2(S_2S_3-S_2S_3+S_4S_2-S_4S_2 \\ \quad +S_6S_1-S_6S_1+S_2S_5-S_2S_5+S_4S_6-S_4S_6 \\ \quad +S_6S_7-S_6S_7)] \\ +\tfrac{1}{4}D[4(S_4^2-S_4^2)+8(S_2S_6-S_2S_6)] \\ +\tfrac{1}{4}E[4S_0^2+8(S_2^2+S_4^2+S_6^2)] \end{array}\right\} \quad (205)$$

The terms containing A, B, C, D reduce to zero, and the value of the factor of E is [16], so that the second member of (205) becomes $4E$; hence

$$E = \frac{\Sigma_1[\frac{1}{2}(\delta_0 \dots \delta_{31})(S_0 \dots S_8)]}{4}. \quad . \quad . \quad (206)$$

Equations (194), (199), (200), (204), and (206) constitute the mathematical foundation of **Form 10** for analyzing the deviations given on pages 920 and 921. Those equations give the value in degrees and minutes of the coefficients A, B, C, D, E in terms of the observed deviations δ_0, δ_1, $\delta_2 \dots \delta_{31}$ and the natural sines S_0, S_1, $S_2 \dots S_8$ of the 32 compass courses (ζ') on which the observations were made. Similar formulas are obtained in the same way for observations on 16 or 8 equidistant points.

311. Computation of the Exact Coefficients.—Eq. (149) is based on the retention of the third order of small quantities which are defined in Art. 303, and is exact to that degree; eq. (190), which forms the basis of the computation of the coefficients in Art. 310, is identical with (149) to the same number of terms in both, but, having only five, the coefficients of Art. 310 are approximate. Eqs. (148) and (149) are identical, the quantities within brackets of the former being represented by single letters in the latter; these several equalities—that is, corresponding terms in (148) and (149) connected with the same function and multiple of ζ'—will now be reproduced: $\mathfrak{A}$, A, $\mathfrak{E}$, E are small quantities of the second order, and $\mathfrak{B}$, B, $\mathfrak{C}$, C, $\mathfrak{D}$, D of the first order. Then from (148) and (149), equating corresponding terms, we have:

$$A = \mathfrak{A}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (207)$$

$$B = \mathfrak{B} + \tfrac{1}{8}\mathfrak{B}.\mathfrak{E}^2 + \tfrac{1}{8}\mathfrak{B}^3 + \tfrac{1}{4}\mathfrak{B}.\mathfrak{D}^2 - \tfrac{1}{2}\mathfrak{B}.\mathfrak{D} - \tfrac{1}{2}\mathfrak{C}.\mathfrak{E}. \quad (208)$$

$$C = \mathfrak{C} - \tfrac{1}{2}\mathfrak{B}.\mathfrak{E} + \tfrac{1}{8}\mathfrak{B}^2.\mathfrak{C} + \tfrac{1}{2}\mathfrak{C}.\mathfrak{D} + \tfrac{1}{4}\mathfrak{C}.\mathfrak{D}^2 + \tfrac{1}{8}\mathfrak{C}^3. \quad (209)$$

$$D = \mathfrak{D}. \qquad (210)$$

$$E = \mathfrak{A}.\mathfrak{D} + \mathfrak{E}. \qquad (211)$$

$$F = \tfrac{1}{2}\mathfrak{B}.\mathfrak{D} - \tfrac{3}{8}\mathfrak{B}.\mathfrak{D}^2 - \tfrac{1}{24}\mathfrak{B}^3 + \tfrac{1}{8}\mathfrak{B}.\mathfrak{C}^2 - \tfrac{1}{2}\mathfrak{C}.\mathfrak{E}. \qquad (212)$$

$$G = \tfrac{1}{2}\mathfrak{B}.\mathfrak{E} - \tfrac{1}{8}\mathfrak{B}^2.\mathfrak{C} + \tfrac{1}{2}\mathfrak{C}.\mathfrak{D} + \tfrac{3}{8}\mathfrak{C}.\mathfrak{D}^2 + \tfrac{1}{24}\mathfrak{C}^3. \qquad (213)$$

$$H = \tfrac{1}{2}\mathfrak{D}^2. \qquad (214)$$

$$K = \mathfrak{E}.\mathfrak{D}. \qquad (215)$$

$$L = \tfrac{3}{8}\mathfrak{B}.\mathfrak{D}^2. \qquad (216)$$

$$M = \tfrac{3}{8}\mathfrak{C}.\mathfrak{D}^2. \qquad (217)$$

$$N = \tfrac{1}{3}\mathfrak{D}^3. \qquad (218)$$

Remembering that the five coefficients A, B, C, D, E are expressed in degrees and minutes, it is seen by eqs. (126) and (127), page 882, that the values of the sine and cosine of an arc in terms of the arc itself, to the third order of small quantities inclusive, are

$\sin B = B - \tfrac{1}{6}B^3$. . (219); and $\cos B = 1 - \tfrac{1}{2}B^2$. . (220)

From these and eq. (23), page 856, we have

$$\frac{B^2}{2} = 1 - \cos B = \text{versin } B \qquad (221)$$

and

$$B = \sin B + (B)\left(\frac{1}{3}.\frac{B^2}{2}\right); \qquad (222)$$

whence

$$B = \sin B + \left(\sin B + \frac{B^3}{6}\right)\left(\frac{1}{3}.\frac{B^2}{2}\right)$$

$$= \sin B + \sin B.\frac{1}{3}.\frac{B^2}{2} + \left(\frac{B^3}{6}.\frac{1}{3}.\frac{B^2}{2}\right). \qquad (223)$$

The last term, being of the fifth order, must be omitted; hence

$$B = \sin B\left(1 + \frac{1}{3}.\frac{B^2}{2}\right) = \sin B(1 + \tfrac{1}{3}\text{ versin } B). \qquad (224)$$

$$C = \sin C(1 + \tfrac{1}{3} \operatorname{versin} C) \quad . \quad . \quad . \quad . \quad . \quad (225)$$

$$D = \sin D(1 + \tfrac{1}{3} \operatorname{versin} D). \quad . \quad . \quad . \quad . \quad (226)$$

eqs. (224), (225), and (226) when necessary, and ues of $\mathfrak{A}$ and $\mathfrak{E}$ which will be determined first, we proceed to deduce the *Exact Coefficients* in terms of A, B, C, D, E from eqs. (207) to (218), retaining small quantities to the third order inclusive.

From (207) and (219) by analogy

$$\mathfrak{A} = A = \sin A. \quad . \quad . \quad . \quad . \quad . \quad . \quad (227)$$

From (210) and (226)

$$\mathfrak{D} = D = \sin D(1 + \tfrac{1}{3} \operatorname{versin} D). \quad . \quad . \quad . \quad (228)$$

From (211), (227), and (228)

$$\mathfrak{E} + \mathfrak{A}.\mathfrak{D} = E; \text{ hence } \mathfrak{E} = \sin E - \sin A.\sin D. \quad (229)$$

Factoring (208), and rearranging its terms, it is

$$\mathfrak{B}\left(1 - \frac{\mathfrak{D}}{2} + \frac{\mathfrak{B}^2}{8} + \frac{\mathfrak{C}^2}{8} + \frac{\mathfrak{D}^2}{4}\right) - \frac{\mathfrak{C}.\mathfrak{E}}{2} = B. \quad . \quad (230)$$

If we square both members of (230), and reject in the result the fourth and subsequent orders, it becomes

$$\mathfrak{B}^2 - \mathfrak{B}^2.\mathfrak{D} = B^2; \quad . \quad . \quad . \quad . \quad . \quad . \quad (231)$$

whence

$$\mathfrak{B}^2(1 - \mathfrak{D}) = B^2, \text{ and } \mathfrak{B}^2 = \frac{B^2}{1 - \mathfrak{D}} = B^2 + B^2.\mathfrak{D}. \quad (232)$$

Substituting this value of $\mathfrak{B}^2$ in (230), and transposing $\frac{\mathfrak{C}.\mathfrak{E}}{2}$, we have

$$\mathfrak{B}\left(1 - \frac{\mathfrak{D}}{2} + \frac{B^2}{8} + \frac{B^2.\mathfrak{D}}{8} + \frac{\mathfrak{C}^2}{8} + \frac{\mathfrak{D}^2}{4}\right) = B + \frac{\mathfrak{C}.\mathfrak{E}}{2}. \quad (233)$$

On dividing both members of (233) by the factor of $\mathfrak{B}$ within parentheses, we obtain eq. (234), *after* rejecting quantities of the fourth and higher orders:

$$\mathfrak{B} = B + \frac{B.\mathfrak{D}}{2} - \frac{B^3}{8} - \frac{B.\mathfrak{C}^2}{8} + \frac{\mathfrak{C}.\mathfrak{E}}{2}$$
$$= B\left(1 + \frac{\mathfrak{D}}{2} - \frac{B^2}{8} - \frac{\mathfrak{C}^2}{8}\right) + \frac{\mathfrak{C}.\mathfrak{E}}{2}. \quad (234)$$

By (232) it is seen that as a result of squaring (230), $\mathfrak{B}^2$ is equal to B^2 plus a term $B^2.\mathfrak{D}$; but that in (234)—which is the *quotient* of the second member of (233) by the factor of $\mathfrak{B}$ in the first member—the term $B^2.\mathfrak{D}$ has disappeared, because it produced terms of the fourth and higher orders.

Now, comparing the quantity within parentheses of the last member of (234) with the factor of $\mathfrak{B}$ in (230), it is seen that, with some changes of sign, they are the same—only, that B^2 has replaced $\mathfrak{B}^2$, and $\frac{\mathfrak{D}^2}{4}$ has disappeared, because in passing from (233) to (234), *it* also produced terms of the fourth and higher orders.

If (209) be arranged as (208) was, an equation for $\mathfrak{C}$, symmetrical with (230) for $\mathfrak{B}$, will result; and operating on this in the same way as on (230), we should obtain an equation for $\mathfrak{C}$ entirely analogous to (234) for $\mathfrak{B}$: therefore, $\mathfrak{B}^2$ and $\mathfrak{C}^2$—as well as $\mathfrak{B}$ and $\mathfrak{C}$ which are coupled respectively with $\mathfrak{E}$ in (208) and (209)—can be replaced directly by B^2, C^2, B, and C; because, while their values when found as in (232) are connected with other terms, still these disappear in the operations, on account of producing higher orders than the third. As a result of these facts, we may therefore write at once an equation for $\mathfrak{C}$ derived from (209), analogous to (234) for $\mathfrak{B}$ derived from (208); and at the same time we may replace $\mathfrak{D}$ by D and $\mathfrak{E}$ by E; the equa-

d $\mathfrak{C}$ respectively, both similar to (234), then are

$$\mathfrak{B}=B\left(1+\frac{D}{2}-\frac{B^2}{8}-\frac{C^2}{8}\right)+\frac{C.E}{2} \quad . \quad . \quad . \quad (235)$$

$$\mathfrak{C}=C\left(1-\frac{D}{2}-\frac{B^2}{8}-\frac{C^2}{8}\right)+\frac{B.E}{2} \quad . \quad . \quad . \quad (236)$$

Substituting in (235) the necessary values from (224) to (229) and then performing the indicated operations on (235), we obtain (237), *after*, however, rejecting all terms of the fourth and higher orders, of which several will result from squaring and multiplying as required in (235). The sum of the exponents of a term determines its order: sin B, sin C, sin D are each of the first order and first power, so that the product of all three would constitute a term of the third order; sin A, sin E, and the *versines* of B, C, D, are each of the second order, and any two, or one of them connected with two of the first order, would constitute a term of the fourth order; it is such terms, and others containing higher orders, that are rejected in passing from (235) to the following equation:

$$\mathfrak{B}=\sin B\,[1+\tfrac{1}{3}\text{ versin }B+\tfrac{1}{2}\sin D-\tfrac{1}{8}\sin^2 B$$
$$-\tfrac{1}{8}\sin^2 C]+\tfrac{1}{2}\sin C.\sin E. \quad (237)$$

Substituting in this the arc for its function in the case of $\sin^2 B$ and $\sin^2 C$, it is

$$\mathfrak{B}=\sin B\left[1+\frac{1}{3}\text{ versin }B+\frac{1}{2}\sin D-\frac{1}{4}.\frac{B^2}{2}-\frac{1}{4}\frac{C^2}{2}\right]$$
$$+\tfrac{1}{2}\sin C.\sin E. \quad (238)$$

This, by means of eq. (23), page 856, becomes

$$\mathfrak{B}=\sin B\,[1+\tfrac{1}{3}\text{ versin }B+\tfrac{1}{2}\sin D-\tfrac{1}{4}\text{ versin }B$$
$$-\tfrac{1}{4}\text{ versin }C]+\tfrac{1}{2}\sin C.\sin E. \quad (239)$$

whence

$$\mathfrak{B}=\sin B\,[1+\tfrac{1}{2}\sin D+\tfrac{1}{12}\text{ versin }B-\tfrac{1}{4}\text{ versin }C]$$
$$+\tfrac{1}{2}\sin C.\sin E. \quad (240)$$

And by a similar procedure with eq. (236) we obtain the following:

$$\mathfrak{C} = \sin C \, [1 - \tfrac{1}{2} \sin D + \tfrac{1}{12} \text{ versin } C - \tfrac{1}{4} \text{ versin } B] + \tfrac{1}{2} \sin B \, . \sin E. \quad (241)$$

Eqs. (227), (228), (229), (240), and (241) are those given on "Form 10, Analysis of Deviations," for computing the Exact Coefficients; see page 921.

The value of the remaining *exact coefficients* $F, G, H \ldots N$, in terms of A, B, C, D, E, will now be obtained from eqs. (212) to (218), which, as already stated, are the comparison of the coefficients of corresponding terms in (148) and (149); the procedure need not be given in detail—it is entirely analogous to that practised upon eq. (208) to obtain (240). Since $\mathfrak{B} . \mathfrak{D} = B(D + \frac{1}{2}D^2)$, from multiplying (228) and (235), member by member, and rejecting in the result the fourth and subsequent orders, we hence have from (212)

$$F = B[\tfrac{1}{2}D - \tfrac{1}{24}B^2 + \tfrac{1}{8}C^2 - \tfrac{1}{8}D^2] - \tfrac{1}{2}E.C. \quad . \quad (242)$$

Similarly, from (228) and (236), $\mathfrak{C} . \mathfrak{D} = C(D - \frac{1}{2}D^2)$, whence, by substitution in (213), this becomes

$$G = C[\tfrac{1}{2}D + \tfrac{1}{24}C^2 - \tfrac{1}{8}B^2 + \tfrac{1}{8}D^2] + \tfrac{1}{2}E.B. \quad . \quad (243)$$

Then from (214) to (218), respectively, we have directly, to third order of small quantities included:

$$H = \tfrac{1}{2}D^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (244)$$

$$K = E.D. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (245)$$

$$L = \tfrac{3}{8}B.D^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (246)$$

$$M = \tfrac{3}{8}C.D^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (247)$$

$$N = \tfrac{1}{3}D^3. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (248)$$

The computation of the coefficients $F, G, H \ldots N$ will be illustrated in the next section.

Section Three: Analysis and Computation of the Deviations.

312. Compass Report—Form 7.—This was devised in the Compass Office in 1882, and, until recently, was known as Form [I].

It was designed to be a convenient form for recording all the data relative to the observations and deviations, either while steaming in a circle or swinging at anchor, using the Sun or a terrestrial mark as the object of observation. With but few alterations consequent upon the changed conditions on shipboard, it is the same that has been in use in the Navy for the past twenty years, and therefore requires no special explanation. Besides, the Form itself indicates the mode of procedure: it is reproduced on pages 918–'19 with a complete series of observations made at sea on board the U. S. S. CONCORD.

313. Method of analyzing the Deviations—Form 10. Like the Form of the preceding article, Form 10 has long been in use in the Navy, and known as Form [IV].

Comparing **Form 10** with eqs. (195) to (201), Art. 310, it will be seen that its columns are but a tabular arrangement of the quantities in those equations: the mathematical basis of the Form being thus pointed out, its practical use will now be described.

A series of observations having been made for Form 7, pages 918–'19, the resulting deviations, col. (9) of Form 7 are transcribed to cols. (2) and (4) of Form 10, pages 920–'21: to illustrate the use of the Form, the deviations of the steering compass of the U. S. S. ATLANTA will be analyzed. This ship is a protected steel cruiser of 3000 tons, with a battery of eight heavy guns: she was swung at the buoys of the Compass Station, Newport, R. I., Sept. 18 and 19, 1886, and the observations were made on the 32 compass-points.

The case of the steering compass is selected because of its *un*favorable location as regards directive force and deviations, the latter attaining a maximum of 50°; it will therefore be a severe test of certain matters to be illustrated.

Form 10 consists of three parts, headed Tables I, II, and III; and the chief liability to error in its use is the manipulation of the algebraic signs: therefore it must be remembered that plus multiplied by plus, and minus multiplied by minus, both produce plus; that plus multiplied by minus produces minus; and that the "sum" of quantities means their numerical difference when of opposite sign, but their addition when of the same sign—the result in the former case receiving the sign of the greater quantity. On account of the varied signs of the parts composing the exact coefficients, especial care is necessary in Table III of Form 10. Upon entering the deviations in cols. (2) and (4), Form 10, pages 920 and 921, mark all that are easterly plus, and all that are westerly minus. The multipliers S_0, S_1, $S_2 \ldots S_8$ have their signs determined by the quadrants in which their angles fall—Fig. 511.

In the case of the ATLANTA, sin B and sin C are minus, although both apparently belong to angles less than 90°, and should therefore be in the first quadrant, where the sine is plus; this seeming inconsistency is not real; for the starboard angle α, calculated from B and C, is 199° 53′, therefore in the third quadrant, where the sine is minus, and thus the signs of B and C are in accord with the Trigonometrical fact.

By eq. (23), page 856, versin $x = 1 - \cos x$: as the cosine is always less than unity, the versin is therefore plus, whatever be the actual sign of x.

Column (5), Form 10, p. 920, is formed by taking half the *algebraic* sum of the quantities on the same horizontal line in cols. (2) and (4), and giving the result the sign of the greater: col. (11) is obtained from (9) and (10) in the same way.

Form 7. **Compass Report.**

U. S. S. CONCORD: Observations for this series of deviations made on the 6th day of February, 1892, at sea, in latitude 34° 50′ S. and longitude 54° 20′ W.; object observed, sun; altitude on beginning set of observations,

Ship's Head by Bridge Compass No. 173.	Times of Observation by Watch.	Watch Correction on l. ap. t.	Local Apparent Times of Observation.	Azimuth of Sun by Bridge Compass. No. 173.
(1)	(2)	(3)	(4)	(5)
N.	7^h 58^m 0^s	Error of watch on local apparent time: 0h. 11m. 09s. To be added to watch time of observation. To be subtracted from watch time of observation.	7^h 46^m 51^s	N. 84° 30′ E.
N. by E.	7 54 30		7 43 21	N. 84 45 E.
N. NE.	7 51 0		7 39 51	N. 85 30 E.
NE. by N.	7 47 30		7 36 21	N. 86 30 E.
NE.	7 44 0		7 32 51	N. 87 30 E.
NE. by E.	7 40 30		7 29 21	N. 88 30 E.
E. NE.	7 37 0		7 25 51	S. 89 30 E.
E. by N.	7 33 30		7 22 21	S. 88 15 E.
E.	7 30 0		7 18 51	S. 87 0 E.
E. by S.	7 26 30		7 15 21	S. 86 15 E.
E. SE.	7 23 0		7 11 51	S. 85 15 E.
SE. by E.	7 19 30		7 08 21	S. 85 15 E.
SE.	7 16 0		7 04 51	S. 85 30 E.
SE. by S.	7 12 30		7 01 21	S. 87 15 E
S. SE.	7 09 0		6 57 51	S. 89 30 E.
S. by E.	7 05 30		6 54 21	N. 88 15 E.
S.	7 02 0		6 50 51	N. 85 30 E.
S by W.	6 58 30		6 47 21	N. 82 30 E.
S. SW.	6 55 30		6 43 51	N. 80 30 E.
SW. by S.	6 51 30		6 40 21	N. 79 30 E.
SW.	6 48 0		6 36 51	N. 78 0 E.
SW. by W.	8 36 30		8 25 21	N. 63 0 E
W. SW.	8 33 0		8 21 51	N. 64 30 E.
W. by S.	8 29 30		8 18 21	N. 67 15 E.
W.	8 26 0		8 14 51	N. 69 15 E.
W. by N.	8 22 30		8 11 21	N. 72 30 E.
W. NW.	8 19 0		8 07 51	N. 75 30 E.
NW. by W.	8 15 30		8 04 21	N. 78 30 E.
NW.	8 12 0		8 0 51	N. 80 30 E.
NW. by N.	8 08 30		7 57 21	N. 82 30 E.
N. NW.	8 05 0		7 53 51	N. 83 0 E.
N. by W.	8 01 30		7 50 21	N. 84 15 E.

Watch Comparisons Before and After Series of Observation.				Local Apparent Time, by Chron			
Before obs:				Chron. No.	12^h	17^m	30^s
Chron. No.	10^h	10^m	0^s	C. C.+	0	07	32
Watch.	6	37	05	G. m. t.	12	25	02
Dif.	3	32	55	Eq. t.	—	14	16
After obs:				G. ap. t. ⊙.	12	10	46
Chron. No.	12	17	30	Long. W.	3	37	20
Watch.	8	44	35	Local ap. t.	8	33	26
Dif.	3	32	55	Watch.	8	44	35
				Error of w. on l. a. t.		11	09

Compass Report. Form 7.

15° 50′; altitude on completing set of observations, 39° 11′; mean declination, 15° 41′ S.; sea smooth; weather clear; under steam.

True Azimuth of Sun by Tables.	Errors of Compass No. 173 while Upright on every Point.	Variation.	Deviations of Compass No. 173.	Ship's Head by Bridge Compass No. 173.
(6)	(7)	(8)	(9)	(1)
S. 91° 35′ E.	3° 55′ E.	Mean of 32 equidistant azimuths, Col. 5 = S. 96° 37′ E. Mean of the corresponding azimuths of Col. 6 = S. 89° 33′ E. Difference = 7° 04′ E. = Variation by observation + constant *A*. Variation by chart = 6° 40′ E.	3° 09′ W.	N.
S. 91 11 E.	4 04 E.		3 0 W.	N. by E.
S. 90 43 E.	3 47 E.		3 17 W.	N. NE.
S. 90 10 E.	3 20 E.		3 44 W.	NE. by N.
S. 89 35 E.	2 55 E.		4 09 W.	NE.
S. 89 08 E.	2 22 E.		4 42 W.	NE. by E.
S. 88 44 E.	0 46 E.		6 18 W.	E. NE.
S. 88 07 E.	0 08 E.		6 56 W.	E. by N.
S. 87 44 E.	0 44 W.		7 48 W.	E.
S. 87 11 E.	0 56 W.		8 0 W.	E. by S.
S. 86 45 E.	1 30 W.		8 34 W.	E. SE.
S. 86 14 E.	0 59 W.		8 03 W.	SE. by E.
S. 85 50 E.	0 20 W.		7 24 W.	SE.
S. 85 15 E.	2 0 E.		5 04 W.	SE. by S.
S. 84 52 E.	4 38 E.		2 26 W.	S. SE.
S. 84 19 E.	7 26 E.		0 22 E.	S. by E.
S. 83 55 E.	10 35 E.		3 31 E.	S.
S. 83 19 E.	14 11 E.		7 07 E.	S. by W.
S. 82 55 E.	16 35 E.		9 31 E.	S. SW.
S. 82 25 E.	18 05 E.		11 01 E.	SW. by S.
S. 82 01 E.	19 59 E.		12 55 E.	SW.
S. 97 36 E.	19 24 E.		12 20 E.	SW. by W.
S. 97 06 E.	18 24 E.		11 20 E.	W. SW.
S. 96 28 E.	16 17 E.		9 13 E.	W. by S.
S. 95 59 E.	14 46 E.		7 42 E.	W.
S. 95 21 E.	12 09 E.		5 05 E.	W. by N.
S. 94 59 E.	9 31 E.		2 27 E.	W. NW.
S. 94 19 E.	7 11 E.		0 07 E.	NW. by W.
S. 93 49 E.	5 41 E.		1 23 W.	NW.
S. 93 14 E.	4 16 E.		2 48 W.	NW. by N.
S. 92 47 E.	4 13 E.		2 51 W.	N. NW.
S. 92 12 E.	3 33 E.		3 31 W.	N. by W.

Location of Compasses.

Designation of Compass.	Height Above Main Deck.	Distance from the Stern.	Compensated or Not.
Bridge No. 173	11′ 2½″	178′ 5½″	No.

The CONCORD: is a steel cruiser of 1710 tons, twin screws, and battery of 6 guns.

Form 10.—U. S. S. Atlanta: Analysis of Deviations of Steering Compass.

TABLE I.—COMPUTATION OF COEFFICIENTS B AND C.

(1)	(2)	(3)	(4)	(5)	(6)	(7)		(8)	
					Half Sum of Cols. (2) & (4) [changing Sum in Col. (4)].	Computation of B.		Computation of C	
Ship's Head by Steering Compass No. ——.	Deviation. Easterly + Westerly −	Ship's Head by Steering Compass No. ——.	Deviation. Easterly + Westerly −	Half Sum of Cols. (2) and (4).	*Semicircular Deviation.*	Multipliers.	Products of Col. (6) by Multipliers.	Multipliers.	Products of Col. (6) by Multipliers.
	° ′		° ′	° ′	° ′		° ′		° ′
NORTH	−18 00	**SOUTH**	+18 30	+ 0 15	−18 15	0	0 00	1	−18 15
N. by E.	−20 30	S. by W.	+32 15	+ 5 52	−26 22	S_1	− 5 08	S_7	−25 52
N. NE.	−23 45	S. SW.	+43 00	+ 9 37	−33 22	S_2	−12 47	S_6	−30 50
NE. by N.	−25 00	SW. by S.	+48 40	+11 50	−36 50	S_3	−20 28	S_5	−30 38
NE.	−26 20	**SW.**	+50 00	+11 50	−38 10	S_4	−26 59	S_4	−26 59
NE. by E.	−27 45	SW. by W.	+48 15	+10 15	−38 00	S_5	−31 36	S_3	−21 06
E. NE.	−29 15	W. SW.	+44 50	+ 7 47	−37 02	S_6	−34 13	S_2	−14 10
E. by N.	−30 40	W. by S.	+40 00	+ 4 46	−35 20	S_7	−34 40	S_1	− 6 54
EAST	−32 00	**WEST**	+33 45	+ 0 52	−32 52	1	−32 52	0	0 00
E. by S	−33 10	W. by N.	+26 50	− 3 10	−30 00	S_7	−29 26	$-S_1$	+ 5 52
E. SE.	−33 10	W. NW.	+19 20	− 6 55	−26 15	S_6	−24 15	$-S_2$	+10 02
SE. by E.	−31 45	NW. by W.	+12 45	− 9 30	−22 15	S_5	−18 30	$-S_3$	+12 22
SE.	−28 45	**NW.**	+ 6 00	−11 22	−17 22	S_4	−12 16	$-S_4$	+12 16
SE. by S.	−23 30	NW. by N.	0 00	−11 45	−11 45	S_3	− 6 32	$-S_5$	+ 9 46
S. SE.	−12 50	N. NW.	− 6 40	− 9 45	− 3 05	S_2	− 1 11	$-S_6$	+ 2 51
S. by E.	+ 3 00	N. by W.	−13 00	− 5 00	+ 8 00	S_1	+ 1 34	$-S_7$	− 7 51

This form is adapted to a computation from observations on the 32 points, but it may be used for 16 or 8 regular points by omitting the intermediate points and taking as divisors one-half or one-fourth of the divisors here given—that is, for 16 points, the divisor for A, B, and C is to be 4, and for D and E, 2; and for 8 points, the divisor for A, B, and C is 2.	Sum of +terms = + 1° 34′	+ 53° 09′
	Sum of −terms = −290 53	−182 35
	Divisor 8) −289° 19′	8) −129° 26′
	B = −36 10	C = −16 11

Form 10.—U. S. S. ATLANTA: ANALYSIS OF DEVIATIONS OF STEERING COMPASS.

TABLE II.—COMPUTATION OF COEFFICIENTS A, D, E.

(9)	(10)	(11)	(12)	(13)		(14)		Multipliers for Computing B, C, D, E.
Upper Half of Col. (5), Table I.	Lower Half of Col. (5), Table I.	Half Sum of Cols. (9) and (10). — *Constant Deviation.*	Half Sum of Cols. (9) and (10) [changing Sign in Col. (10)]. — *Quadrantal Deviation.*	Computation of D. Multipliers.	Products of Col. (12) by Multipliers.	Computation of E. Multipliers.	Products of Col. (12) by Multipliers.	
° ′	° ′	° ′	° ′		° ′		° ′	S_1= .195
+ 0 15	+ 0 52	+ 0 33	− 0 18	0	0 00	1	− 0 18	S_2= .383
+ 5 52	− 3 10	+ 1 21	+ 4 31	S_2	+ 1 43	S_6	+ 4 11	S_3= .556
+ 9 37	− 6 55	+ 1 21	+ 8 16	S_4	+ 5 51	S_4	+ 5 51	S_4= .707
+11 50	− 9 30	+ 1 10	+10 40	S_6	+ 9 51	S_2	+ 4 05	S_5= .837
+11 50	−11 22	+ 0 14	+11 36	1	+11 36	0	0 00	S_6= .924
+10 15	−11 45	− 0 45	+11 00	S_6	+10 10	$-S_2$	− 4 13	S_7= .981
+ 7 47	− 9 45	− 0 59	+ 8 46	S_4	+ 6 12	$-S_4$	− 6 12	S_8=1
+ 4 40	− 5 00	− 0 10	+ 4 50	S_2	+ 1 51	$-S_6$	− 4 28	S_0=0
Sum of + terms = +4° 39′ Sum of − terms = − 1 54 Divisor 8) + 2 45 A = +0° 21			Sum of + terms = +47° 14′ Sum of − terms = − 0 00 Divisor 4) + 47 14 D = +11 48			+14° 07′ −15 11 4) − 1 04 E = − 0 16		

Form 10.—U. S. S. ATLANTA: EXACT COEFFICIENTS: STEERING COMPASS.

TABLE III.—COMPUTATION OF EXACTC OEFFICIENTS $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$,

	A	B	C	D	E
	° ′	° ′	° ′	° ′	° ′
Angles	.+ 0 21	−36 10	−16 11	+11 48	− 0 16
Sines	+.0061	−.5901	−.2787	+.2045	−.0040
Versines.	* * *	+.1927	+.0396	+.0211	* * *

$\mathfrak{A}=\sin A=+.0061;$

$\mathfrak{B}=\sin B[1+\frac{1}{2}\sin D+\frac{1}{12}\text{ versin } B-\frac{1}{4}\text{ versin } C]+\frac{1}{2}\sin C\sin E$

$=(-.5901)[1+.1022+.0160-.0099]+[\frac{1}{2}(-.2787)(-.0046)]=-.6531;$

$\mathfrak{C}=\sin C[1-\frac{1}{2}\sin D+\frac{1}{12}\text{versin } C-\frac{1}{4}\text{ versin } B]+\frac{1}{2}\sin B\sin E;$

$=(-.2787)[1-.1022+.0033-.0482]+[\frac{1}{2}(-.5901)(-.0046)]=-.2363;$

$\mathfrak{D}=\sin D[1+\frac{1}{3}\text{ versin } D]=(+.2045)[1+.0070]=+.2059;$

$\mathfrak{E}=\sin E-\sin A\sin D=(-.0046)-[(+.0061)(+.2045)]=-.0058,$

$\tan g\ a=\frac{\mathfrak{C}}{\mathfrak{B}}=\frac{-.2363}{-.6531}\qquad\frac{\log.-9.3735}{\log.-9.8150}$

$a=199°\ 53'$ log. tan 9.5585

Still referring to Form 10, col. (2) contains the total deviation for one half the circle, and col. (4) that for the other half: by changing signs of the latter and taking the mean for each point, we obtain the deviation whose *Period* is a circle, that is, col. (6); it is proper to the eastern semicircle, and becomes applicable to the western, by change of sign.

That the process of obtaining col. (6) causes the constant and quadrantal deviations to disappear—leaving only that whose period is a circle—will be evident when we consider that those other deviations will, in the process, have different signs in each semicircle and therefore cancel. To illustrate this feature: the half-sum of cols. (2) and (4) *as they stand*, causes the semicircular deviation to disappear, because it has opposite signs and equal values in the eastern and western semicircles; this leaves in col. (5) the mean of the quadrantal deviation for each *half* of the circle, combined, however, with the constant deviation; col. (5) being further arranged by quadrants as in cols. (9) and (10), and added together, causes the quadrantal deviation to cancel, because its signs are different in opposite quadrants, and leaves only the constant deviation, col. (11).

On the other hand, by changing the signs of col. (10), the constant deviation will have different signs in opposite quadrants, and cancel, leaving the quadrantal deviation alone, whose mean value for the four quadrants appears in col. (12): it is that belonging to the N.E. and S.W. quadrants, and becomes proper to the S.E. and N.W. quadrants by change of sign. Its Period is a semicircle.

Cols. (7), (8), (13), and (14) are obtained by multiplying each angle in cols. (6) and (12) by the value of S_0, S_1, S_2 . . . S_8 opposite it; or the products may be taken directly from Table X of Comdr. Diehl's work on the compass. When the angle, as 38° 10′, exceeds the tabulated amount, it may be divided into any two parts, as 30° and 8° 10′, and the sum of the products for both parts taken.

In Table III of Form 10, the starboard angle α is derived by its tangent from $\mathfrak{B}$ and $\mathfrak{C}$, and the signs of these determine the final value of α: if both be plus, it is in the first quadrant; if $-\mathfrak{B}$ and $+\mathfrak{C}$, it is in the second; if $-\mathfrak{B}$ and $-\mathfrak{C}$, in the third; and if $+\mathfrak{B}$ and $-\mathfrak{C}$, in the fourth.

The function *tangent* takes every possible value from 0 to ∞, while its angle increases from $0°$ to $90°$; therefore, in all cases, the angle taken from the tables will be numerically less than $90°$: for the ATLANTA, the angle corresponding to log. tan. 9.5585 is $19° 53'$; but $\mathfrak{B}$ and $\mathfrak{C}$ being both negative, the angle is really in the third quadrant, whence $\alpha = 180° + 19° 53' = 199° 53'$.

On account of the frequency of the following operations, the rules regarding them are inserted here.

"*To square a number:* multiply the logarithm of the number by 2: the product is the logarithm of the number required. When the number is a decimal fraction, subtract the index (after being doubled) from 10 multiplied by 2 (or 20), diminish the remainder by 1, and prefix the number of ciphers indicated by this remainder to the number corresponding to the logarithm."

"*To extract the square root of a number:* divide the logarithm of the given number by 2: the quotient is the logarithm of the square root required. When the given number is a decimal fraction, that is, when the index of its logarithm is 9, 8, 7, etc., increase the index by 10."

To find the third, fourth, or any other power of a number; or to extract the third, fourth, or any other root thereof; is performed in a similar manner to squaring, and extracting the square root—using in each case the index suitable to the requisite operation.

"*To find the logarithm of a given number.* When the number consists of a whole number, with or without decimals, the index is 1 less than the number of figures in the whole number. When the number consists of deci-

mals only, count the number of ciphers between the decimal point and the first significant figure after it, and subtract this number from 9: the remainder is the index."

"*To find the natural number of a given logarithm.* Look for the decimal part of the given logarithm in the body of the table, and take out the number from the side column and top. To place the decimal point, add 1 to the given index of the logarithm, and mark off to the left this number of figures; these will constitute the whole number; the rest, if any, will be the decimal part. If the index is 9, put the dot before the first figure; if it is 8, prefix one cipher to the first figure and place the dot before the cipher; if it is 7, prefix two ciphers and place the dot before them; and so on."

314. Accuracy of the Coefficients derived from various numbers of observations.—The coefficients A, B, and C can be determined by observations on any four *equidistant* points, and as there are eight such different groups in the circle, we may hence obtain as many separate values of them: thus, we may observe on the cardinal points; or on the group N. by E.—E. by S.—S. by W.—W. by N.; or on the group N.NE.—E.SE.—S.SW.—W.NW.; and so on. The value of A for each such group appears in col. (11) of Form 10; they are transcribed to Table 73 under the heading 4: by taking the mean of pairs of this series, that is, of the 1st and 5th, 2d and 6th, etc., we get the series under head 8, or four values, each the mean of observations on 8 points; again, pairs of *this* series, that is, the 1st and 3d, 2d and 4th, give two values, each the mean of observations on 16 points; and finally, the mean of these last is the value of A for 32 points.

By a similar process, we obtain from cols. (7) and (8), Form 10, values of B and C for 4, 8, 16, and 32 points; they are transcribed to Table 73 under those headings: thus the value of B is the algebraic sum of the pair on lines 1 and 9

of col. (7), Form 10, that is, its value on the cardinal points; its value on line 10 is the algebraic sum of the pair on lines 2 and 10 of col. (7), or of the group N. by E.—E. by S.—S. by W.—W. by N.; and so on. And similarly for the values of C, D, and E.

Table 73.

COEFFICIENTS OF THE U. S. S. ATLANTA FROM OBSERVATIONS ON DIFFERENT NUMBERS OF COMPASS POINTS.

Coefficient.	Number of Points. 4 (1)	8 (2)	16 (3)	32 (4)
A	+0° 33′ +1 21 +1 21 +1 10 +0 14 −0 45 −0 59 −0 10	+0° 24′ +0 18 +0 11 +0 30	+0° 18′ +0 24	+0° 21′
B	−32° 52′ −34 34 −37 02 −38 58 −39 15 −38 08 −35 24 −33 06	−36° 03′ −36 21 −36 13 −36 02	−36° 08′ −36 12	−36° 10′

Coefficient.	Number of Points. 4 (1)	8 (2)	16 (3)	32 (4)
C	−18° 15′ −20 00 −20 48 −18 16 −14 43 −11 20 −11 19 −14 45	−16° 29′ −15 40 −16 03 −16 30	−16° 16′ −16 05	−16° 11′
D	Quad'l +11° 36′	+11° 36′ +11 53 +12 03 +11 42	+11° 49′ +11 47	+11° 48′
E	Card'l −0° 18′	−0° 18′ −0 02 −0 21 −0 23	−0° 19′ −0 13	−0° 16′

The coefficient D cannot be determined by observations on the cardinal points, nor E on the quadrantal; because they respectively reduce to zero on those groups. On examining Table 73, it is seen that for any coefficient, there is some difference between the values deduced from different groups of *four* points: considering the abnormal amount of the deviations, and consequent difficulty of accurate observation, perhaps the differences are no greater than the utmost care could prevent—they certainly would not be *as* great with equal care if the maximum deviation were 25° or 30° instead of 50°, as here: still, the discrepancies point to the fact that coefficients determined from obser-

vations on *only four equidistant points are not accurate enough for use.*

On the other hand, regarding the values obtained from 32 points as the most accurate, those from 8 and 16 points differ but little among themselves, and also but slightly from the value for 32 points: therefore, the coefficients from 8 points, and still more those from 16, are trustworthy, even with large deviations; and while it is desirable to swing ship periodically on 32 points, still it will suffice to observe on 16, or even 8, to keep informed of changes in the coefficients.

315. Computation of the deviations, and comparison of results with observation.—Values of the coefficients may be substituted in eqs. (120), (125), or (149), Arts, 302 and 303, and the deviation thence computed for any point or number of points; and the quantities of the equation chosen may be arranged in any tabular form that will facilitate the work.

Fourier's Series—eq. (149)—has been employed for computing the deviations for 32 points of the ATLANTA's steering compass, and Table 74 contains the principal parts of the work; the headings of the columns nearly suffice to indicate the procedure, but as the table may be used as a general FORM, some points will be explained.

The deviations of the ATLANTA are so large—the maximum being 50°—that nine terms of the series were found necessary to obtain accurate results: these terms are arranged above the headings of the columns, pages 928–'31.

The table presents a formidable front, and at first view may well deter any navigator from undertaking its computation as a substitute for observation; but in reality it is neither as forbidding nor as laborious as it appears. In the first place, the most troublesome part was to determine the proper *multiplier* (S_0, S_1, $S_2 \ldots S_8$) and its sign for each function of the single or multiple angle ζ' corre-

sponding to each of the 32 compass courses: this being done in cols. (5), (7), (11), etc., they constitute a permanent FORM which may be followed mechanically by any other computer. In the second place, only a few *actual* computations are necessary, as the same values recur again and again, and may be copied from the first ones: thus, in cols. (6) and (8), only the first five values—from N. to NE.—differ, the remainder being a repetition of these; so, in cols. (12), (14), (17), and (19), actual calculation is necessary only for points from N. to E., as these recur down each column; lastly, in cols. (22) and (24) only three points need be computed, all others being identical. Moreover, the multiplication need not be actually made, as the products in cols. (6), (8), (12), and (14) may be taken from tables for that purpose. For primarily determining the proper multiplier and its sign, Fig. 511 is most useful. Suppose we want the multiplier for S.SW., for which, by Fig. 511, $\zeta' = 202° 30'$: the *sine* is S_2 and *cosine* S_6, both minus, and such they appear in cols. (11) and (13) opposite S.SW. Again: for this point, $2\zeta' = 405°$, or 45°, since all functions recur after 360°; sin 45° = cos 45° = S_4, as in cols. (5) and (7). And again: for S.SW., $3\zeta' = 607° 30'$, or 247° 30', by subtracting 360°, and sin 247° 30' $= -S_6$ and cos $= -S_2$, both which are found in cols. (16) and (18). And similarly for $4\zeta'$ for cols. (21) and (23); and for each point of the compass.

Two methods offer for computing the coefficients F, G, H, K: first, using the coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$ of Table III, Form 10, and substituting them in eqs. (212) to (215), Art. 311; this is the direct, and probably better, method.

Second, by means of the coefficients A, B, C, D, E, determined in Tables I and II of Form 10, which must be substituted in eqs. (242) to (245), Art. 311; this is a little circuitous: these coefficients are expressed in degrees and minutes—all must be reduced to minutes, and multiplied by the value given in eq. (21), page 854, to convert them

TABLE 74.

ION OF THE DEVIATIONS OF THE STEERING COM-
(:FFICIENTS, AND COMPARISON OF COMPUTED WITH

$$\delta = A + D \sin 2\zeta' + E \cos 2\zeta' + B \sin \zeta' + C \cos \zeta'$$

Ship's Head by Steering Compass.	Observed Deviations. (1)	Deviations Computed from A, B, C, D, E, F, G, H, and K: Sum of Cols. (10), (15), (20), (25). (2)	Differences Between Cols. (1) and (2). (3)	Constant Deviation. A. (4)	Multi-pliers for D. (5)	$D = +11° 48'$ Product of D by Multi-pliers of Col. (5). (6)
N.	−18° 00′	−17° 04′	+0° 56′	+0° 21′	S_0	0° 00′
N. by E.	−20 30	−20 00	+0 30	+0 21	$+S_2$	+ 4 31
N. NE.	−23 45	−22 25	+1 20	+0 21	$+S_4$	+ 8 21
NE. by N.	−25 00	−24 34	+0 26	+0 21	$+S_6$	+10 54
NE.	−26 20	−26 31	−0 11	+0 21	$+S_8$	+11 48
NE. by E.	−27 45	−28 19	−0 34	+0 21	$+S_6$	+10 54
E. NE.	−29 15	−29 50	−0 35	+0 21	$+S_4$	+ 8 21
E. by N.	−30 40	−29 59	−0 33	+0 21	$+S_2$	+ 4 31
E.	−32 00	−32 17	−0 17	+0 21	S_0	0 00
E. by S.	−33 10	−33 09	+0 01	+0 21	$-S_2$	− 4 31
E. SE.	−33 10	−33 22	−0 12	+0 21	$-S_4$	− 8 21
SE. by E.	−31 45	−32 07	−0 22	+0 21	$-S_6$	−10 54
SE.	−28 45	−28 31	+0 14	+0 21	$-S_8$	−11 48
SE. by S.	−23 30	−21 46	+1 44	+0 21	$-S_6$	−10 54
S. SE.	−12 50	−11 05	+1 45	+0 21	$-S_4$	− 8 21
S. by E.	+ 3 00	− 4 16	+1 16	+0 21	$-S_2$	− 4 31
S.	+18 30	+17 08	−1 22	+0 21	S_0	0 00
S. by W.	+32 15	+31 02	−1 13	+0 21	$+S_2$	+ 4 31
S. SW.	+43 00	+41 59	−1 01	+0 21	$+S_4$	+ 8 21
SW. by S.	+48 40	+48 46	+0 06	+0 21	$+S_6$	+10 54
SW.	+50 00	+50 55	+0 55	+0 21	$+S_8$	+11 48
SW. by W.	+48 15	+49 15	+1 00	+0 21	$+S_6$	+10 54
W. SW.	+44 50	+45 04	+0 14	+0 21	$+S_4$	+ 8 21
W. by S.	+40 00	+39 31	−0 29	+0 21	$+S_2$	+ 4 31
W.	+33 45	+33 25	−0 20	+0 21	S_0	0 00
W. by N.	+26 50	+27 03	+0 13	+0 21	$-S_2$	− 4 31
W. NW.	+19 20	+20 16	+0 56	+0 21	$-S_4$	− 8 21
NW. by W.	+12 45	+13 07	+0 22	+0 21	$-S_6$	−10 54
NW.	+ 6 00	+ 5 43	−0 17	+0 21	$-S_8$	−11 48
NW. by N.	00 00	− 0 18	−0 18	+0 21	$-S_6$	−10 54
N. NW.	− 6 40	− 7 49	−1 09	+0 21	$-S_4$	− 8 21
N. by W.	−13 00	−13 00	0 00	+0 21	$-S_2$	− 4 31

TABLE 74.—*Continued.*

PASS OF THE U. S. S. ATLANTA FROM THE MAGNETIC OBSERVED VALUES.

$+F \sin 3\zeta' + G \cos 3\zeta' + H \sin 4\zeta' + K \cos 4\zeta'$.

Multipliers for E.	$E = -0° 16'$. Product of E by Multipliers of Col. (7).	Sum of Cols. (6) and (8): Quadrantal Deviation.	Sum of Cols. (4) and (9): Unchanging Part of Deviation.	Multipliers for B.	$B = -36° 10'$. Product of B by Multipliers of Col. (11).
(7)	(8)	(9)	(10)	(11)	(12)
$+S_8$	−0° 16′	− 0° 16′	+ 0° 05′	S_0	0° 00′
$+S_6$	−0 14	+ 4 17	+ 4 38	$+S_1$	− 7 04
$+S_4$	−0 11	+ 8 10	+ 8 31	$+S_2$	−13 50
$+S_2$	−0 06	+10 48	+11 09	$+S_3$	−20 06
S_0	0 00	+11 48	+12 09	$+S_4$	−25 34
$-S_2$	+0 06	+11 00	+11 21	$+S_5$	−30 04
$-S_4$	+0 11	+ 8 32	+ 8 53	$+S_6$	−33 24
$-S_6$	+0 14	+ 4 45	+ 5 06	$+S_7$	−35 29
$-S_8$	+0 16	+ 0 16	+ 0 37	$+S_8$	−36 10
$-S_6$	+0 14	− 4 17	− 3 56	$+S_7$	−35 29
$-S_4$	+0 11	− 8 10	− 7 49	$+S_6$	−33 24
$-S_2$	+0 06	−10 48	−10 27	$+S_5$	−30 04
S_0	0 00	−11 48	−11 27	$+S_4$	−25 34
$+S_2$	−0 06	−11 00	−10 39	$+S_3$	−20 06
$+S_4$	−0 11	− 8 32	− 8 11	$+S_2$	−13 50
$+S_6$	−0 14	− 4 45	− 4 24	$+S_1$	− 7 04
$+S_8$	−0 16	− 0 16	+ 0 05	S_0	0 00
$+S_6$	−0 14	+ 4 17	+ 4 38	$-S_1$	+ 7 04
$+S_4$	−0 11	+ 8 10	+ 8 31	$-S_2$	+13 50
$+S_2$	−0 06	+10 48	+11 09	$-S_3$	+20 06
S_0	0 00	+11 48	+12 09	$-S_4$	+25 34
$-S_2$	+0 06	+11 00	+11 21	$-S_5$	+30 04
$-S_4$	+0 11	+ 8 32	+ 8 53	$-S_6$	+33 24
$-S_6$	+0 14	+ 4 45	+ 5 06	$-S_7$	+35 29
$-S_8$	+0 16	+ 0 16	+ 0 37	$-S_8$	+36 10
$-S_6$	+0 14	− 4 17	− 3 56	$-S_7$	+35 29
$-S_4$	+0 11	− 8 10	− 7 49	$-S_6$	+33 24
$-S_2$	+0 06	−10 48	−10 27	$-S_5$	+30 04
S_0	0 00	−11 48	−11 27	$-S_4$	+25 34
$+S_2$	−0 06	−11 00	−10 39	$-S_3$	+20 06
$+S_4$	−0 11	− 8 32	− 8 11	$-S_2$	+13 50
$+S_6$	−0 14	− 4 45	− 4 24	$-S_1$	+ 7 04

TABLE 74.—*Continued.*

COMPUTATION OF THE DEVIATIONS OF THE STEERING COM- COEFFICIENTS, AND COMPARISON OF COMPUTED WITH

$\delta = A + D \sin 2\zeta' + E \cos 2\zeta' + B \sin \zeta' + C \cos \zeta'$

Ship's Head by Steering Compass.	Multipliers for C	$C=-16° 11'$. Product of C by Multipliers of Col. (13).	Sum of Cols. (12) and (14). Semicircular Deviation.	Multipliers for F.	$F=-.058$. Product of F by Multipliers of Col. (16).	Multipliers for G.	$G=-.016$. Product of G by Multipliers of Col. (18).
	(13)	(14)	(15)	(16)	(17)	(18)	(19)
N.	$+S_8$	−16° 11′	−16° 11′	S_0	0° 00′	$+S_8$	−0° 55′
N. by E.	$+S_7$	−15 52	−22 56	$+S_3$	−1 50	$+S_5$	−0 45
N. NE.	$+S_6$	−14 57	−28 47	$+S_6$	−3 05	$+S_2$	−0 20
NE. by N.	$+S_5$	−13 28	−33 34	$+S_7$	−3 16	$-S_1$	+0 10
NE.	$+S_4$	−11 26	−37 00	$+S_4$	−2 21	$-S_4$	+0 38
NE. by E.	$+S_3$	− 9 00	−39 04	$+S_1$	−0 38	$-S_7$	+0 55
E. NE.	$+S_2$	− 6 11	−39 35	$-S_2$	+1 16	$-S_6$	+0 52
E. by N.	$+S_1$	− 3 09	−38 38	$-S_5$	+2 45	$-S_3$	+0 31
E.	S_0	0 00	−36 10	$-S_8$	+3 19	S_0	0 00
E. by S.	$-S_1$	+ 3 09	−32 20	$-S_5$	+2 45	$+S_3$	−0 31
E. SE.	$-S_2$	+ 6 11	−27 13	$-S_2$	+1 16	$+S_6$	−0 52
SE. by E.	$-S_3$	+ 9 00	−21 04	$+S_1$	−0 38	$+S_7$	−0 55
SE.	$-S_4$	+11 26	−14 08	$+S_4$	−2 21	$+S_4$	−0 38
SE. by S.	$-S_5$	+13 28	− 6 38	$+S_7$	−3 16	$+S_1$	−0 10
S. SE.	$-S_6$	+14 57	+ 1 07	$+S_6$	−3 05	$-S_2$	+0 20
S. by E	$-S_7$	+15 52	+ 8 48	$+S_3$	−1 50	$-S_5$	+0 45
S.	$-S_8$	+16 11	+16 11	S_0	0 00	$-S_8$	+0 55
S. by W.	$-S_7$	+15 52	+22 56	$-S_3$	+1 50	$-S_5$	+0 45
S. SW.	$-S_6$	+14 57	+28 47	$-S_6$	+3 05	$-S_2$	+0 20
SW. by S.	$-S_5$	+13 28	+33 34	$-S_7$	+3 16	$+S_1$	−0 10
SW.	$-S_4$	+11 26	+37 00	$-S_4$	+2 21	$+S_4$	−0 38
SW. by W.	$-S_3$	+ 9 00	+39 04	$-S_1$	+0 38	$+S_7$	−0 55
W. SW.	$-S_2$	+ 6 11	+39 35	$+S_2$	−1 16	$+S_6$	−0 52
W. by S.	$-S_1$	+ 3 09	+38 38	$+S_5$	−2 45	$+S_3$	−0 31
W.	S_0	0 00	+36 10	$+S_8$	−3 19	S_0	0 00
W. by N.	$+S_1$	− 3 09	+32 20	$+S_5$	−2 45	$-S_3$	+0 31
W. NW.	$+S_2$	− 6 11	+27 13	$+S_2$	−1 16	$-S_6$	+0 52
NW. by W.	$+S_3$	− 9 00	+21 04	$-S_1$	+0 38	$-S_7$	+0 55
NW.	$+S_4$	−11 26	+14 08	$-S_4$	+2 21	$-S_4$	+0 38
NW. by N.	$+S_5$	−13 28	+ 6 38	$-S_7$	+3 16	$-S_1$	+0 10
N. NW.	$+S_6$	−14 57	− 1 07	$-S_6$	+3 05	$+S_2$	−0 20
N. by W.	$+S_7$	−15 52	− 8 48	$-S_3$	+1 50	$+S_5$	−0 45

TABLE 74.—*Continued.*

PASS OF THE U. S. S. ATLANTA FROM THE MAGNETIC OBSERVED VALUES.

$+F \sin 3\zeta' + G \cos 3\zeta' + H \sin 4\zeta' + K \cos 4\zeta'$.

Sum of Cols. (17) and (19): Sextanial Deviation.	Multipliers for H.	$H = +.022$. Product of H by Multipliers of Col. (21).	Multipliers for K.	$K = -.001$. Product of K by Multipliers of Col. (23)	Sum of Cols. (22) and (24): Octantal Deviation.
(20)	(21)	(22)	(23)	(24)	(25)
−0° 55′	S_0	0° 00′	$+S_8$	−0° 03′	−0° 03′
−2 35	$+S_4$	+0 55	$+S_4$	−0 02	+0 53
−3 25	$+S_8$	+1 16	S_0	0 00	+1 16
−3 06	$+S_4$	+0 55	$-S_4$	+0 02	+0 57
−1 43	S_0	0 00	$-S_8$	+0 03	+0 03
+0 17	$-S_4$	−0 55	$-S_4$	+0 02	−0 53
+2 08	$-S_8$	−1 16	S_0	0 00	−1 16
+3 16	$-S_4$	−0 55	$+S_4$	−0 02	−0 57
+3 19	S_0	0 00	$+S_8$	−0 03	−0 03
+2 14	$+S_4$	+0 55	$+S_4$	−0 02	+0 53
+0 24	$+S_8$	+1 16	S_0	0 00	+1 16
−1 33	$+S_4$	+0 55	$-S_4$	+0 02	+0 57
−2 59	S_0	0 00	$-S_8$	+0 03	+0 03
−3 26	$-S_4$	−0 55	$-S_4$	+0 02	−0 53
−2 45	$-S_8$	−1 16	S_0	0 00	−1 16
−1 05	$-S_4$	−0 55	$+S_4$	−0 02	−0 57
+0 55	S_0	0 00	$+S_8$	−0 03	−0 03
+2 35	$+S_4$	+0 55	$+S_4$	−0 02	+0 53
+3 25	$+S_8$	+1 16	S_0	0 00	+1 16
+3 06	$+S_4$	+0 55	$-S_4$	+0 02	+0 57
+1 43	S_0	0 00	$-S_8$	+0 03	+0 03
−0 17	$-S_4$	−0 55	$-S_4$	+0 02	−0 53
−2 08	$-S_8$	−1 16	S_0	0 00	−1 16
−3 16	$-S_4$	−0 55	$+S_4$	−0 02	−0 57
−3 19	S_0	0 00	$+S_8$	−0 03	−0 03
−2 14	$+S_4$	+0 55	$+S_4$	−0 02	+0 53
−0 24	$+S_8$	+1 16	S_0	0 00	+1 16
+1 33	$+S_4$	+0 55	$-S_4$	+0 02	+0 57
+2 59	S_0	0 00	$-S_8$	+0 03	+0 03
+3 26	$-S_4$	−0 55	$-S_4$	+0 02	−0 53
+2 45	$-S_8$	−1 16	S_0	0 00	−1 16
+1 05	$-S_4$	−0 55	$+S_4$	−0 02	−0 53

into parts of radius; the results are to be used in eqs. (242) to (245), Art. 311; then the products of the coefficients by the multipliers of cols. (16), (18), (21), (23) are to be found; and finally, these products must be converted into degrees and minutes by means of eq. (19), page 854, in order to get the quantities of cols. (17), (19), (22), (24). In all this the greatest care must be exercised with regard to the algebraic signs.

Some matters illustrated by Table 75 will now be pointed out. In col. (2) the deviations computed from the five terms usually employed are given: compared with observation, the differences appear in col. (5), the sign of each difference being that which will make the observed value of col. (1) *numerically* identical with the computed value of col. (2).

The differences or residuals of col. (5) present six clearly defined maxima and minima—a sextantal deviation of considerable amount: therefore the terms $F.\sin 3\zeta' + G.\cos 3\zeta'$ were computed as in cols. (16) to (19), Table 74, and added to the deviations of col. (2), Table 75, whence col. (3): this is compared with col. (1), and the differences appear in col. (6); with but slight irregularities they present eight maxima and minima—an octantal deviation. The terms $H.\sin 4\zeta' + K.\cos 4\zeta'$ were now computed, as in cols. (21) to (24), Table 74, and added to col. (3), Table 75, whence col. (4) of latter: the differences between this and col. (1) appear in col. (7); they indicate a vestige of recurrent maxima and minima, but with much irregularity both of sign and numerical amount: they are probably errors of observation, and are very moderate, considering the unfavorable conditions midst which the compass was placed.

Under such circumstances, the directive force is so weakened, that on some points the compass-card scarcely comes to rest, but moves listlessly through several degrees,

TABLE 75.

U. S. S. ATLANTA; STEERING COMPASS; COMPARISON OF OBSERVED AND COMPUTED DEVIATIONS; WITH RESIDUALS AFTER THE QUADRANTAL, SEXTANTAL, AND OCTANTAL COMPONENTS, ARE SUCCESSIVELY INCLUDED IN THE TOTAL DEVIATION.

Ship's Head by Steering Compass.	Deviations Observed.	Deviations Computed from Coefficients *A, B, C, D*, and *E*: Sum of Cols. (10) and (15) of Table 74.	Deviations Computed from Coefficients *A, B, C, D, E, F*, and *G*: Sum of Col. (2) this Table and Col. (20) of Table 74.	Deviations Computed from Coefficients *A, B, C, D, E, F, G, H*, and *K*: Sum of Col. (3) this Table and Col. (25) of Table 74.	Difference Between Observed and Computed Deviations, Latter Including Quadrantal: Difference of Cols. (1) and (2).	Difference Between Observed and Computed Deviations, Latter Including Sextantal: Difference of Cols. (1) and (3).	Difference Between Observed and Computed Deviations, Latter Including Octantal: Difference of Cols. (1) and (4).
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
N.	−18° 00′	−16° 06′	−17° 01′	−17° 04′	+1° 54′	+0° 59′	+0° 56′
N. by E.	−20 30	−18 18	−20 53	−20 00	+2 12	−0 23	+0 30
N. NE.	−23 45	−20 16	−23 41	−22 25	+3 29	+0 04	+1 20
NE. by N.	−25 00	−22 25	−25 31	−24 34	+2 35	−0 31	+0 26
NE.	−26 20	−24 51	−26 34	−26 31	+1 29	−0 14	−0 11
NE. by E.	−27 45	−27 43	−27 26	−28 19	+0 02	+0 19	−0 34
E. NE.	−29 15	−30 42	−28 34	−29 50	−1 27	+0 41	−0 35
E. by N.	−30 40	−33 32	−30 16	−29 59	−2 52	+0 24	−0 33
E.	−32 00	−35 33	−32 14	−32 17	−3 33	−0 14	−0 17
E. by S.	−33 10	−36 16	−34 02	−33 09	−3 06	−0 52	+0 01
E. SE.	−33 10	−35 02	−34 38	−33 22	−1 52	−1 28	−0 12
SE. by E.	−31 45	−31 31	−33 04	−32 07	+0 14	−1 19	−0 22
SE.	−28 45	−25 35	−28 34	−28 31	+3 10	+0 11	+0 14
SE. by S.	−23 30	−17 17	−20 43	−21 46	+6 13	+2 47	+1 44
S. SE.	−12 50	− 7 04	− 9 49	−11 05	+5 46	+3 01	+1 45
S. by E.	+ 3 00	+ 4 24	− 3 19	− 4 16	+1 24	+0 19	+1 16
S.	+18 30	+16 16	+17 11	+17 08	−2 14	−1 19	−1 22
S. by W.	+32 15	+27 34	+30 09	+31 02	−4 41	−2 06	−1 13
S. SW.	+43 00	+37 18	+40 43	+41 59	−5 42	−2 17	−1 01
SW. by S.	+48 40	+44 43	+47 49	+48 46	−3 57	−0 51	+0 06
SW.	+50 00	+49 09	+50 52	+50 55	−0 51	+0 52	+0 55
SW. by W.	+48 15	+50 25	+50 08	+49 15	+2 10	+1 53	+1 00
W. SW.	+44 50	+48 28	+46 20	+45 04	+3 38	+1 30	+0 14
W. by S.	+40 00	+43 44	+40 28	+39 31	+3 44	+0 28	−0 29
W.	+33 45	+36 47	+33 28	+33 25	+3 02	−0 17	−0 20
W. by N.	+26 50	+28 24	+26 10	+27 03	+1 34	−0 40	+0 13
W. NW.	+19 20	+19 24	+19 00	+20 16	+0 04	−0 20	+0 56
NW. by W.	+12 45	+10 37	+12 10	+13 07	−2 08	−0 35	+0 22
NW.	+ 6 00	+ 2 41	+ 5 40	+ 5 43	−3 19	−0 20	−0 17
NW. by N.	0 00	− 4 01	+ 0 35	− 0 18	−4 01	+0 35	−0 18
N. NW.	− 6 40	− 9 18	− 6 33	− 7 49	−2 38	+0 07	−1 09
N. by W.	−13 00	−13 12	−12 07	−13 00	−0 12	+0 35	0 00

with a tendency to a larger arc of swing on the slightest provocation.

Cols. (4) and (7) of Table 75—the final summation of all the calculated deviations and their differences from those observed—are given in cols. (2) and (3) of Table 74.

Both the sextantal and octantal deviations—cols. (20) and (25), Table 74—may be due either to the ship or to the length of the needles, or partly to both: their source is not easy to discover, as the mathematical expression for them is the same whether caused by Ship or Compass; this was illustrated in Part Second dealing with that instrument.

While it is mathematically demonstrated that the arrangement of the needles on the card prevents these errors, still it is true, as stated in connection with that demonstration in Part Second, that iron may be placed so near the needles as to destroy this advantage.

The whole theory of this subject is based upon the hypothesis that the needles are mere particles—of no appreciable length: but they *must have some* length, and the actual one they have is deemed the best compromise between the ideal and practical, in view of the magnetic strength essential to them.

That the ATLANTA's steering compass was surrounded by most vicious influences is undoubted; but that such was a necessity of design and construction seems incredible: the instrument upon which the safety of the ship depends, deserves more suitable environment than one that will cause it to err 50°.

Tables 74 and 75 further show that with abnormal deviations, the customary five terms of Fourier's Series, embracing only the coefficients A, B, C, D, E, will not suffice: for mere safety, the sextantal terms must be included, and for ordinary accuracy, the octantal also.

Attention is directed to the relative proportions of all

the components of the Total Deviation in Figs. 501 to 507: though not drawn actually to represent cols. (10), (15), (20), and (25) of Table 74, they illustrate them.

Section Four: What the Coefficients Represent, and Why They Fluctuate in Value.

316. Conditions that affect the coefficients.—Although the hypothesis of the theory of the deviations is, that iron is only of two kinds—hard and soft—still in actual construction many degrees of both enter the ship, and hence we have to deal with much variety of magnetic susceptibility: this condition the coefficients represent, and as it changes with time and place, so do they.

In Arts. 268 to 272 the hypothetical conditions are represented by rods and magnets: they constitute the framework of the coefficients, and that of each coefficient will now be described. It should be understood, however, that these rods and magnets represent *force* in two directions—horizontal and vertical; but that the iron of the ship is never such, owing to her rolling and pitching: still it is with horizontal and vertical forces that the formulas deal, except in the case of a steady list, which is treated as the heeling error.

317. The Constant deviation: 𝔄 or A.—This coefficient arises from two distinct sources—one accidental and varied, the other regular and single. Of the first are systematic personal errors in observing; instrumental defects of the compass aboard, or that ashore in the case of reciprocal bearings; local deviating causes near latter compass; error in the magnetic direction of a distant object when that is used; and inaccuracy in the Variation by chart when employed to separate the deviation from compass errors.

In a round of time-azimuths at sea, when the procedure

or 'orm 7, is followed, the constant, if any, is combi.. the Variation, and must be separated from it by is of the deviations: the Variation thence found .orrect one.

 above, and similar errors, may be either plus or n is, or partly each; and all being entirely adventitious, may vary with time or place.

The second source of the constant is the ship herself. From page 879, we have

$$\mathfrak{A} = \frac{1}{\lambda}\left(\frac{d-b}{2}\right) \quad . \quad (113); \text{ and } \mathfrak{E} = \frac{1}{\lambda}\left(\frac{d+b}{2}\right). \quad . \quad (115).$$

The coefficient $\mathfrak{A}$ thus depends on d and b—two rods of soft iron in various positions, Figs. 436 and 438, Part Third: if the compass be on the midship-line, and the soft iron symmetrically located, this source reduces to zero, for by symmetry there are equal influences of opposite sign.

But suppose absence of symmetry, and that soft iron surrounds the compass as in Figs. 518 and 519: it may easily

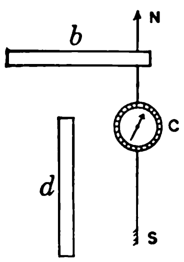

FIG. 518.

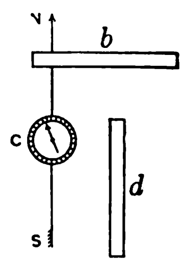

FIG. 519

be inferred that the first will produce $+\mathfrak{A}$ and the second $-\mathfrak{A}$: if either rod be removed, the other will cause a combination of $\mathfrak{A}$ and $\mathfrak{E}$; for by eq. (115) the latter depends on these rods also. The case of Figs. 518 and 519 is frequently realized in a compass placed on each side of a

steering wheel whose spindle is of wrought iron. The effects of b and d are shown by Experiments 3, 4, and 5, Part Third.

The portion of $\mathfrak{A}$ arising from soft iron does not change with time or place.

Thus the *Constant* is the resultant of many possible causes, and its sign will be that of the predominant one: it is generally small—less than 1°, and if more its source should be investigated.

318. The Semicircular deviation; and $\mathfrak{B}$, $\mathfrak{C}$, or B, C.—From page 879 we have

$$\mathfrak{B} = \frac{1}{\lambda}\left(c.\tan\theta + \frac{P}{H}\right) \quad . \quad . \quad . \quad . \quad . \quad (116)$$

and

$$\mathfrak{C} = \frac{1}{\lambda}\left(f.\tan\theta + \frac{Q}{H}\right). \quad . \quad . \quad . \quad . \quad (117)$$

Each of these coefficients is seen to have two terms, whose nature will be better understood by reference to Figs. 434, 441, and 442, and the text explanatory of them: these terms depend on hard and soft iron, the former represented by P and Q, the latter by c and f; these last are vertical, and having tan θ for a factor, will vary according to the Dip—that is, have maximum values in high latitudes, minimum in low, and be zero at the magnetic equator, with change of sign on crossing from north to south: $\mathfrak{B}$ and $\mathfrak{C}$ will undergo corresponding fluctuations. The terms containing P and Q represent hard iron, or permanent magnetism. The hull acquires a stable magnetic character only after two or three years from launching; and during the dissipation of the surcharge, considerable decrease will be experienced in $\mathfrak{B}$ and $\mathfrak{C}$: after that, minor fluctuations occur from transient magnetism, whose sources have been set forth in Art. 292.

It is seen by eqs. (116) and (117) that P and Q vary inversely as H—or, that $\mathfrak{B}$ and $\mathfrak{C}$ *in*crease in polar regions, and *de*crease toward the magnetic equator.

Thus both *terms* in each coefficient conspire to increase or decrease $\mathfrak{B}$ and $\mathfrak{C}$ in the same geographical locality: this matter is treated more at length in Chapter VII. It would hence seem that changes in the deviations from fluctuation of $\mathfrak{B}$ and $\mathfrak{C}$ could be predicted, and both their parts separated; this is true, and it can be done with considerable accuracy—the details of the procedure will be given in the next chapter.

These coefficients represent the force toward bow and side, respectively: together they produce the semicircular deviation, whose resultant and its direction are treated in Art. 299. This direction is the neutral line between the two grand regions of opposite polarity in an *iron* ship—its magnetic equator: it coincides approximately with the magnetic direction of the keel while building; it is the zero line of semicircular deviation due to the magnetism of hard iron. The zero point of that due to vertical soft iron, c and f, eqs. (116) and (117), is magnetic *north*, so that the combination—that due to *both* hard and soft iron—is somewhere between them—not much removed from that due to P and Q, however.

The neutral line is determined by a magnetic survey, or it may be deduced from a table of deviations: from it the direction in which the ship was built, may be inferred with much confidence; on the other hand, knowing the latter, the neutral line may be approximately drawn.

$\mathfrak{B}$ and $\mathfrak{C}$ have such variety and extremes of value in various types of ship, and in different parts of the same ship, that it would be futile to give numerical examples as a guide to their amounts.

319. The Quadrantal derivation; and $\mathfrak{D}$, $\mathfrak{E}$, or D, E.— From page 879 we have

$$\mathfrak{D}=\frac{1}{\lambda}\left(\frac{a-e}{2}\right) \quad . \quad . \quad (114); \quad \text{and} \quad \mathfrak{E}=\frac{1}{\lambda}\left(\frac{d+b}{2}\right). \quad . \quad (115)$$

Both $\mathfrak{D}$ and $\mathfrak{E}$ thus owe their existence wholly to horizontal soft iron, represented by rods a, e, d, and b. The nature and changes of $\mathfrak{E}$ have been explained in Art. 317, in connection with $\mathfrak{A}$: $\mathfrak{E}$ is always small in value.

The rods upon which $\mathfrak{D}$ depends are represented in Figs. 435 and 439, Part Third: a little study of these, and bearing in mind how induction converts them into magnets as the ship swings round, will show the deviation each produces, and whether the rod adds to the directive force or detracts from it. Thus a in positions (1) and (3), Fig. 435, causes a positive deviation and *in*crease of directive force; while in (2) it causes negative deviation and *de*crease of force: similarly, e at (16) and (18), Fig. 439, produces a positive deviation with *in*crease of force, and at (17) a negative, with *de*crease. The coefficients $\mathfrak{D}$ and $\mathfrak{E}$ are the components toward bow and side, respectively, of the force that produces quadrantal deviation: their resultant and its direction are treated in Art. 299; $\mathfrak{D}$ is the principal part, $\mathfrak{E}$ contributing but little.

When $\mathfrak{D}$ and λ have been determined, a and e may be found as follows: from eqs. (110) and (114), page 879, we have $2\lambda = 2 + a + e$. (249); and $2\mathfrak{D}.\lambda = a - e$; . (250) whence

$a = -1 + \lambda(1 + \mathfrak{D})$. (251); and $e = -1 + \lambda(1 - \mathfrak{D})$. . (252)

For the locality of the ATLANTA's *steering* compass, whose deviations are analyzed in Arts. 313, 314, and 315, $\mathfrak{D} = +.206$ and $\lambda = +.86$: substituting these in (251) and (252), they become

$$a = -1 + .86(1 + 0.206) = +.037 \quad . \quad . \quad (253)$$

and

$$e = -1 + .86(1 - .206) = -.317. \quad . \quad . \quad (254)$$

Then from (250) we have

$$\mathfrak{D} = \frac{a-e}{2\lambda} = \frac{a}{2\lambda} - \frac{e}{2\lambda}. \quad . \quad . \quad . \quad . \quad (255)$$

In this $\frac{a}{2\lambda}$ represents induction in the longitudinal soft iron, and $-\frac{e}{2\lambda}$ that in the transverse.

Taking the first, and substituting in it the value of a from (251), and then the values of $\mathfrak{D}$ and λ, it becomes

$$\frac{a}{2\lambda} = \frac{-1+\lambda(1+\mathfrak{D})}{2\lambda} = \frac{-1+(.86)(1+.206)}{2(.86)}$$
$$= +.022 = +1^\circ\ 15'. \quad . \quad . \quad . \quad . \quad . \quad (256)$$

Similarly, for $-\frac{e}{2\lambda}$ by means of (252) we have

$$-\frac{e}{2\lambda} = \frac{1-\lambda(1-\mathfrak{D})}{2\lambda} = \frac{1-(.86)(1-.206)}{2(.86)}$$
$$= +.184 = +10^\circ\ 36'. \quad . \quad . \quad . \quad . \quad (257)$$

The quantities .022 and .184 are the natural sines of the angles that follow them; their meaning is this: that at the location of the ATLANTA's *steering* compass, the force due to transverse induction is nearly nine times greater than that due to longitudinal. The condition may be illustrated by Fig. 320: $-e$ is a bundle of nine short, thick, wrought-iron wires beneath the compass, and $-a$ is a single such wire, one half forward and one half abaft it; as the ship swings out of the meridian, both $-a$ and $-e$ cause the needle to deviate to the eastward; but while $-a$ adds slightly to the directive force, $-e$ greatly detracts from it.

For the same spot on board, the quadrantal deviation is practically invariable with either time or geographical change.

330. **The mean directive force to magnetic north: λ.** By eq. (110), page 8[illegible], this coefficient depends on soft iron:

$$\lambda = 1 - \frac{a+e}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (110)$$

It is of such importance that it will be illustrated at length. There are four different combinations of *a* and *e*: First,

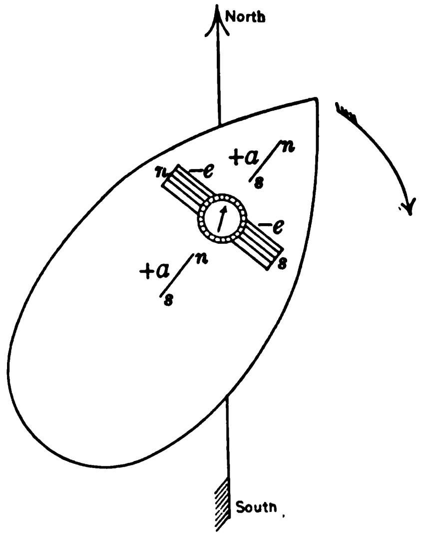

FIG. 520.

iron may be disposed all round the compass—Fig. 521—as the coaming of a hatch where beams and fore-and-aft

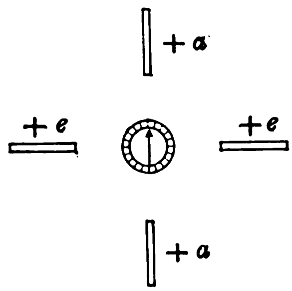

FIG. 521.

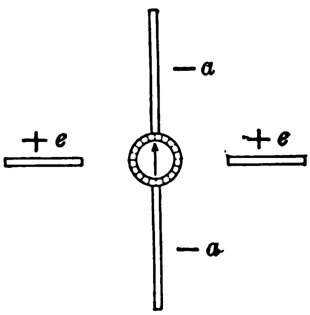

FIG. 522.

pieces are cut away; when unequal on different sides, *a* and *e* produce opposite deviations, whose algebraic sum is the result obtained. Second, the beams only may be

cut out, and a long mass, like the propeller-shaft, or the keel, may extend below the compass, exemplifying $-a$ and $+e$, Fig. 522; the former will *de*crease the directive force, the latter *in*crease it. Third, a section of the fore-and-aft piece may be removed, and an iron bridge extend across, Fig. 523, giving $+a$ and $-e$; the former increases, and the latter diminishes, the directive force; and both produce positive deviation. Fourth, both bridge and shaft may be the influencing

FIG. 523.

FIG. 524.

masses; or the compass may be located on a small superstructure deck; or directly over the boilers and engines—all which would be typified by $-a$ and $-e$, Fig. 524: when unequal on different sides they produce conflicting deviations. In the first and fourth cases, if the influences are equal all round, no deviation will occur; but the effect in each is very different: in Fig. 521 the directive force is increased—in 524, diminished.

Still another condition arises from permanent magnetism: it will readily be seen that as the steel magnet M, Fig. 525—representative of the two polar regions of the hull—swings round the compass, it will increase the directive force in the upper semicircle, and decrease it in the lower.

The *natural* directive force upon the compass is the

horizontal component of the Earth's magnetism, so that all the foregoing variously modifies it: the square of the number of oscillations of a small magnetic needle in the same time in each condition of field—natural or modified—is an index of its intensity.

Now, for a particular spot on the ship, the directive force upon the compass will vary with the *heading:* if it be

FIG. 525.

determined on four or more equidistant points, and each value be resolved into the magnetic meridian by multiplying it by the cosine of the deviation proper to the course upon which it was determined, then we obtain individual values of the directive force toward the magnetic north: taking the mean of these, and dividing it by the value of the natural field, we get a *ratio;* this RATIO is λ.

The natural field is arbitrarily assumed as unity; and λ is generally less than this, showing that the ship diminishes the directive force; but λ, being a mean value, may be the mean of wide extremes as well as of values differing but little from each other: the former would indicate a bad location for the compass,—the latter a good one. *Practically*, the value of λ does not vary with geographical change.

For calculating it, we have from eq. (118), page 879:

$$\lambda = \frac{H'}{H}.\cos\delta\left(\frac{1}{1+\mathfrak{B}.\cos\zeta-\mathfrak{C}.\sin\zeta+\mathfrak{D}.\cos 2\zeta-\mathfrak{E}.\sin 2\zeta}\right). \quad (258)$$

In this, H is the Earth's horizontal intensity; H' that of Ship and Earth combined; and δ the deviation proper to the *magnetic* course ζ: thus the factor $\frac{H'}{H}.\cos\delta$ expresses the procedure above described for obtaining λ.

The numerical work will be illustrated by data relative to the steering compass of the U. S. S. SAN FRANCISCO before it was compensated. The Ship was swung and a Table of Deviations obtained; this showed a maximum of $-29^\circ\ 30'$ on compass course SE. by S. The table was analyzed on Form 10, and the coefficients obtained as follows:

$A=-0^\circ\ 25'$; $\mathfrak{A}=-.007$; $B=-11^\circ\ 44'$; $\mathfrak{B}=-.233$;
$C=+5^\circ\ 46'$; $\mathfrak{C}=+.085$; $D=+16^\circ\ 38'$; $\mathfrak{D}=+.29$;
$E=+0^\circ\ 11'$; $\mathfrak{E}=+.005$.

While the ship's head was N. $38^\circ\ 30'$ W. magnetic, with a westerly deviation of $\delta=4^\circ$ (whose $\cos=+.9976$), a horizontal needle was oscillated, making ten oscillations in $26^s.36=T'$; on shore, it made the same number in $22^s.5=T$; hence $\frac{(22.5)^2}{(26.36)^2}=\frac{T^2}{T'^2}=\frac{H'}{H}=.728$; from the direction of the ship's head, $\zeta=321^\circ\ 30'$, of which the sine is $-.6225$, and $\cos=+.7826$; $2\zeta=643^\circ$, or $283^\circ(+360^\circ)$; the sine of this is $-.9744$, and cosine $+.2249$. Substituting these values in (258) it becomes

$$\lambda=(.728)(+.9976)\left\{\frac{1}{1+(-.233)(+.7826)-(+.085)(-.6225)+(+.29)(+.2249)-(+.005)(-.9744)}\right\}$$

Hence $\lambda = .772$, which indicates considerable decrease of directive force by the ship.

When oscillations are made with the ship heading successively on four or more equidistant *magnetic* courses, the quantities in eq. (258) containing functions of the courses will reduce to zero, as explained in Art. 296—[6], leaving only the factor $\frac{H'}{H} . \cos \delta$; denoting this for the several headings by 0, 1, 2 . . . n, eq. (258) becomes

$$\lambda = \frac{\frac{H_0'}{H} . \cos \delta_0 + \frac{H_1'}{H} . \cos \delta_1 + \frac{H_2'}{H} . \cos \delta_2 + \ldots + \frac{H_n'}{H} . \cos \delta_n}{n} \quad . \quad . \quad (259)$$

Table 76 is an example of this from experiments on the U. S. S. ALBATROSS, on eight magnetic courses.

TABLE 76.

DETERMINATION OF λ ON THE U. S. S. ALBATROSS.

$T = 25.24$ Sec., on Shore.

ζ	δ	T	$\frac{H'}{H} = \frac{T^2}{T'^2}$	$\cos \delta$	$\frac{H'}{H} \cos \delta$
(1)	(2)	(3)	(4)	(5)	(6)
		s.			
0°	+ 2°	31.46	.644	.9994	.6436
45	−10	31.26	.652	.9848	.6421
90	−19	27.07	.8693	.9455	.8219
135	−16	23.40	1.163	.9613	1.1180
180	− 3	22.07	1.308	.9986	1.3062
225	+15	22.93	1.211	.9659	1.1697
270	+19	26.60	.9003	.9455	.8512
315	+14	29.93	.7111	.9703	.6900
					8)7.2427
					$\lambda = .9053$

Although, on this ship, λ has a fairly good value—still the extremes in col. (6), Table 76, of which λ is the mean, show the compass to have indifferent surroundings.

From the foregoing, it is seen that λ can be determined in two principal ways—1st, by oscillation experiments on *one magnetic* course, and ascertaining the deviation proper to that, and also the deviations on 8, 16, or 32 equidistant *compass*-courses, from which to deduce the coefficients; then computing λ by eq. (258): 2d, by oscillation experiments on *four* or *more equidistant magnetic* headings, and determining the deviations proper to them, when the computation is by eq. (259). The latter affords a view of the individual values of λ as the ship swings through 360°, and is the best mode of determining it; the first method gives no such view, and is delusive in so far that it may indicate a fairly good *average*, without showing those quarters in which the directive force may be very weak.

CHAPTER XXIV.

VARIOUS METHODS OF DETERMINING THE MAGNETIC COEFFICIENTS; THEIR FLUCTUATION WITH GEOGRAPHICAL CHANGE; AND COMPUTATION OF THE DEVIATIONS.

Section One: $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ from Observations for Deviation on Three Specific Points of a Quadrant.

321. Northeast quarter.—The coefficients $\mathfrak{A}$ and $\mathfrak{E}$ being generally small, and $\mathfrak{D}$ practically constant, it is desirable to have a short and speedy method for determining fluctuations in the coefficients that are most liable to change, viz., $\mathfrak{B}$ and $\mathfrak{C}$: such is the method of this section.

Furthermore, it is a means of determining $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ *ab initio* from three very convenient points—the two cardinal and principal quadrantal points of any quadrant; and from them computing a table of deviations by equations to be given in Art. 327: in this case $\mathfrak{A}$ and $\mathfrak{E}$ are to be considered zero.

The great advantage of acquiring at first a thorough knowledge of the ship's magnetic character by survey in dock, oscillation experiments with horizontal and vertical needles, and swinging on 32 points, should never be lost sight of or omitted: all short methods—such as those of this chapter—should be deemed a means of detecting *change* in original conditions, rather than replacing a complete investigation.

…nulas for each quadrant, with numerical exam-
p… given in this and the next three articles.
…m eq. (124), page 880, we have

$$\ldots = \mathfrak{A}.\cos\delta + \mathfrak{B}.\sin\zeta' + \mathfrak{C}.\cos\zeta' + \mathfrak{D}.\sin(2\zeta'+\delta) + \mathfrak{E}.\cos(2\zeta'+\delta). \quad (260)$$

Expanding $\sin(2\zeta'+\delta)$ and $\cos(2\zeta'+\delta)$ by means of (30) and (31), Art. 296, eq. (260) becomes

$$\left.\begin{aligned}\sin\delta = \mathfrak{A}.\cos\delta + \mathfrak{B}.\sin\zeta' + \mathfrak{C}.\cos\zeta' \\ + \mathfrak{D}.\sin 2\zeta'.\cos\delta + \mathfrak{D}.\cos 2\zeta\ .\sin\delta \\ + \mathfrak{E}.\cos 2\zeta'.\cos\delta - \mathfrak{E}\ \sin 2\zeta'.\sin\delta\end{aligned}\right\}. \quad (261)$$

The quantities δ, S, and ζ' applicable to the points employed will receive the designations they have on Fig. 511; and it should be recalled that S_0, S_1, etc., may be used for both sines and cosines, and that their values are specific for 0° and 90°. For NORTH, NORTHEAST, and EAST, (261) becomes successively

$$\begin{aligned}\sin\delta_0 = \mathfrak{A}.\cos\delta_0 + \mathfrak{B}.S_0 + \mathfrak{C}.S_8 + \mathfrak{D}.S_0\cos\delta_0 \\ + \mathfrak{D}.S_8\sin\delta_0 + \mathfrak{E}.S_8\cos\delta_0 - \mathfrak{E}.S_0\sin\delta_0.\end{aligned} \quad (262)$$

$$\begin{aligned}\sin\delta_4 = \mathfrak{A}.\cos\delta_4 + \mathfrak{B}.S_4 + \mathfrak{C}.S_4 + \mathfrak{D}.S_8\cos\delta_4 \\ + \mathfrak{D}.S_0\sin\delta_4 + \mathfrak{E}.S_0\cos\delta_4 - \mathfrak{E}.S_8\sin\delta_4.\end{aligned} \quad (263)$$

$$\begin{aligned}\sin\delta_8 = \mathfrak{A}.\cos\delta_8 + \mathfrak{B}.S_8 + \mathfrak{C}.S_0 - \mathfrak{D}.S_0\cos\delta_8 \\ - \mathfrak{D}.S_8\sin\delta_8 - \mathfrak{E}.S_8\cos\delta_8 - \mathfrak{E}.S_0\sin\delta_8.\end{aligned} \quad (264)$$

From (264) we obtain the value of $\mathfrak{B}$, and from (262) that of $\mathfrak{C}$ as follows, since $S_0 = 0$, and $S_8 = 1$:

$$\begin{aligned}\mathfrak{B} &= (1+\mathfrak{D})\sin\delta_8 - (\mathfrak{A}-\mathfrak{E})\cos\delta_8 \\ &= (1+\mathfrak{D})\sin\delta_8, \text{ if } \mathfrak{A} \text{ and } \mathfrak{E} = \text{zero}.\end{aligned} \quad (265)$$

$$\begin{aligned}\mathfrak{C} &= (1-\mathfrak{D})\sin\delta_0 - (\mathfrak{A}+\mathfrak{E})\cos\delta_0 \\ &= (1-\mathfrak{D})\sin\delta_0, \text{ if } \mathfrak{A} \text{ and } \mathfrak{E} = \text{zero}.\end{aligned} \quad (266)$$

Substituting these values of $\mathfrak{B}$ and $\mathfrak{C}$ in (263)—transposing terms—multiplying throughout by S_4—and remembering that by Trig. $S_4^2=\frac{1}{2}$, we have

$$\left.\begin{aligned}\mathfrak{D}[S_4.\cos\delta_4-\tfrac{1}{2}\sin\delta_0+\tfrac{1}{2}\sin\delta_8]\\ =(S_4\sin\delta_4-\tfrac{1}{2}\sin\delta_0-\tfrac{1}{2}\sin\delta_8)\\ +\mathfrak{A}(-S_4\cos\delta_4+\tfrac{1}{2}\cos\delta_0+\tfrac{1}{2}\cos\delta_8)\\ +\mathfrak{E}(+S_4\sin\delta_4+\tfrac{1}{2}\cos\delta_0-\tfrac{1}{2}\cos\delta_8)\end{aligned}\right\}. \quad (267)$$

If $\mathfrak{A}$ and $\mathfrak{E}$ are zero, this becomes, after dividing by the coefficient of $\mathfrak{D}$,

$$\mathfrak{D}=\frac{S_4.\sin\delta_4-\frac{1}{2}(\sin\delta_0+\sin\delta_8)}{S_4.\cos\delta_4-\frac{1}{2}(\sin\delta_0-\sin\delta_8)}. \quad . \quad . \quad (268)$$

By (265) to (268), $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ may be computed either with or without values of $\mathfrak{A}$ and $\mathfrak{E}$. An example from the U. S. S. ATLANTA will be taken, and $\mathfrak{A}$ and $\mathfrak{E}$ will first be considered either zero or unknown.

The deviations on three points of the NE. quadrant, by Table 74, are, for N. $=\delta_0=-18°\ 0'$; NE. $=\delta_4=-26°\ 20'$; and E. $=\delta_8=-32°\ 00'$.

Substituting the requisite data in the above equations, they become

$$\mathfrak{D}=\frac{(+.707)(-.443)-\frac{1}{2}\{(-.309)+(-.530)\}}{(+.707)(+.896)-\frac{1}{2}\{(-.309)-(-.530)\}}$$
$$=\frac{-.313-\{-.419\}}{+.633-\{+.110\}}=\frac{-.313+.419}{+.633-.110}=\frac{+.106}{+.523}=+.200 \quad (269)$$

$$\mathfrak{B}=(1+.200)(-.530)=-.636. \quad . \quad . \quad . \quad (270)$$

$$\mathfrak{C}=(1-.200)(-.309)=-.247. \quad . \quad . \quad . \quad (271)$$

$$\tan\alpha=\frac{\mathfrak{C}}{\mathfrak{B}}=\frac{-.247}{-.636}=+.3887 \text{ and}$$

$$\alpha=21°\ 15'+180°=201°\ 15'. \quad (272)$$

On account of the liability to error by one who does not deal frequently with algebraic signs, the various stages of the numerical work have been set down in eq. (269); and it will be found conducive to accuracy to follow that practice and to enclose the individual quantities in parentheses: then, plus outside means no change in the sign inside, as $\frac{1}{2}\{(-.309)+(-.530)\}$; but minus outside indicates change of sign within, as $\frac{1}{2}\{(-.309)-(-.530)\}$; this is really $\frac{1}{2}\{-.309+.530\}$, or $+.110$. It should be remembered that by Trig. $\sin(-y)=-\sin y$, but that $\cos(-y)=+\cos y$; whence $\sin\delta_4=\sin(-26° 20')=-.443$, and $\cos\delta_4=+.896$. Comparing the arcs corresponding to the values of $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ just found with those in Table 73 obtained from *four equidistant* points, it will be seen that the former differ by about the same amount as the latter from the values deduced from 32 points: the chief advantage, then, of observing on three points in a quadrant over four equidistant points, is, that the ship has to swing through only a small arc instead of a full circle; it saves time and labor, but is not more accurate.

The comparison is made, however, in a case of excessive deviations—a maximum of 50°; it is therefore not a fair illustration of the quadrant method: a better test will be made in Art. 323.

The starboard angle by the quadrant method differs only 1° 22′ from that determined by 32 points—an amount that warrants the method being used for this purpose.

As the values of $\mathfrak{A}$ and $\mathfrak{E}$ for the ATLANTA's steering compass are known from observations on 32 points (Form 10, p. 921), it will be useful to see what change their introduction into eqs. (265), (266), and (267) will make in the values just obtained for $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$: performing the numerical work, the results and comparison are shown in Table 77; *and the change is very small*, but so are $\mathfrak{A}$ and $\mathfrak{E}$ in this case.

TABLE 77.

U. S. S. ATLANTA.

Values of	$\mathfrak{B}$	$\mathfrak{C}$	$\mathfrak{D}$
Without $\mathfrak{A}$ and $\mathfrak{E}$...	−.636	−.247	+.200
With $\mathfrak{A}$ and $\mathfrak{E}$......	−.645	−.248	+.202
Difference.........	+.009	+.001	+.002

322. Southeast quarter.—Formulas for this quadrant and those of the western semicircle are deduced in the same way as the preceding, and from the same primary equation, (124), page 880: they are similar to formulas (265), (266), and (268), and may be written at once by inserting the values of S and δ proper to each quadrant. For EAST, SOUTHEAST, and SOUTH they are:

$$\begin{aligned}\mathfrak{B} &= +(1+\mathfrak{D})\sin\delta_8-(\mathfrak{A}-\mathfrak{E})\cos\delta_8\\ &=(1+\mathfrak{D})\sin\delta_8,\text{ if }\mathfrak{A}\text{ and }\mathfrak{E}=\text{zero.}\end{aligned} \quad (273)$$

$$\begin{aligned}\mathfrak{C} &= -(1-\mathfrak{D})\sin\delta_{16}+(\mathfrak{A}+\mathfrak{E})\cos\delta_{16}\\ &=-(1-\mathfrak{D})\sin\delta_{16},\text{ if }\mathfrak{A}\text{ and }\mathfrak{E}=\text{zero.}\end{aligned} \quad (274)$$

$$\left.\begin{aligned}&\mathfrak{D}[S_4.\cos\delta_{12}-\tfrac{1}{2}(\sin\delta_8-\sin\delta_{16})]\\ &\quad=-S_4.\sin\delta_{12}+\tfrac{1}{2}(\sin\delta_8+\sin\delta_{16})\\ &\quad\quad+\mathfrak{A}[+S_4\cos\delta_{12}-\tfrac{1}{2}(\cos\delta_8+\cos\delta_{16})]\\ &\quad\quad+\mathfrak{E}[+S_4\sin\delta_{12}+\tfrac{1}{2}(\cos\delta_8-\cos\delta_{16})]\end{aligned}\right\}. \quad (275)$$

323. Southwest quarter.—For SOUTH, SOUTHWEST, and WEST the formulas are:

$$\begin{aligned}\mathfrak{B} &= -(1+\mathfrak{D})\sin\delta_{24}+(\mathfrak{A}-\mathfrak{E})\cos\delta_{24}\\ &=-(1+\mathfrak{D})\sin\delta_{24},\text{ if }\mathfrak{A}\text{ and }\mathfrak{E}=\text{zero.}\end{aligned} \quad (276)$$

$$\begin{aligned}\mathfrak{C} &= -(1-\mathfrak{D})\sin\delta_{16}+(\mathfrak{A}+\mathfrak{E})\cos\delta_{16}\\ &=-(1-\mathfrak{D})\sin\delta_{16},\text{ if }\mathfrak{A}\text{ and }\mathfrak{E}=\text{zero.}\end{aligned} \quad (277)$$

$$
\left.\begin{aligned}
&\ldots \sin\delta_{16}-\sin\delta_{24})]\\
&=S_4\sin\delta_{20}-\tfrac{1}{2}(\sin\delta_{16}+\sin\delta_{24})\\
&\quad+\mathfrak{A}[-S_4\cos\delta_{20}+\tfrac{1}{2}(\cos\delta_{16}+\cos\delta_{24})]\\
&\quad+\mathfrak{E}[+S_4\sin\delta_{20}-\tfrac{1}{2}(\cos\delta_{16}-\cos\delta_{24})]
\end{aligned}\right\}. \qquad (278)
$$

TABLE 78.

COEFFICIENTS OF U. S. S. CONCORD FROM DIFFERENT NUMBERS OF COMPASS POINTS.

Coefficients from Analysis of Deviations on 32 Points and Oscillation Experiments	4 Points (1)	8 Points (2)	16 Points (3)	32 Points (4)
		B		
$\mathfrak{A}=-.0038$ $A=-0°\ 13'$ $\mathfrak{B}=-.364$ $B=-20°\ 11'$ $\mathfrak{C}=-.153$ $C=-9°\ 23'$ $\mathfrak{D}=+.107$ $D=+6°\ 08'$ $\mathfrak{E}=+.0048$ $E=+0°\ 15'$ $\lambda=.857$ $\alpha=202°\ 44'$	−19° 30′ −19 57 −20 07 −20 49 −21 06 −20 35 −20 06 −19 19	−20° 18′ −20 16 −20 07 −20 04	−20° 12′ −20 10	−20° 11′
		C		
	−10° 00′ −10 34 −10 15 − 9 33 − 8 50 − 8 20 − 8 20 − 9 12	−9° 25′ −9 27 −9 18 −9 23	−9° 22′ −9 25	−9° 23′
		D		
	Quadrantal Points +6° 15′	+6° 15′ +6 03 +6 10 +6 05	+6° 12′ +6 04	+6° 08′
	Maximum deviation = +27° 20′ on SW			

Table 78 contains magnetic information relative to the U. S. S. CONCORD, derived from analysis of a very careful swinging on 32 points. The maximum deviation was

$+27° 20'$ on SW.; as this and the other data indicate the customary conditions of compass locations on ships of the Navy, formulas (276), (277), and (278) will be tested by means of observations in the SW. quarter: S. $=\delta_{16}=+10° 00'$; SW. $=\delta_{20}=+27° 20'$; and W. $=\delta_{24}=+19° 00'$. $\mathfrak{A}$ and $\mathfrak{E}$ are seen by Table 78 to be very small: they would affect the results but little—will be considered zero—and hence the terms containing them disappear. Inserting the requisite data in the formulas, they become

$$\mathfrak{B}=-(1+.107)(+.326)=-.361. \qquad (279)$$

$$\mathfrak{C}=-(1-\{+.107\})(+.174)=.155. \qquad (280)$$

$$\mathfrak{D}=\frac{(+.707)(+.459)-\frac{1}{2}\{(+.174)+(+.326)\}}{(+.707)(+.888)-\frac{1}{2}\{(+.174)-(+.326)\}}$$

$$=\frac{+.325-.250}{+.628+.076}=\frac{+.075}{+.704}=+.107. \qquad (281)$$

TABLE 79.

U. S. S. ATLANTA: STEERING COMPASS.

Method of Swinging.	$\mathfrak{B}$	$\mathfrak{C}$	$\mathfrak{D}$	Maximum Deviation.	λ
(1)	(2)	(3)	(4)	(5)	(6)
32 Points.......	−.653	−.236	+.206		
Quadrant.......	−.636	−.247	+.200	50°	.86
Difference......	.017	.011	.006		

U. S. S. CONCORD: POOP-DECK COMPASS.

Method of Swinging.	$\mathfrak{B}$	$\mathfrak{C}$	$\mathfrak{D}$	Maximum Deviation.	λ
32 Points.......	−.364	−.153	+.107		
Quadrant.......	−.361	−.155	+.107	27° 20′	.857
Difference......	.003	.002	.000		

In Table 79 the coefficients from three points of a quadrant are compared with those from 32 points: in the case

of the ATLANTA, whose deviations were abnormal, the differences are too great to warrant very much confidence in results from *three* points; and such, too, is the case with values of B and C from *four equidistant* points in Table 73: but for the CONCORD, whose deviations were about the average, the results from *three* points are seen to be almost identical with those from 32 points (Table 79); also, by Table 78, there is quite a fair approximation of the values of B and C from *four equidistant* points to their values from 32 points. Therefore it would seem that when the deviations do not exceed 27°, either the quadrant method or that for four equidistant points will give fairly trustworthy results; but a swing on at least eight equidistant points should always be practicable; and the matter is too important to let either time or labor be a factor in its curtailment.

324. Northwest quarter.—For the points NORTH, NORTHWEST, and WEST the formulas are:

$$\mathfrak{B} = -(1-\mathfrak{D})\sin\delta_{24} - (\mathfrak{A}-\mathfrak{E})\cos\delta_{24}. \qquad (282)$$

$$\mathfrak{C} = -(1-\mathfrak{D})\sin\delta_{0} - (\mathfrak{A}+\mathfrak{E})\cos\delta_{0}. \qquad (283)$$

$$\left.\begin{aligned}\mathfrak{D}[S_4\cos\delta_{28} + \tfrac{1}{2}(\sin\delta_0 - \sin\delta_{24})]&\\ -S_4\sin\delta_{28} - \tfrac{1}{2}(\sin\delta_0 - \sin\delta_{24})&\\ +\mathfrak{A}[-S_4\cos\delta_{28} - \tfrac{1}{2}(\cos\delta_0 - \cos\delta_{24})]&\\ +\mathfrak{E}[-S_4\sin\delta_{28} - \tfrac{1}{2}(\cos\delta_0 - \cos\delta_{24})]&\end{aligned}\right\}. \qquad (284)$$

Section Two: $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, and λ from Observations of Deviation and Force on any Compass Points.

325. Two headings.—Throughout this work frequent occasion has arisen for treating of the determination of magnetic force by means of the oscillation of a small needle. The principle has been fully explained and illustrated

in the following Articles: 108 to 110, 128, 135, 136, 171, 230, 239, 242 to 252, 265, 266, and 320; Table 37; and Fig. 417. The principle is an important one, and the articles cited should be read to ensure its proper application.

The formulas for observations of deviation and force on two headings will now be deduced. From eqs. (110), (113), (114), (115), (116), and (117), page 879, we obtain the following:

$$a = -1 + \lambda(1 + \mathfrak{D}). \quad . \quad (285); \qquad -b = \lambda(\mathfrak{A} - \mathfrak{E}). \quad . \quad (286)$$

$$e = -1 + \lambda(1 - \mathfrak{D}). \quad . \quad (287); \qquad d = \lambda(\mathfrak{A} + \mathfrak{E}). \quad . \quad . \quad (288)$$

$$c.\tan\theta + \frac{P}{H} = \lambda.\mathfrak{B}. \quad . \quad (289); \qquad f.\tan\theta + \frac{Q}{H} = \lambda.\mathfrak{C}. \quad (290)$$

Substituting the second members of (285) to (290) for their equivalents in eqs. (94) and (95), page 875, these become, *after* dividing throughout by λ:

$$\frac{H'}{\lambda.H}.\cos\zeta' = (1 + \mathfrak{D})\cos\zeta + (\mathfrak{A} - \mathfrak{E})\sin\zeta + \mathfrak{B}. \quad (291)$$

$$-\frac{H'}{\lambda.H}.\sin\zeta' = (\mathfrak{A} + \mathfrak{E})\cos\zeta - (1 - \mathfrak{D})\sin\zeta + \mathfrak{C}. \quad (292)$$

As shown in Art. 321 and Table 77, $\mathfrak{A}$ and $\mathfrak{E}$ are generally small and affect the final results but little; regarding them as zero, (291) and (292) become after transposition:

$$\mathfrak{B} = +\frac{1}{\lambda}.\frac{H'}{H}.\cos\zeta' - (1 + \mathfrak{D})\cos\zeta. \quad . \quad . \quad (293)$$

$$\mathfrak{C} = -\frac{1}{\lambda}.\frac{H'}{H}\sin\zeta' + (1 - \mathfrak{D})\sin\zeta. \quad . \quad . \quad . \quad (294)$$

From (285) and (287) we have

$$(1 + \mathfrak{D}) = \frac{1}{\lambda}(1 + a). \quad . \quad (295); \qquad (1 - \mathfrak{D}) = \frac{1}{\lambda}(1 - e). \quad . \quad (296)$$

... the second members of (295) and (296) in ...-94), these become

$$\lambda.\mathfrak{B} = +\frac{H'}{H}.\cos\zeta' - (1+a)\cos\zeta. \quad . \quad . \quad (297)$$

$$\lambda.\mathfrak{C} = -\frac{H'}{H}.\sin\zeta' + (1-e)\sin\zeta. \quad . \quad . \quad . \quad (298)$$

Let ζ_1' and ζ_2' be any two *compass* courses; ζ_1 and ζ_2 the corresponding *magnetic* courses; T_1' and T_2' the times of ten oscillations of a small horizontal needle in the location of the compass with the ship on the respective headings ζ_1' and ζ_2'; and T the time of ten oscillations of the same needle on shore in a spot free from iron: then $\frac{T^2}{T_1'^2}=\frac{H_1'}{H}$, and $\frac{T^2}{T_2'^2}=\frac{H_2'}{H}$. For each heading, a bearing of the sun by compass is to be taken and the local apparent time noted; from these and the declination and latitude, the sun's true bearing can be calculated, and thence the magnetic headings ζ_1 and ζ_2 obtained. Substituting the above quantities in (297) and (298) we form four equations, two for each heading, thus:

$$\lambda.\mathfrak{B} = +\frac{H_1'}{H}\cos\zeta_1' - (1+a)\cos\zeta_1. \quad . \quad . \quad (299)$$

$$\lambda.\mathfrak{B} = +\frac{H_2'}{H}\cos\zeta_2' - (1+a)\cos\zeta_2. \quad . \quad . \quad (300)$$

$$\lambda.\mathfrak{C} = -\frac{H_1'}{H}\sin\zeta_1' + (1+e)\sin\zeta_1. \quad . \quad . \quad (301)$$

$$\lambda.\mathfrak{C} = -\frac{H_2'}{H}\sin\zeta_2' + (1+e)\sin\zeta_2. \quad . \quad . \quad (302)$$

Subtracting (300) from (299), and (302) from (301), both λ... and $\lambda.\mathfrak{C}$ disappear; then deducing the values of $(1+a)$

and $(1+e)$, and dividing both numerator and denominator of the second members of the resulting equations by 2, which will not alter the value of the first members, while rendering the second more convenient for subsequent use, we have

$$(1+a) = \frac{\frac{1}{2}\left[\frac{H_1'}{H}\cos\zeta_1' - \frac{H_2'}{H}\cos\zeta_2'\right]}{\frac{1}{2}[\cos\zeta_1 - \cos\zeta_2]} \quad . \quad . \quad . \quad (303)$$

$$(1+e) = \frac{\frac{1}{2}\left[\frac{H_1'}{H}\sin\zeta_1' - \frac{H_2'}{H}\sin\zeta_2'\right]}{\frac{1}{2}[\sin\zeta_1 - \sin\zeta_2]} \quad . \quad . \quad . \quad (304)$$

Adding (299) to (300), and (301) to (302), first members of each pair together, and also second members together, the values of $\mathfrak{B}$ and $\mathfrak{C}$ are thence deduced:

$$\mathfrak{B} = +\frac{1}{\lambda}\left[\frac{1}{2}\left\{\frac{H_1'}{H}.\cos\zeta_1' + \frac{H_2'}{H}\cos\zeta_2'\right\} - (1+a)\frac{1}{2}\{\cos\zeta_1 + \cos\zeta_2\}\right]. \quad (305)$$

$$\mathfrak{C} = -\frac{1}{\lambda}\left[\frac{1}{2}\left\{\frac{H_1'}{H}\sin\zeta_1' + \frac{H_2'}{H}\sin\zeta_2'\right\} - (1+e)\frac{1}{2}\{\sin\zeta_1 + \sin\zeta_2\}\right]. \quad (306)$$

From (285) and (287) we have

$$(1+a) = \lambda + \lambda.\mathfrak{D}. \quad . \quad (307); \qquad (1+e) = \lambda - \lambda.\mathfrak{D}; \quad . \quad (308)$$

whence
$$\lambda = \tfrac{1}{2}\{(1+a) + (1+e)\} \quad . \quad . \quad . \quad . \quad (309)$$

and
$$\mathfrak{D} = \frac{1}{\lambda}.\tfrac{1}{2}\{(1+a) - (1-e)\}. \quad . \quad . \quad . \quad (310)$$

From observations on the two headings, the quantities in the second members of (303) and (304) are all known, whence $(1+a)$ and $(1+e)$ become known; from these, λ

is found directly by (309), then $\mathfrak{D}$ by (310), and finally, $\mathfrak{B}$ and $\mathfrak{C}$ by (305) and (306). Also

$$\tan \alpha = \frac{\mathfrak{C}}{\mathfrak{B}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (311)$$

Although the title of this section conveys the idea that *any* compass-points may be used, still there are restrictions: When ζ_1' is east and ζ_2' west, eq. (303) reduces to $\frac{0}{0}$, because cos 90° (east) = cos 270° (west) = 0; similarly, when ζ_1' is north, and ζ_2' south, both whose sines are 0, eq. (304) becomes $\frac{0}{0}$; thus the problem is indeterminate when the two headings are on the cardinal points or close to them. When ζ_1 and ζ_2 are *equally distant* from one of the cardinal points, as NE. *and* NW., or NE. *and* SE., the denominator of (303) reduces to zero with the first pair, and that of (304) with the second pair: similarly when ζ_1 and ζ_2 are equally distant from west and south. This bars the use of the following pairs of magnetic headings: N. and S.; E. and W.; NE. and SE.; SE. and SW.; SW. and NW.; NE. and NW.

If, otherwise, the magnetic headings are diametrically opposite, as N.NE. and S.SW., then

$$\cos \zeta_1 + \cos \zeta_2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (312)$$

and

$$\sin \zeta_1 + \sin \zeta_2 = 0; \quad . \quad . \quad . \quad . \quad . \quad . \quad (313)$$

whence

$$\cos \zeta_1 = -\cos \zeta_2 \text{ or } \tfrac{1}{2}(\cos \zeta_1 - \cos \zeta_2) = \cos \zeta_1, \quad (314)$$

and similarly

$$\tfrac{1}{2}(\sin \zeta_1 - \sin \zeta_2 = \sin \zeta_1. \quad . \quad . \quad . \quad . \quad (315)$$

These simplify (303), (304), (305), and (306), as can readily be seen. While *any* two magnetic headings that are *un-*

equally distant from a cardinal point will make the solution of the problem possible, yet there is a choice; and the greatest accuracy will result from those near two adjacent cardinal points, as N. and E., or diametrically opposite quadrantal points, as NE. and SW.

Docking a ship affords an excellent opportunity for using this method: accurate observations can then be made *in* dock for one heading; and on coming out, the second heading may be made favorable by suitably tying up the ship to a wharf. Or, if at first alongside a wharf, she may be warped into another direction for the second heading: in very still water good oscillation experiments can be made.

The case of two headings at *un*equal distances from a cardinal point is the one most likely to occur, and will be illustrated by an example from experiments made in 1898 with the SCORESBY.

The vessel and its vicinity were first cleared of all iron, and a small horizontal needle was oscillated in the place of the compass, the latter having been removed; the mean of ten sets of ten oscillations each was $15.98^s = T$, the amplitude of swing for each set being $20°$ and $5°$ at beginning and ending. Then the compass was put in place, and the vessel swung on the eight principal points, the bearing of the "electric sun" being taken on each; it was the same for all, N. $33° 0'$ W. $= 327°$, and this constituted the *magnetic* line of bearing with which subsequent compass-bearings of the electric sun were compared, when the vessel, loaded with soft iron and magnets, was swung for a series of deviations.

The "electric sun" is a small incandescent lamp fixed in an upper corner of the room; and as the compass is mounted so that its pivot is in the vertical line through the pivot on which the vessel swings, there is practically no parallax in the bearings. Magnets and soft iron were now suitably disposed on the SCORESBY, and she was swung

on sixteen points, resting four minutes on each; the maximum deviation was $-27° 15'$ on SW., and the neutral points were about N. 60° E. and S. 30° W.: thus a table of deviations was obtained, from which the coefficients were deduced, to be used as standards with which to compare those computed by other methods, as, for instance, the one of this article.

Next, the vessel, with everything as above, was successively put on two headings, and the following observations made:

	FIRST HEADING.	SECOND HEADING.
Bearing of electric sun, magnetic..	N. 33° 0′ W. = 327°	N. 33° 0′ W. = 327°
Bearing of electric sun by compass.	N. 51 0 W. = 309	N. 39 0 W. = 321
Deviation......	+18 0 E.	+6 0 E.
Ship's head by compass.......	N. 0° 0 W. = 351 = ζ_1'	S. 54 0 W. = 234 = ζ_2'
Deviation.......	18 0 E.	6 0 E.
Ship's head, magnetic..........	N. 9 0 E. = 9 = ζ_1	S. 60 0 W. = 240 = ζ_2
Mean of five sets of ten oscillations each, horizontal needle, through arc of 20° at beginning to 5° at ending.	$T_1 = 16.00^s$ $\frac{T^2}{T_1^2} = \frac{H_1'}{H} = \frac{(15.08)^2}{(16.00)^2} = 0.804$	$T_2' = 13.95^s$ $\frac{T^2}{T_2^2} = \frac{H_2'}{H} = \frac{(15.08)^2}{(13.95)^2} = 1.321$

$\sin \zeta_1' = -.1564$; $\cos \zeta_1' = +.9877$; $\sin \zeta_2' = -.8090$; $\cos \zeta_2' = -.5878$;
$\sin \zeta_1 = +.1564$, $\cos \zeta_1 = +.9877$; $\sin \zeta_2 = -.8660$; $\cos \zeta_2 = -.5000$.

Substituting the above data in (303) and (304), they become

$$(1-\lambda) = \frac{\frac{1}{2}[(-.804)(+.9877) - (-1.321)(-.5878)]}{\frac{1}{2}[(+.9877) - (-.5000)]}$$

$$= \frac{-.8280}{-.7438} = +1.115. \quad (316)$$

$$(1+e)=\frac{\frac{1}{2}[(+.894)(-.1564)-(+1.321)(-.809)]}{\frac{1}{2}[(+.1564)-(-.866)]}$$

$$=\frac{+.4646}{+.5112}=+0.909. \quad (317)$$

Then substituting these values of $(1+a)$ and $(1+e)$ in (309) we have

$$\lambda=\tfrac{1}{2}\{(+1.115)+(+.909)\}=1.012. \quad . \quad . \quad (318)$$

Substituting the values of (316), (317), and (318) in (310) it becomes

$$\mathfrak{D}=\left(\frac{1}{1.012}\right)\tfrac{1}{2}\{(+1.115)-(+.909)\}$$

$$=(+.988)\{+.103\}=+.1017. \quad (319)$$

We now have all the data for computing $\mathfrak{B}$ and $\mathfrak{C}$; and inserting the requisite quantities in (305) and (306), they become

$$\mathfrak{B}=+(+.988)[\tfrac{1}{2}\{(+.894)(+.987)+(+1.321)(-.587)\} \\ -(+1.115)\tfrac{1}{2}\{(+.9877)+(-.5000)\}]; \text{ or}$$

$$\mathfrak{B}=+(+.988)[\{+.0534\}-(+1.115)\{+.2438\}] \\ =+(.988)[+.0534-.2718]=-.2154. \quad (320)$$

$$\mathfrak{C}=-(+.988)[\tfrac{1}{2}\{(+.894)(-.1564)+(+1.321)(-.809)\} \\ -(+.909)\tfrac{1}{2}\{(+.1564)+(-.866)\}]; \text{ or}$$

$$\mathfrak{C}=-(+.988)[\{-.604\}-(+.909)\{-.355\}] \\ =-(+.988)[-.604+.322]=+.2786. \quad (321)$$

From (311), (320), and (321)

$$\tan\alpha=\frac{+.2786}{-.2154}=-1.2934;$$

$$\therefore\ \alpha=52^\circ\ 18'+90^\circ=142^\circ\ 18'. \quad . \quad . \quad (322)$$

In order to compare the results of (318) to (322) with those from 16 points, the deviations on the latter were analyzed, and all the coefficients and the starboard angle calculated; also, to get λ by a different method, formula (258), p. 944, was employed; substituting in it the exact coefficients from the observations on 16 points and the following data from the *first* of the two headings worked out above. $\delta = 18^\circ\ 0'$ E.; $\frac{H_1'}{H} = +.894$; $\zeta_1 = 9^\circ\ 0'$; $2\zeta_1 = 18^\circ\ 0'$; we have

$$\lambda = (+.894)(+.951)\left[\frac{1}{\begin{array}{l}1 + (-.223)(+.988) - (+.284)(+.156) \\ \quad + (+.1035)(+.9511) \\ \quad - (+.0005)(+.309)\end{array}}\right];$$

or
$$\lambda = \frac{+.8502}{+.8343} = 1.019. \quad . \quad . \quad . \quad . \quad (323)$$

The results of both methods—on two headings and on 16 equidistant points—will be best compared as in Table 80:

TABLE 80.

RESULTS OF EXPERIMENTS.

$\mathfrak{A}$	$\mathfrak{B}$	$\mathfrak{C}$	$\mathfrak{D}$	$\mathfrak{E}$	[illegible]	λ
− [illegible]	− [illegible]	− [illegible]	− [illegible]	− [illegible]	[illegible]	1.019
− [illegible]	− [illegible]	− [illegible]	− [illegible]	− [illegible]		

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	Angles of Two Headings	Difference
[illegible]	− [illegible]	− [illegible]	[illegible]	− [illegible]	− [illegible]	[illegible]
[illegible]	− [illegible]	− [illegible]	[illegible]	− [illegible]	− [illegible]	[illegible]
[illegible]	[illegible]	− [illegible]	[illegible]	− [illegible]	− [illegible]	[illegible]
[illegible]	− [illegible]	[illegible]	[illegible]			
[illegible]	[illegible]	[illegible]	[illegible]			

Considering the quantities in cols. (2) and (3) natural *sines*, their angles are in cols. (5) and (6); the swing on 16 points is unquestionably the more accurate method, but it is remarkable how closely both methods agree; the observations were made with great care, however, and also they were susceptible of more accuracy with the SCORESBY than is generally the case with a ship in the water; a set in dry-dock should be equally good.

The method of two headings is available when others are not; it can be practised without interfering with the ship's work—while in dock, or coaling, or repairing; it affords the important coefficients by means of which approximate deviations on *all* points can be computed for urgent need, by a formula to be given in Art. 327; and the compass can be fairly well compensated from its results: altogether, it is very useful and worthy of confidence when extreme care is taken with the observations and the conditions are favorable.

It will be seen that some of the quantities are repeated in formulas (303) to (310), and it will occur to the navigator to arrange them in a convenient form for computation.

326. One heading.—Reproducing equations (293) and (294), they are:

$$\mathfrak{B} = +\frac{1}{\lambda}\left(\frac{H'}{H}\right)\cos\zeta' - (1 + \mathfrak{D})\cos\zeta. \quad . \quad . \quad (324)$$

$$\mathfrak{C} = -\frac{1}{\lambda}\left(\frac{H'}{H}\right)\sin\zeta' + (1 - \mathfrak{D})\sin\zeta. \quad . \quad . \quad (325)$$

$\mathfrak{A}$ and $\mathfrak{E}$ do not enter these, and, as has been shown, with *normal* deviations they do not affect the results greatly.

After completion of the ship, λ and $\mathfrak{D}$ (for the same spot on board) change but little with either time or place, but individual values of λ do vary with different headings: an examination of λ and $\mathfrak{D}$ for different types of vessel of our Navy shows that they vary considerably; in one ship the

value of λ was 0.64 on a NE. course, and 1.3 on a south course, and from ship to ship of different types it had mean values ranging from 0.69 to 0.93; $\mathfrak{D}$ in the same variety of vessel ranged from 0.04 to 0.15. Supposing nothing known of the ship's magnetism, it would therefore be somewhat wild guessing to *assume* values of λ and $\mathfrak{D}$: with λ and $\mathfrak{D}$ *assumed*, and $\mathfrak{A}$ and $\mathfrak{E}$ omitted, eqs. (324) and (325) must be regarded as affording only rough approximations of the quantities computed by them; their results should be relied upon only until the earliest possible opportunity for replacing them by those of an accurate method. Still they are useful in these cases: *first*, for following the development of the ship's magnetic character while building; *second*, after completion, for speedily determining *changes* in $\mathfrak{B}$ and $\mathfrak{C}$, using λ and $\mathfrak{D}$ from the original investigation of the ship's magnetism; and *third*, for deciding upon a site for the compass.

With the ship, then, in dry dock, or quietly moored to a wharf—the only two conditions in which this method should ever be employed—and with no other iron vessel or large mass of iron near enough to exercise any influence, the procedure is this: set up the compass and observe the direction of the ship's head, and take a time-azimuth of the sun from which to compute the true bearing and obtain ζ and δ; then remove the compass and oscillate the horizontal needle in its place (T'), and also on shore (T), by which we get $\frac{T^2}{T'^2}=\frac{H'}{H}$; assume the most probable values of λ and $\mathfrak{D}$; and then insert the requisite data in (324) and (325), whence $\mathfrak{B}$ and $\mathfrak{C}$ become known; from these α is computed by (311). The compass may now be roughly compensated by either the resultant or the component method. Finally, an approximate table of deviations may be computed by the formulas of Art. 327.

Section Three: Computation of the Deviations and Changes in the Ship's Magnetism that Affect Them.

327. Computation of the deviations by means of either the exact or approximate coefficients.—Formulas for observations on five headings, on two headings, and on one heading will be given. From eq. (261), p. 948, we have

$$\left.\begin{aligned}\sin\delta = \mathfrak{A}.\cos\delta + \mathfrak{B}.\sin\zeta' + \mathfrak{C}.\cos\zeta' \\ +\mathfrak{D}.\sin 2\zeta' \cos\delta + \mathfrak{D}.\cos 2\zeta' \sin\delta \\ +\mathfrak{E}.\cos 2\zeta' \cos\delta - \mathfrak{E}.\sin 2\zeta' \sin\delta\end{aligned}\right\}. \quad (326)$$

Considering $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, and $\sin\delta$ quantities of the first order, and $\mathfrak{A}$ and $\mathfrak{E}$ of the second, we may write $\cos\delta = 1$: then $\mathfrak{E}.\sin 2\zeta'.\sin\delta$, being of the third order, disappears, since only those of the second are to be retained, and (326) becomes

$$\sin\delta - \mathfrak{D}.\cos 2\zeta'.\sin\delta = \mathfrak{A} + \mathfrak{B}\sin\zeta' + \mathfrak{C}\cos\zeta' \\ + \mathfrak{D}\sin 2\zeta' + \mathfrak{E}\cos 2\zeta'; \quad (327)$$

whence

$$\sin\delta = \frac{\mathfrak{A} + \mathfrak{B}\sin\zeta' + \mathfrak{C}\cos\zeta' + \mathfrak{D}\sin 2\zeta' + \mathfrak{E}\cos 2\zeta'}{1 - \mathfrak{D}.\cos 2\zeta'}. \quad (328)$$

This is very nearly an exact formula: by observing the deviations ($\delta_1 \ldots \delta_5$) on *any five compass courses* ($\zeta_1' \ldots \zeta_5'$) we get data for as many equations similar to (328); these solved by any algebraic process, as there are only five unknown quantities, afford values of the coefficients, and substituting them in (328), the deviation (δ) can then be calculated for the 32 points (ζ'): to facilitate the work, it may be arranged in a Form similar to Table 74.

If the compass is in the midship line, or so favorably located that the symmetry of the horizontal soft iron with

is perfect or nearly so, then $\mathfrak{A}$ and $\mathfrak{E}$ will be so very small that their influence on the deviasafely neglected: under these conditions, eq. 880, becomes

$$n\ \delta = \frac{\mathfrak{B}.\sin\zeta + \mathfrak{C}.\cos\zeta + \mathfrak{D}.\sin 2\zeta}{1 + \mathfrak{B}.\cos\zeta - \mathfrak{C}.\sin\zeta + \mathfrak{D}.\cos 2\zeta}. \quad (329)$$

bservations on *two headings*, Art. 325, the coefficients , and $\mathfrak{D}$ are obtained; substituting them in (329), we can successively calculate the deviations on the 32 *magnetic* points (ζ); and then by means of a Napier diagram obtain the deviations on the compass courses (ζ').

With the compass the center of symmetry of horizontal soft iron, as just stated, and $\mathfrak{A}$ and $\mathfrak{E}$ therefore negligible; and further, considering $\cos\delta = 1$, as above, eq. (326) becomes

$$\sin\delta = \mathfrak{B}.\sin\zeta' + \mathfrak{C}.\cos\zeta' + \mathfrak{D}\,(\sin 2\zeta' \cos\delta + \cos 2\zeta' \sin\delta); \quad (330)$$

or $$\sin\delta = \mathfrak{B}.\sin\zeta' + \mathfrak{C}.\cos\zeta' + \mathfrak{D}\,\sin(2\zeta' + \delta). \quad (331)$$

Or, making a further approximation, (331) reduces to

$$\delta = B.\sin\zeta' + C.\cos\zeta' + D.\sin 2\zeta'. \quad (332)$$

$\mathfrak{B}$ and $\mathfrak{C}$ are found by means of (324) and (325) from observations on *one heading*, Art. 326; then assuming a value for $\mathfrak{D}$, and inserting these quantities in (331) or (332), we may calculate the deviations on the 32 *compass* points (ζ'): but to arrive at these equations for use on *one heading*, that is, (324), (325), (331), and (332), so many assumptions, approximations, and omissions had to be made, that the statement in Art. 326 relative to their yielding only crude results is entirely justified. Eqs. (324) and (325) are fully adequate, however, for the purposes specified in Art. 326.

328. Changes in a ship's magnetism after launching.— While building, an iron ship is much like a steel bar under the influence of a powerful magnet, or an accumulator connected with a battery—it is being charged up; and like every mass of iron or steel under similar circumstances, it receives more than it can retain. The *sur*-charge dissipates in time—a long or short period according to the hardness of the metal—and eventually only the quantity of magnetism remains that gives distinctive character to the ship.

The coefficients 𝔅 and ℭ are the most mobile of the features to indicate magnetic change; and their phases in two cases will be presented to illustrate the matter.

First, the ACHILLES of the British Navy: built and plated in dock, head S. 52° E.; floated Dec., 1863, and secured in River Medway with head S. 62° E., for purpose of taking in machinery, iron masts, and to equip for sea; in March, 1864, made a short trial trip in river and then resumed former moorings, but with head N. 62° W.; in October, 1864, being completed, was swung and began sea service: her record from date of launching for six years is given in

TABLE 81.

CHANGE IN SHIP'S MAGNETISM AFTER LAUNCHING.

(From Capt. Evans, R. N.)

Date.	Condition of Ship.	𝔅	Angles.	ℭ	Angles.
(1)	(2)	(3)	(4)	(5)	(6)
Dec., 1863	In dock, head S. 52° E.	+.464	27° 42′	+.323	18° 50′
Sept., 1864	Moored, head N. 62° W.	+.377	22 12	+.037	2 12
Oct. 12, 1864	Moored, head S. 55° E.	+.355	20 48	+.062	3 36
Oct. 13, 1864	Completely swung	+.362	21 15	+.051	2 54
Dec., 1864	" "	+.361	21 12	+.123	7 05
April, 1865	" "	+.322	18 48	+.191	11 0
June, 1865	" "	+.288	16 45	+.124	7 06
Aug., 1866	" "	+.308	17 55	+.138	7 55
April, 1867	" "	+.309	18 0	+.146	8 24
April, 1868	" "	+.314	18 18	+.132	7 36
June, 1869	" "	+.305	17 45	+.127	7 18

Table 81. The coefficients 𝔅 and ℭ are given in cols. (3) and (5); but their fluctuations will be better appreciated by considering them the sines of angles, and giving the corresponding values of these in cols. (4) and (6) respectively.

It will be seen by cols. (3) and (4) that for the first year and a half 𝔅 slowly and steadily decreased, and then during the following four years had quite a uniform value of about 18°: ℭ, on the contrary, cols. (5) and (6), dropped suddenly in ten months to one-ninth its original value—fluctuated about this minimum for a year—then rose—and finally fell again, acquiring a uniform value of about 7° 30′ during the last four years of the record.

Second, the ROYAL CHARTER: this was a full square-rigged (iron) ship, 326 feet long, 41.5 feet beam, with auxiliary steam power and a hoisting propeller; she was built head N. 50° W.; launched Sept., 1855; equipped and loaded at Liverpool; and then sailed for Australia. Dr. Scoresby took passage for the special purpose of making observations of her magnetism; the data of Table 82 are

TABLE 82.

CHANGE IN SHIP'S MAGNETISM AFTER LAUNCHING

Place of Swing	Date of Swing	Coefficients					
		A	*B*	*C*	*D*	*E*	λ
Liverpool...	Jan., 1856	−0° 27′	−3° 48′	−19° 42′	+6° 59′	−0° 52′	.936
Melbourne.	May, 1856	−1 27	−1 11	− 8 59	+6 23	−0 16	Mean of
Liverpool...	Aug., 1856	−0 03	−1 06	− 3 22	+6 10	+0 56	16 points.

from him, and may therefore be relied upon. The ship sailed January, 1856—reached Australia in May—and returned to Liverpool in August, all in the same year: she was completely swung just before going—upon arrival at Melbourne—and again after return; the coefficients derived from analysis of the deviations are given in Table 82. The passage out was a stormy one, and it will be se

values of B and C that it sufficed to shake most of the *sur*-charge out of the ship during the first four months of her sea life: on the other hand, the values of D denote a very uniform quadrantal deviation, notwithstanding extreme geographical change and the rough treatment by the sea; A and E show considerable change—*for them.*

329. Separation of the two parts of 𝔅 and ℭ.—From eqs. (116) and (117), p. 879, we have

$$\mathfrak{B} = \frac{1}{\lambda} . c . \tan \theta + \frac{1}{\lambda} \cdot \frac{P}{H}. \quad (333); \quad \mathfrak{C} = \frac{1}{\lambda} . f . \tan \theta + \frac{1}{\lambda} \cdot \frac{Q}{H}. \quad (334)$$

By Fig. 434 and Art. 268, it will be seen that P and Q represent the magnetic effect of hard iron—a disturbing force that is practically constant.

By Figs. 441 and 442, page 757, c and f represent the effect of vertical soft iron—transient magnetism, whose power depends on the vertical component of the Earth's intensity, and therefore increases or decreases with the Dip.

For two reasons, it is desirable to know the degree of each effect: first, to compensate the compass properly, for we cannot counteract the effect of soft iron by a permanent magnet, nor that of hard iron by a Flinders' bar; and second, for predicting changes in the deviations as the ship traverses different latitudes.

Of the quantities in (333) and (334), θ and H are given on Magnetic Charts; λ is practically constant—it may be determined once for all on completion of the ship; and 𝔅 and ℭ are computed from tables of deviations. Suppose the ship swung in two widely separated places, and designate the quantities proper to each by $\mathfrak{B}_1$, $\mathfrak{C}_1$, θ_1, H_1, and $\mathfrak{B}_2$, $\mathfrak{C}_2$, θ_2, H_2; then from (333) and (334) we have

$$\mathfrak{B}_1 = \frac{c}{\lambda} . \tan \theta_1 + \frac{P}{\lambda} \cdot \frac{1}{H_1}. \quad (335); \quad \mathfrak{B}_2 = \frac{c}{\lambda} . \tan \theta_2 + \frac{P}{\lambda} \cdot \frac{1}{H_2}. \quad (336)$$

$$\mathfrak{C}_1 = \frac{f}{\lambda} . \tan \theta_1 + \frac{Q}{\lambda} \cdot \frac{1}{H_1}. \quad (337); \quad \mathfrak{C}_2 = \frac{f}{\lambda} . \tan \theta_2 + \frac{Q}{\lambda} \cdot \frac{1}{H_2}. \quad (338)$$

se to get $\frac{P}{\lambda}, \frac{Q}{\lambda}, \frac{c}{\lambda}$, and $\frac{f}{\lambda}$, we obtain:

$$\frac{P}{\lambda}=\frac{\{H_1H_2\}[\mathfrak{B}_2 \tan \theta_1-\mathfrak{B}_1 \tan \theta_2]}{H_1 \tan \theta_1-H_2 \tan \theta_2}. \quad . \quad (339)$$

$$\frac{Q}{\lambda}=\frac{\{H_1H_2\}[\mathfrak{C}_2 \tan \theta_1-\mathfrak{C}_1 \tan \theta_2]}{H_1 \tan \theta_1-H_2 \tan \theta_2}. \quad . \quad . \quad (340)$$

$$\frac{c}{\lambda}=\frac{H_1\mathfrak{B}_1-H_2\mathfrak{B}_2}{H_1 \tan \theta_1-H_2 \tan \theta_2}. \quad . \quad . \quad . \quad . \quad . \quad (341)$$

$$\frac{f}{\lambda}=\frac{H_1\mathfrak{C}_1-H_2\mathfrak{C}_2}{H_1 \tan \theta_1-H_2 \tan \theta_2}. \quad . \quad . \quad . \quad . \quad . \quad (342)$$

In (333) and (334) the terms representing *hard* iron are

$$\frac{P}{\lambda . H} \text{ and } \frac{Q}{\lambda . H}, \quad . \quad . \quad . \quad . \quad . \quad (343)$$

components toward the bow and side respectively: squaring each, we have

$$\left(\frac{P}{\lambda . H}\right)^2 \quad . \quad (344) \quad \text{and} \quad \left(\frac{Q}{\lambda . H}\right)^2; \quad . \quad (345)$$

whence their resultant R', or the total effect of *hard* iron, is

$$R'=\sqrt{\left(\frac{P}{\lambda . H}\right)^2+\left(\frac{Q}{\lambda . H}\right)^2}. \quad . \quad . \quad . \quad (346)$$

Similarly the terms representative of *soft* iron in (333) and (334) are

$$\frac{c}{\lambda} \tan \theta \text{ and } \frac{f}{\lambda} \tan \theta, \quad . \quad . \quad . \quad (347)$$

to bow and side respectively; whence their resultant R'' is

$$R''=\sqrt{\left(\frac{c}{\lambda} . \tan \theta\right)^2+\left(\frac{f}{\lambda} . \tan \theta\right)^2}. \quad . \quad . \quad (348)$$

Since the resultant of the forces that produce semicircular deviation lies in the direction whose angle is given by $\tan\alpha = \frac{\mathfrak{C}}{\mathfrak{B}}$, by analogy the direction of the resultant of each kind, that of hard iron denoted by α', and that of soft by α'', will be given by

$$\tan\alpha' = \frac{\frac{1}{\lambda}\cdot\frac{Q}{H}}{\frac{1}{\lambda}\cdot\frac{P}{H}}. \quad . \quad . \quad (349); \qquad \tan\alpha'' = \frac{\frac{1}{\lambda}.f.\tan\theta}{\frac{1}{\lambda}.c.\tan\theta}. \quad . \quad . \quad (350)$$

Eqs. (339) to (342) give the separate effects of hard and soft iron, and (346) to (350) give the *resultant* of each and its direction.

The matter will be illustrated by observations of the Navigator of the U. S. S. SAN FRANCISCO, made near the port of San Francisco, California, and at Payta, Peru: the ship is a steel cruiser of 4000 tons with a primary battery of 12 guns:

Data for San Francisco, Cal.	Data for Payta, Peru.
$\mathfrak{B}_1 = -.154$	$\mathfrak{B}_2 = -.349$
$\mathfrak{C}_1 = +.049$	$\mathfrak{C}_2 = +.106$
$H_1 = +.258$	$H_2 = +.332$
$\theta_1 = +62°$; $\tan = +1.881$	$\theta_2 = +5°$; $\tan = +.087$

Substituting these in formulas (339) to (342), they become:

$$\frac{P}{\lambda} = \frac{\{(+.258)(+.332)\}[(-.349)(+1.881)-(-.154)(+.087)]}{(+.258)(+1.881)-(+.332)(+.087)}$$

$$= \frac{-.0546}{+.456} = -.120. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (351)$$

$$\frac{Q}{\lambda} = \frac{\{(+.258)(+.332)\}[(+.106)(+1.881)-(+.049)(+.087)]}{(+.258)(+1.881)-(+.332)(+.087)}$$

$$= \frac{+.01657}{+.456} = +.0363. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (352)$$

$$\frac{c}{\lambda}=\frac{(+.258)(-.154)-(+.332)(-.349)}{(+.258)(+1.881)-(+.332)(+.087)}=\frac{+.076136}{+.456}$$
$$=+.166. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (353)$$

$$\frac{f}{\lambda}=\frac{(+.258)(+.049)-(+.332)(+.106)}{(+.258)(+1.881)-(+.332)(+.087)}=\frac{-.02255}{+.456}$$
$$=-.0494. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (354)$$

To determine the effect, separately, of hard and of soft iron at San Francisco, substitute the data for that place, and the values of $\frac{P}{\lambda}$, $\frac{Q}{\lambda}$, $\frac{c}{\lambda}$, and $\frac{f}{\lambda}$ just found, in (335) and (337), whence

$$\mathfrak{B}_1=(+.166)(+1.881)+(-.120)\left(\frac{1}{+.258}\right)$$
$$=(+.312)+(-.465). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (355)$$

$$\mathfrak{C}_1=(-.0494)(+1.881)+(+.0363)\left(\frac{1}{+.258}\right)$$
$$=(-.092)+(+.141). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (356)$$

To interpret these results: in (355) the quantity

$$(+.312)=\frac{c}{\lambda}.\tan\theta_1 \quad . \quad . \quad . \quad . \qquad (357)$$

is the longitudinal component of the magnetic effect of vertical soft iron; and as it is plus, c is in the position of (26) or (23) of Fig. 441, that is, it is either forward of the compass with the upper pole acting, or abaft with the lower pole acting: the quantity

$$(-.465)=\frac{P}{\lambda}\cdot\frac{1}{H_1} \quad . \quad . \quad . \quad . \quad . \qquad (358)$$

is the longitudinal component of the magnetic effect of hard iron; and as it is minus, P is abaft the compass, or the pull is toward the stern.

Similarly, in (356) the quantity

$$(-.092)=\frac{f}{\lambda}\tan\theta_1 \quad . \quad . \quad . \quad . \quad . \quad (359)$$

is the transverse component of the soft-iron effect; and as it is minus, f is in the position (28) or (29) of Fig. 442—either to starboard of the compass with lower pole acting, or to port with upper pole acting: the quantity

$$(+.141)=\frac{Q}{\lambda}\cdot\frac{1}{H_1} \quad . \quad . \quad . \quad . \quad . \quad (360)$$

is the transverse component of hard-iron effect, and as it is plus, Q is to starboard of the compass, or the pull is toward that side.

To obtain the resultant magnetic effect of hard and of soft iron separately, and the direction of each, insert the numerical quantities of (357) to (360), which are applicable to the port of San Francisco, in (346) to (350); whence

$$R'=\sqrt{(-.465)^2+(+.141)^2}=.485. \quad . \quad (361)$$

$$\tan\alpha'=\frac{+.141}{-.465}=3.032;\ \therefore\ \alpha'=161^\circ\ 45'. \quad . \quad (362)$$

$$R''=\sqrt{(+.312)^2+(-.092)^2}=.325. \quad . \quad . \quad (363)$$

$$\tan\alpha''=\frac{-.092}{+.312}=0.295;\ \therefore\ \alpha''=343^\circ\ 35'. \quad . \quad (364)$$

Eqs. (361) and (362) give the total magnetic force of hard iron and its direction; and (363) and (364) give the same quantities for the vertical soft iron.

The angles α' and α'' found in the tables corresponding to tangents 3.032 and 0.295 are $71^\circ\ 45'$ and $16^\circ\ 25'$: but as $\tan=\frac{\sin}{\cos}$, since this is $\frac{+\sin}{-\cos}$ in (362), the angle α' is really in the second quadrant, or $90^\circ+71^\circ\ 45'$; and since it is

$\frac{-\sin}{+\cos}$ in (364) the angle α'' is in the fourth quadrant, or $360° - 16° \ 25' = 343° \ 35'$.

330. To compute a Table of Deviations for change of geographical position.—With the data afforded by eqs. (351) to (354), the navigator can prepare in advance a table of deviations for any region it is contemplated visiting; and instances are not rare where it would be advisable to do so. Suppose a cruise in Alaskan waters were in view: fog and rain prevail there, and the opportunities for swinging ship are not as good as in middle latitudes. From the swing at San Francisco, the exact coefficients are known; $\mathfrak{A}$ and $\mathfrak{E}$ vary but little with change of place, and in any event they affect the result but slightly; $\mathfrak{D}$ is practically constant; so that $\mathfrak{B}$ and $\mathfrak{C}$ alone are required for the new locality. To determine them for, say Sitka, designate the quantities proper to that place by θ_3, H_3, $\mathfrak{B}_3$, $\mathfrak{C}_3$: then $\theta_3 = 75°$ and $\tan \theta_3 = 3.732$; $H_3 = +0.155$; substituting these and the values from eqs. (351) to (354) in formulas (333) and (334), p. 969, we obtain $\mathfrak{B}_3$ and $\mathfrak{C}_3$: with these and $\mathfrak{A}$, $\mathfrak{D}$, $\mathfrak{E}$ from the swing at San Francisco, a complete table of deviations may be computed by means of formula (120), p. 880, arranged in any convenient tabular Form similar to Table 74.

331. The Gaussin Error.—When an electric current is increasing or diminishing, it experiences a retard, so that its strength at any instant is not equal to the electromotive force; this is due to inductance, or self-induction, as it was formerly called—an opposing force in the medium evoked by the current itself: the alternating current is a series of growths and declines, and although current and force have the same frequency, still their curves do not coincide—but are out of step, due to inductance; this is illustrated by Fig. 526, where the solid curve represents electromotive force and the dotted one current.

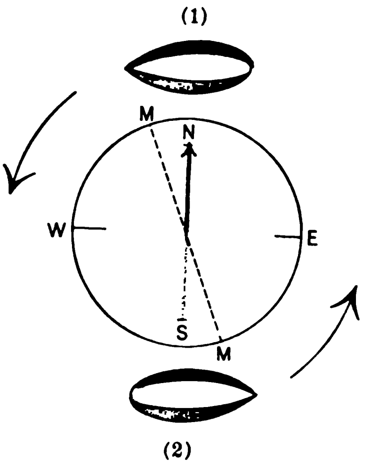

Fig. 527.

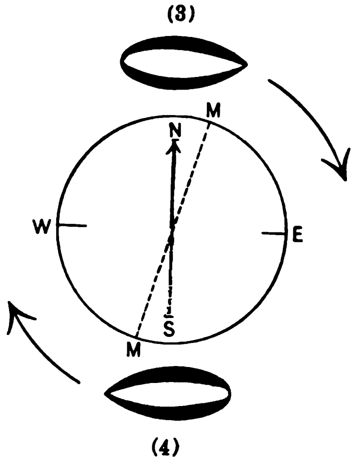

Fig. 528.

(To face p. 974.)

Something analogous to this occurs in the transient magnetism of a ship: whether she steams in a circle or pursues a straight course, the amount induced at any instant is not that of the Earth's field at the time and place,

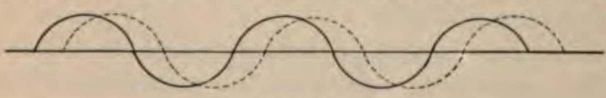

FIG. 526.

but the amount proper to some previous time in a field already passed over—there is a lag in the induction.

In the case of an armature coil revolving between the field magnets of a dynamo, Fig. 131, it cuts the lines of force *normally* in a horizontal line, and the maximum current should pass there, but experience shows it to be otherwise—the brushes must be set at the ends of a line making an angle with the horizontal—there is a lag: so, when a ship swings with either helm, Figs. 527 and 528, there is a lag in the maximum induction—it occurs a little after her course has been *normal* to the direction (H) of the lines of the Earth's horizontal intensity, as at $\overline{MM}$; and the more rapid the swing, the greater the angle $\overline{MM}$ will make with $\overline{NS}$, or the greater the difference between the deviations obtained from each helm.

Therefore, for accuracy, the ship should swing with both helms, and their mean be taken; or if with one only, it should be done slowly, resting four or five minutes on each point, to afford induction full effect.

As the ship thus swings through 360°, it is evident that the polarities of this transient magnetism will take every possible position in the mass of her iron—from bow to stern and the reverse, from side to side, and at every angle with the keel; also, it will have all degrees of influence, according to the heading of the ship: the case is entirely analogous to that of the revolving armature in the field of the dynamo magnets. If the ship steer east or west for

some time, by Figs. 527 and 528, it is seen that on an easterly course, as at (2) and (3), the starboard side becomes invested with blue polarity, and the port side with red, while, on a westerly course, as at (1) and (4), the conditions are the converse of this.

The compass is equally distant from both sides—in line with their polarities—and therefore unaffected while heading east or west; but as soon as the ship changes course, the transient magnetism causes temporary error: the direction of this may be inferred from the figures; for instance, in (1) and (4), heading west if the ship turns north, both the starboard and port sides will repel the north and south ends respectively of the needle, and a westerly deviation may be anticipated; if, on the contrary, she turns south, an easterly deviation should be looked for.

In (2) and (3), ship heading east, the converse of this must be expected for the same turns.

CHAPTER XXV.

GRAPHIC METHODS OF DEALING WITH THE DEVIATIONS.

Section One: Napier's Diagram.

332. Principle and use of the diagram.—Almost every phenomenon admits of representation by coördinates: the tide rises and falls as time goes on, and intervals of the latter may be denoted by abscissas, while heights of the former by ordinates; a curve drawn through the ends of these indicates the successive ebb and flow. So with the deviations: to different azimuths of the ship's head correspond definite degrees of deviation; and the one may be represented by abscissas, the other by ordinates.

Consider Fig. 529: let $\overline{A_0A_4}$ represent a part of the graduated rim of the compass-card straightened out; A_0, A_1, A_2 . . . A_4 are magnetic courses differing from each other by 10°; $\overline{A_1E_1}=4°$, $\overline{A_2E_2}=8°$ . . . $\overline{A_4E_4}=7°\ 30'$; these lines are perpendicular to $\overline{A_0A_4}$, and represent the deviations on the preceding courses: the curve $A_0E_1 \ldots E_4$ drawn through their ends shows the varying distance of the needle from the meridian as the ship swings round. If we consider them compass courses, we might still designate these by A_0, A_1, $A_2 \ldots A_4$, but the deviations being different for the *same* point, must be represented by another system of ordinates: let these be inclined at a common angle of 45° to the line of abcissas $\overline{A_0A_4}$, and make $\overline{A_1B_1}$, $\overline{A_2B_2} \ldots \overline{A_4B_4}$

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magnetic courses, the dotted for compass courses; each system is inclined 60° to the base line and to each other, as shown between N.W and N.W. by N. See Fig. 499.

To trace the curve: suppose the deviations to be for *compass* courses—that is, ζ'_{18} = S.SW. = 202° 30′; δ_{18} = +32° 0′; ζ'_{19} = SW. by S. = 213° 45′; δ_{19} = +33° 45′; ζ'_{8} = E. = 90°; δ_{8} = −18° 0′; ζ'_{9} = E. by S. = 101° 15′; δ_{9} = −21° 30′: take a pair of dividers, open them to a distance of 32° on the vertical line, place one point at S.SW., and where the other falls on the dotted ordinate to the right (because 32° is plus), make a small cross and enclose it by a circle; do the same for +33° 45′ at SW. by S.; and for every other value of the deviation, observing that those marked minus are to be laid off to the left of the base line on the dotted diagonals, from their respective points, as −18° 0′ at East, and −21° 30′ at E. by S.: then by means of a spline draw a curve *through* all the crosses, if it can be done; if not, then a midway line through the space over which they are scattered. A wavy line is the symbol of the development of most phenomena: the changes of temperature and pressure of the air, for example, are appropriately represented by such curves.

The phenomenon of deviations is preëminently typified by an undulatory line, and when a series of values falls irregularly outside such a curve, it is probably due to errors of observation; so that a midway line through the space they cover will more nearly represent what actually occurred than a curve that might be *un*naturally bent through *all* the points of observation.

If the deviations be for magnetic courses, the mode of tracing the curve is the same—using the solid diagonals.

Napier's Diagram may be called an expeditious method of least squares, in so far that it gives the most probable values of the quantities sought; but like all solutions by construction, it lacks the accuracy of methods by com-

equal to the deviations; they all terminate on the same curve, but at different points from those of the magnetic courses, thus showing the difference in the deviations for the *same* point.

Suppose, now, we wish to know the compass course corresponding to a magnetic course of 25°, upon which

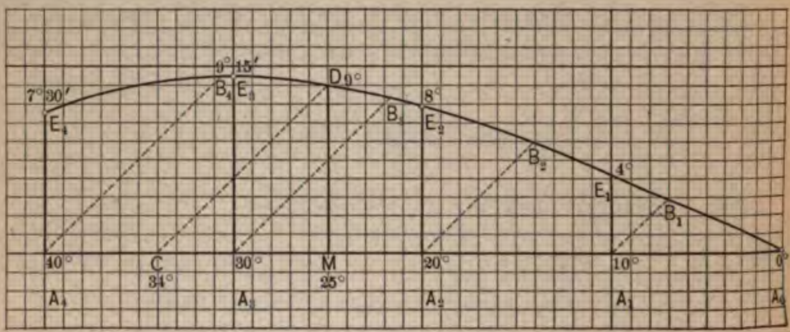


FIG. 529.

there is a westerly deviation of 9°: on the line of azimuths, at $M=\zeta=25°$, erect the perpendicular $\overline{MD}=\delta=-9°$, and draw $\overline{DC}$ at an angle of 45° with $\overline{A_0A_4}$; it meets the latter at $C=34°=\zeta'$; and as $\zeta'=\zeta+\delta$, hence $34°=25°+9°$; we have thus simply "*constructed*" the formula expressing the relation between the compass course, the magnetic course and the deviation.

Instead of one system of ordinates being perpendicular to the base line and the other inclined at an angle of 45° as in Fig. 529, both systems may be equally inclined to and to each other—that is, have an angle of 60° between them—and by this transition Napier's Diagram is formed— Fig. 499. This requires but little explanation: the vertical line represents the rim of the compass-card straightened out; the points and degrees are engraved along its extent as a base line; the solid diagonals are the ordina

putation: it has some advantages, however, over the latter. The method by least squares, which is that of Form 10, requires observations on *equidistant* points—4, 8, 16, or 32; but for Napier's Diagram they may be taken with any irregularity, and even from day to day as the ship swings to wind or tide, or in any other way that produces different headings: but it must be remembered that during the period covered by such adventitious observations, the deviations may change considerably, and that the results can never have the accuracy of a complete series begun and finished within a few hours.

Any number of observations over four will give a curve; but it is liable to great error with only a few; never less than eight should be used for even moderate deviations; while for abnormal ones—say over 40°—the thirty-two observations are requisite: in all cases, they should be pretty evenly distributed around the circle.

333. To convert compass courses into magnetic, and the converse.—The principle of this procedure was explained in the previous article, so that it only remains to give specific rules for ready guidance.

Given—compass course; required—magnetic: look for the compass course on the vertical line—from it proceed to the curve by the *dotted* ordinate passing through it, or by a line parallel to a dotted ordinate; from the point of the curve reached, return to the vertical line by a *solid* ordinate, or a line parallel to one, and the point of the vertical line arrived at is the magnetic course.

Thus: for compass course W.SW. = 247° 30′, Fig. 499, a dotted ordinate does pass through it, but from its point of intersection with the curve no solid ordinate returns, so one is drawn; this meets the vertical line at 277° 30′, which is the corresponding magnetic course.

Given—magnetic; required—compass course: look for the former on the vertical line—from it proceed to the curve

by a *solid* ordinate—and from the point reached, return to the vertical line by a *dotted* ordinate; the point arrived at will be the required compass course. Thus: for magnetic course E. ½ S. = 95° 30′, Fig. 499, no solid ordinate passes through it, so one is drawn; from its point of intersection with the curve no dotted ordinate returns, and one is drawn; it meets the vertical line at 116° 45′, which is the corresponding compass course. GENERALLY: whichever course is given, find it on the vertical line—place one point of a pair of dividers on it—open them until the other point meets the curve in the direction of a *dotted* ordinate if a compass course be given, but in the direction of a *solid* ordinate if a magnetic course be given; let this point of the dividers rest on the curve where it meets it, and, using this point as a center, swing the other point, which is on the given course, until it cuts the vertical line again—as if drawing an arc, of which the intercepted part of the vertical line is a chord; the second point on the vertical line is the course sought: we have thus to deal with an equilateral triangle of which the two sides represent the compass and magnetic courses respectively, and the base the deviation proper to them.

The following couplets will assist the memory in using the rules:

(1) "From compass course, magnetic course to follow,
Depart by *dotted*, and return by *solid*."

(2) "But if you seek to steer a course allotted,
Take *solid* from chart, and keep her head on dotted."

When the deviations are computed by formula (120), p. 880, in which the exact coefficients are expressed in parts of radius, and magnetic courses are used, a curve must be plotted on Napier's Diagram, in order to form a Table of Deviations on compass courses.

Section Two: The Dygogram, or Representation of the Direction and Amount of the Force Acting on the Compass.

334. The principle and value of the Dygogram.—As a steel ship is a magnet, a compass placed on it is under the influence of one of her poles: the strength of the dominant pole is pitted against the Earth's horizontal intensity, and the needle quivers in the balance between them—inclining in different directions according to the varying power of both forces. This condition is portrayed in Fig. 530:

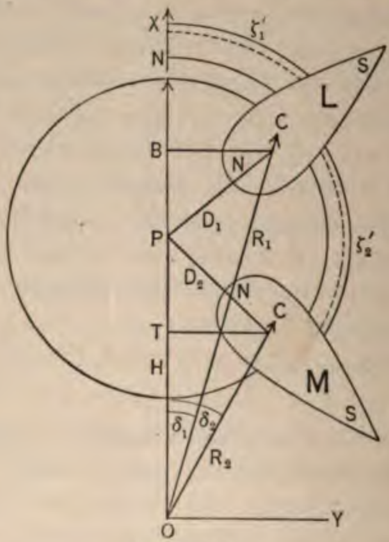


FIG. 530.

$\overline{OP} = H$ = the direction and amount of the Earth's horizontal intensity; $\overline{PC} = D_1$ = that of the ship's disturbing force—supposed due to a north pole in the after body; and $\overline{PC} = D_2$ = the same quantity for the position M of the ship. Drawing $\overline{OC} = R_1$ and $\overline{OC} = R_2$, we thereby compound H with D_1 and D_2, and obtain the resultants of

these forces on the headings ζ_1' and ζ_2'; their directions, δ_1 and δ_2, are the deviations proper to those courses.

For any heading ζ_n' we may by a similar construction obtain R_n and δ_n, and thus arrive at a graphical representation of the force acting upon the compass and its direction—the force-and-angle diagram, or dynamo-goniogram, or Dygogram as it has been abbreviated.

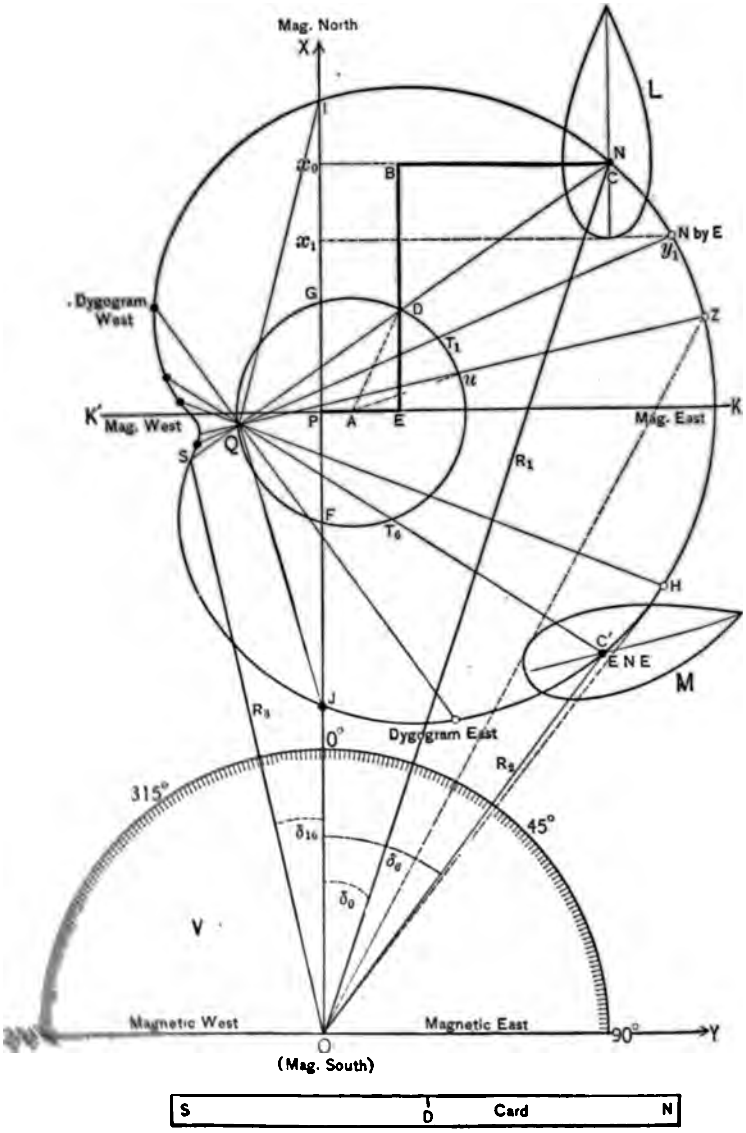
The adaptation of the Dygogram by Captain Colongue of the Russian Navy will form the basis of what follows. Besides affording a speedy solution of many problems of the deviations, the Dygogram appeals more strongly to the understanding, and presents the conditions and forces in action more clearly to view than formulas; also, it is an easy check on computations: if the results of both methods agree, it is fair to presume that the procedure has been correct; if discordant, one or the other is at fault, and it is a warning to inquire into the source of error.

Although its usefulness is great, still it must not be forgotten that it is a method by construction, and carries with it all the little inaccuracies of such: it is an invaluable pictorial solution, but should never replace computation by formulas.

335. Construction of the Dygogram.—The coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, representing the magnetic force of the ship, are used in conjunction with the Earth's intensity H, for the construction; and the mechanical performance of this operation will first be described, leaving the reasons for the procedure to be stated when the curve is before the eye.

In order to be specific, values are given to the coefficients as follows: $\mathfrak{A} = +.055$; $\mathfrak{B} = +.245$; $\mathfrak{C} = +.345$; $\mathfrak{D} = +.165$; and $\mathfrak{E} = +.085$.

In Fig. 531 (*which is constructed on a scale of millimeters*), draw $\overline{OX}$ to magnetic north as a meridian, and $\overline{OY}$ to magnetic east, that is, rectangular axes with origin of coördinates at O. From O lay off $\overline{OP}$ toward the north



Mag. North
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B
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N by E
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Dygogram West
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D
Z
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K'
Mag. West
P
A
E
K
Mag. East
Q
S
R_1
F
T_0
H
C'
E N E
M
J
R_3
Dygogram East
R_2
0°
315°
45°
δ_{16}
δ_6
δ_0
V
Magnetic West
Magnetic East
90°
Y
O
(Mag. South)
S
D
Card
N

FIG. 531.

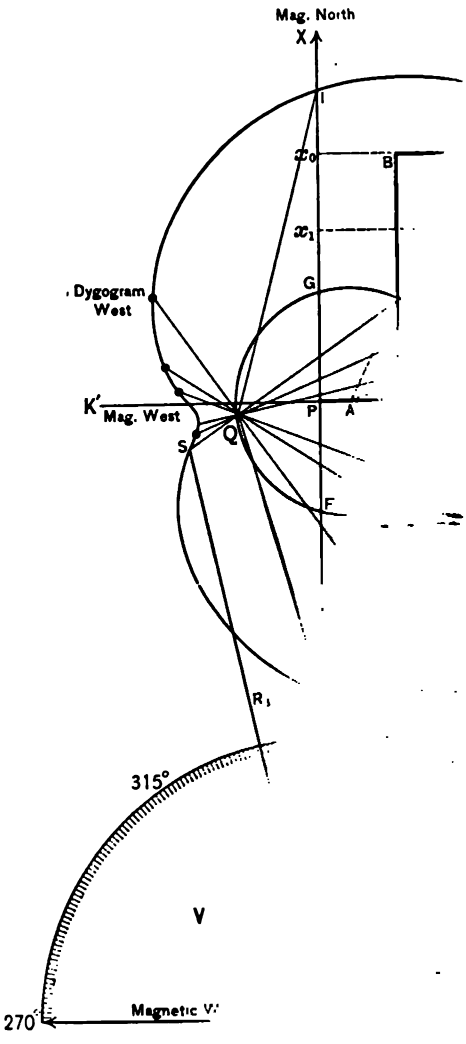
and make it equal to unity = one decimeter = 100 millimeters; and through P draw $\overline{KK'}$ parallel to $\overline{OY}$.

Each coefficient may be either plus or minus.

To construct $+\mathfrak{A}$: take its amount from the millimeter-scale and lay it off on $\overline{KK'}$ from the point P toward the right or east and mark its end A; then $\overline{PA} = +.055$ mm.; if it were $-\mathfrak{A}$, it would be laid off on $\overline{KK'}$ from P toward the left or west. For $+\mathfrak{E}$: measure its amount by scale and lay it off on $\overline{KK'}$ from the point A toward the east and mark its end E; then $\overline{AE} = +.085$ mm.; if it were $-\mathfrak{E}$, it would be laid off from A on $\overline{KK'}$ toward the west, whether the point A fell on the east or west of $\overline{OX}$. For $+\mathfrak{D}$: draw a parallel to $\overline{OX}$ through the point E; lay off the amount of $\mathfrak{D}$ on this parallel toward the north from the point E, and mark the end D; then $\overline{ED} = +.165$; if it were $-\mathfrak{D}$, it would be laid off from the point E toward the south on the parallel to $\overline{OX}$ through the point E, whether E fell east or west of $\overline{OX}$. For $+\mathfrak{B}$: lay off its amount from the point D on the parallel to $\overline{OX}$ through E toward the north and mark its end B; then $\overline{DB} = +.245$; if it were $-\mathfrak{B}$, it would be laid off from the point D toward the south on the parallel through E whether D itself fell above or below $\overline{KK'}$. For $+\mathfrak{C}$: through the point B draw a parallel to $\overline{KK'}$, and on this parallel measure the amount of $\mathfrak{C}$ from the point B toward the east and mark the end N; then $\overline{BN} = +.345$; if it were $-\mathfrak{C}$, it would be laid off from B on the parallel through that point, toward the west, whether B fell north or south of the line $\overline{KK'}$.

Open a pair of pencil dividers to the length of the dotted line $\overline{AD}$, and with A as a center describe the circumference $DFQG$, called the generating circle. Draw the line $\overline{ND}$ and prolong it to S, so that $\overline{DS} = \overline{ND}$; and mark the second intersection of NS with the generating circle, Q.

Take a protractor, such as that shown at V, and place its diameter $\overline{OY}$ along the line $\overline{NS}$ with the center O at Q,



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ight angles to $\overline{PC}$, Fig. 530, extended vertically nd from the compass pivot, with fore-and-aft through a slot in the keel: then as the ship swung —in reality on the circumference of the generating the point of the pencil would trace out on paper the ship's bottom the dygogram curve *NHS*, Fig. and thus superpose the semicircular deviation on the phery of the quadrantal circle, which is what is actually by the construction of Fig. 531.

The contour of the dygogram varies with every ship rding to her magnetic character, and in the same ship different spots; as a result, we have the shifting of the points of the curve: *magnetic* North, South, East, and West are one thing—fixed; the *dygogram* points N., N. by E., N.NE., etc., or the successive magnetic headings of the ship proper to certain parts of the curve are quite another—movable with reference to the coördinate axes.

If, in Fig. 530, we multiply R_1 successively by $\cos\delta_1$ and $\sin\delta_1$, we thereby obtain its components $\overline{OB}$ and $\overline{BC}$ in the axis $\overline{OX}$ and parallel to $\overline{OY}$: by reference to eqs. (118) and (119), p. 879, which are reproduced below, it is seen that in Fig. 530 we have the geometrical construction of those formulas; for $\frac{H'}{\lambda H}$ is the combined force of Earth and Ship (H') in terms of the mean force to magnetic north ($\lambda . H$); therefore $\frac{H'}{\lambda . H}$ corresponds to $\overline{OC} = R_1$ in Fig. 530, and hence $\frac{H'}{\lambda . H} . \cos\delta$ is the same as $R_1 \cos\delta_1 = \overline{OB}$, and $\frac{H'}{\lambda H}\sin\delta$ the same as $R_1 \sin\delta_1 = \overline{BC}$.

Following are the equations above referred to with their numbers on page 879.

$$\frac{H'}{\lambda . H} . \cos\delta = 1 + \mathfrak{B} . \cos\zeta - \mathfrak{C} . \sin\zeta + \mathfrak{D} . \cos 2\zeta - \mathfrak{E} . \sin 2\zeta. \quad (118)$$

$$\frac{H'}{\lambda . H} . \sin \delta = \mathfrak{A} + \mathfrak{B} . \sin \zeta + \mathfrak{C} . \cos \zeta$$
$$+ \mathfrak{D} . \sin 2\zeta + \mathfrak{E} . \cos 2\zeta. \quad (119)$$

If in these we make $\zeta = 0$, that is, have the ship's head north magnetic, they become, since $\sin 0° = 0$ and $\cos 0° = 1$:

$$\frac{H'}{\lambda . H} . \cos \delta = 1 + \mathfrak{B} + \mathfrak{D} = x_0, \quad . \quad . \quad . \quad (365)$$

$$\frac{H'}{\lambda . H} . \sin \delta = \mathfrak{A} + \mathfrak{C} + \mathfrak{E} = y_0, \quad . \quad . \quad . \quad (366)$$

and thus it is further seen that the procedure of Art. 335 was but the geometrical construction of formulas (365) and (366)—the former (365) being equal to $\overline{OP}$ $(=1)$ $+ \mathfrak{D}(=\overline{ED} = +.165$ mm.$) + \mathfrak{B}(=\overline{DB} = +.245$ mm.), all in the direction of the axis $\overline{OX}$; and the latter (366) being equal to $\mathfrak{A}(=\overline{PA} = +.055$ mm.$) + \mathfrak{E}(=\overline{AE} = +.085$ mm.) $+ \mathfrak{C}(=\overline{BN} = +.345$ mm.), all in the direction of the axis $\overline{OY}$. Eq. (365) represents the line $\overline{ON}$, Fig. 531, projected on the meridian, or $R_1 . \cos \delta_0 = \overline{Ox_0}$, and eq. (366) the same line projected to magnetic east or $R_1 . \sin \delta_0 = \overline{x_0 N}$: the dygogram, therefore, is the geometrical construction of formulas (118) and (119).

By making $\zeta = 0$ in these equations, we obtained the coördinates x_0, y_0 in (365) and (366) for one point of the dygogram: substituting in (118) and (119) other values of ζ, we may obtain the necessary number of points for completely tracing the curve; thus the coördinates x_1, y_1 for N. by E. and x_8, y_8 for East are here given as examples:

$$x_1 = 1 + [(+.245)(+.981)] - [(+.345)(+.195)]$$
$$+ [(+.165)(+.924)] - [(+.085)(+.382)] = +1.284 \text{ mm.}$$

$$y_1 = [+.055] + [(+.245)(+.195)] + [(+.345)(+.981)]$$
$$+ [(+.165)(+.382)] + [(+.085)(+.924)] = +.572 \text{ mm.}$$

$$x_8 = 1 + [(+.245)(0)] - [(+.345)(+1.)]$$
$$+ [(+.165)(-1.)] - [(+.085)(0)] = +.490 \text{ mm.}$$

$$y_8 = [+.055] + [(+.245)(+1.)] + [(+.345)(0)]$$
$$+ [(+.165)(0)] + [(+.085)(-1.)] = +.215 \text{ mm.}$$

When, as above, we multiply the coefficients by the functions of ζ connected with them in the formulas, we thereby express *numerically* the modified condition of the field around the compass for that particular heading of the ship, just as it was mechanically illustrated by the telescopic nature of the line $\overline{OC}$, Fig. 530.

The computation of (118) and (119) for a number of values of ζ is tedious though accurate; but there is a speedy method of plotting the curve—that described in Art. 335—which is sufficiently accurate, *when carefully done.*

The point Q is called the pole of the dygogram. As will be seen by Fig. 531, the line $\overline{AD}$ is the resultant of $\overline{AE}$ and $\overline{ED}$, or $\overline{AD} = \sqrt{\mathfrak{C}^2 + \mathfrak{D}^2}$; and similarly $\overline{DN} = \sqrt{\mathfrak{B}^2 + \mathfrak{C}^2}$; the former is the radius of a circle whose circumference forms a *circular* base line upon which another circle, whose radius is $\overline{DN}$, rolls, and traces the dygogram.

337. Values of the deviation and force obtained from the dygogram.—By constructing values of the coefficients, two points of the dygogram are obtained—North and South; and drawing a perpendicular to $\overline{NS}$ through Q gives East and West: if, in the further construction, the dots around the periphery of the protractor be placed at intervals of $11° 15'$, the radii through these dots and the pole Q will intersect the curve (when traced) in the remaining dygogram points, which are the successive headings of the ship by compass—as, for example, in positions L and M, Fig. 531. Drawing lines from O to all these points of intersection, we obtain the deviations thus: place the protractor with its center at O, zero radius along $\overline{OP}$, and

$90° \ldots 270°$-diameter coincident with $\overline{OY}$; then read the angles corresponding to the lines drawn to the points—they are the deviations, those on the right of $\overline{OP}$ being easterly, those on the left, westerly: for example, in Fig. 531, $\delta_0 = 18° \ 30'$ E. for North and $\delta_{16} = 12° \ 45'$ W. for South. Then with a millimeter scale measure the lines from O to the same points, and if the construction is based on the mean force to north as unit of measure, these lengths will represent the combined force of Ship and Earth, for the several headings, in the same unit, thus: $\overline{ON} = R_1 = 1.465$ for head north, and $\overline{OC'} = R_2 = .755$ for head E.NE.

Having obtained the deviations on all the *magnetic* points, they are to be plotted on Napier's Diagram, and the deviations on the *compass* points thence obtained, as explained in Art. 333.

If the deviation and force are desired for *any course*, other than one of the 32 points—say for N. 23° E.—the procedure is as follows: place the protractor with its center O at Q, Fig. 531, and the diameter $\overline{OY}$ along $\overline{NS}$, the semicircle toward the southward; at 23° from $\overline{NS}$ toward the right make a dot, and draw a line through this and Q—it will cut the curve at Z, and NQZ is the course N. 23° E.; draw $\overline{OZ}$, and measure it and also the angle POZ—the latter is the deviation and the former the force proper to the given magnetic heading. This procedure is really identical with that for obtaining the 32 points already described.

The points I and J, Fig. 531, in which the meridian cuts the curve, are those headings on which there is no deviation: to ascertain which ones they are, draw lines from Q to I and J, and prolong them well beyond; lay the protractor V with its center O on Q and diameter along $\overline{NS}$, with the semicircle alternately on each side of this line, and read off the number of degrees from North and from South to the lines $\overline{QI}$ and $\overline{QJ}$ respectively. Thus, in Fig.

531, the headings of the ship on which there is no deviation are NQI = N. 41° W., and SQJ = S. 70° E.; the force on the former is $\overline{OI} = 1.495$; and on the latter, $\overline{OJ} = .525$.

The point of contact of a tangent to the curve from O is that heading of the ship for which the deviation is a maximum; a tangent may be drawn on each side of the meridian, denoting respectively the greatest easterly and westerly deviation: for example, $\overline{OH}$ is tangent at H; the magnetic heading upon which the maximum easterly deviation occurs is NQH = N. 57° E.; the directive force is $\overline{OH} = .895$; and the deviation is POH = 38° E.

Often the coefficients $\mathfrak{A}$ and $\mathfrak{E}$ are almost equal in value and opposite in sign; this is the case with the U. S. S. ATLANTA, which had $\mathfrak{A} = +.0061$ and $\mathfrak{E} = -.0058$; and the U. S. S. CONCORD, with $\mathfrak{A} = -.0038$ and $\mathfrak{E} = +.0048$: they so nearly cancel, that for these ships $\mathfrak{A}$ and $\mathfrak{E}$ may be considered zero. Many other ships have these coefficients so small that they scarcely affect the results obtained from a dygogram. In all such cases the construction is simplified by their omission: both the center of the generating circle and the point D then fall on the meridian $\overline{OX}$, and the radius of the circle is simply $\mathfrak{D}$.

338. To determine $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, and λ from observations on two headings; and thence obtain the Deviations and Force on the 32 compass points.—The mathematical basis of this problem, the observations to be taken for it, the cases in which it is most useful, the conditions that conduce to accuracy, and those in which it fails entirely—have all been set forth in connection with its solution by computation in Art. 325; the method by construction will be given here, and the example of Art. 325 used. Reproducing its data we have ζ_1 = N. 9° E. = 9°; δ_1 = 18° 0′ E.; $\dfrac{H_1'}{\lambda . H} = R_1 = .894$;

ζ_2 = S. 60° W. = 240°; δ_2 = 6° E.; $\dfrac{H_2'}{\lambda . H} = R_2 = 1.321$.

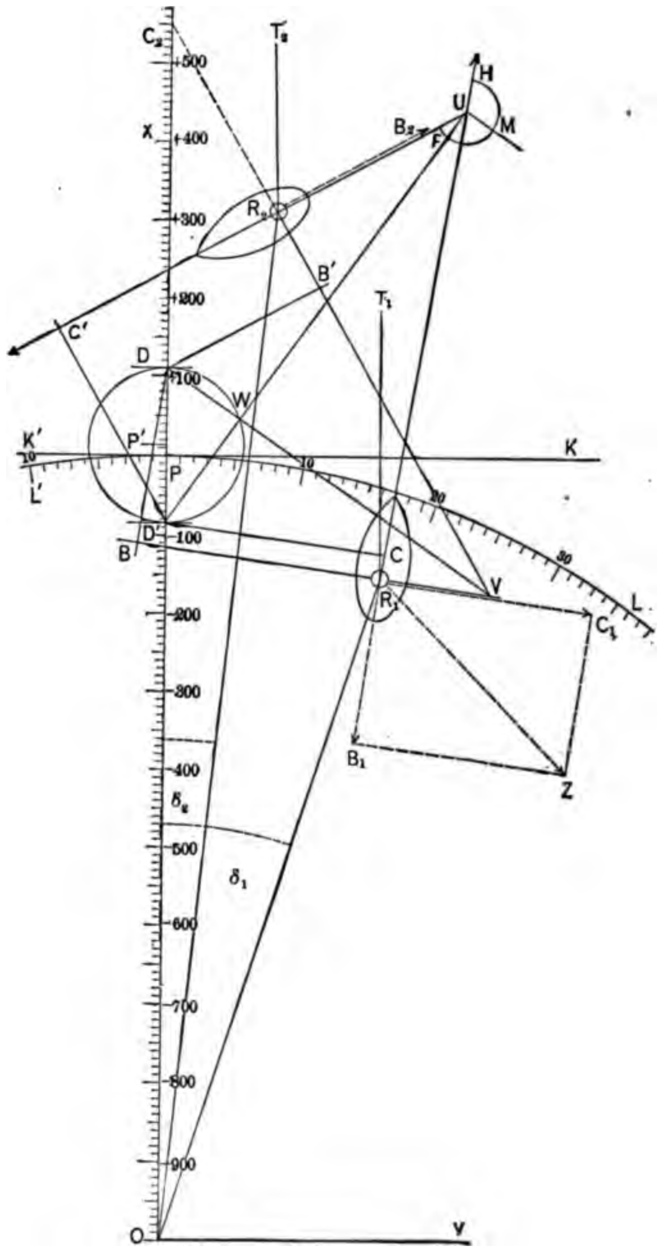
One familiar with constructions of this kind will find some of the following details superfluous, but they will probably prove useful to others.

The Dygogram Form of the Compass Office will be used —Fig. 532. The coördinate axes $\overline{OX}$ and $\overline{OY}$ are directed to magnetic north and east respectively; $\overline{KK'}$ is parallel to $\overline{OY}$; $\overline{OP}=1$ = unity of scale for measuring all lengths; it is divided into 100 parts; the tenths are numbered .100, .200, etc.; the smaller divisions are hundredths, and may be divided into halves, thus enabling values to the third decimal place to be plotted; the arc LPL' has its center at O, and is for plotting the observed deviations, and afterward for determining them when the dygogram has been traced.

The observations for this problem give the force and its direction (which is the deviation) on two headings—they are two points of the dygogram; from them the coefficients are found, the complete curve is drawn, and from this the force and deviation on every *magnetic* point are obtained: plotting the deviations on Napier's Diagram, those on the *compass* points are thence derived.

Following is the procedure of constructing the above data: Lay off the angle $POR_1=\delta_1=18° \ 0'$ E., the deviation on the first heading, and make $\overline{OR_1}=\frac{H_1'}{\lambda.H}=R_1=.894$ of the scale $\overline{OP}$; lay off $POR_2=\delta_2=6° \ 0'$ E., and make $OR_2=\frac{H_2'}{\lambda.H}$ $=R_2=1.321$; westerly deviations should be laid off to the left of $\overline{OP}$.

Through the points R_1 and R_2 draw the parallels $\overline{R_1T_1}$ and $\overline{R_2T_2}$ to $\overline{OX}$; place a protractor with its center successively on the points R_1 and R_2 and its zero-radius on these parallels, and count toward the right the number of degrees of the ship's magnetic heading in each case, and mark it; for R_1 it is $9°=\zeta_1=T_1R_1C$, and for R_2 it is $260°=$



C₂
T₂
H
U
M
B₂
F
X
+500
+400
+300
+200
+100
R₂
B′
T₁
C′
D
W
K′
P′
K
L′
P
10
20
30
D′
-100
B
C
V
R₁
L
C₁
-200
-300
B₁
Z
-400
δ₂
-500
δ₁
-600
-700
-800
-900
O
Y

FIG. 532.

$\zeta_2 = T_2R_2C'$, reckoning from $\overline{R_2T_2}$ toward the right; draw the outline of a ship around R_1 and R_2—each on its proper heading; draw lines through these headings until they meet, and mark the point of intersection U—the lines are $\overline{R_1U}$ and $\overline{R_2U}$.

Draw perpendiculars through the points R_1 and R_2 to the lines $\overline{R_1U}$ and $\overline{R_2U}$, and where they meet mark the point V.

Bisect the angle made by the ship's magnetic headings; in the present case, it is the angle corresponding to the arc $HMF = 260° - 9° = 231°$, whose half is $115°\ 30'$: place the center of a protractor at U, and its zero-radius coincident with $\overline{UH}$—the direction of the ship's head at R_1, and count $115°\ 30'$ to the right, which determines the point M for drawing the bisectrix; then $HUM = 115°\ 30'$.

Through V draw a line *parallel* to this bisectrix $\overline{UM}$ and call it $\overline{VW}$; from the point U let fall a perpendicular upon $\overline{VW}$ and mark the intersection W. Prolong $\overline{VW}$ to $\overline{OX}$ and mark the point D; produce $\overline{UW}$ to cut $\overline{OX}$ and mark the intersection D'. Measure the distance $\overline{DD'}$; it is .194, and $\frac{1}{2}(\overline{DD'}) = .097$, which is the radius of the generating circle; lay this off from either D or D'—it cuts $\overline{OX}$ in P', which is the center of that circle, and from which it is to be drawn.

Prolong $\overline{VR_1}$ and from D let fall a perpendicular upon it—to B; also, let fall a perpendicular from D upon $\overline{VR_2}$, meeting it at B'; then $\overline{DB} = \overline{DB'}$—they represent the longitudinal disturbing force: only one of these lines need be drawn, but as their equality is a test of the accuracy of the construction, it is well to draw both. From D' let fall perpendiculars upon $\overline{UR_1}$ and $\overline{UR_2}$, and mark the points of meeting these lines C and C' respectively; then $\overline{D'C} = \overline{D'C'}$—they represent the transverse disturbing force, and only one need be drawn.

As both systems of forces are separated—one pair ema-

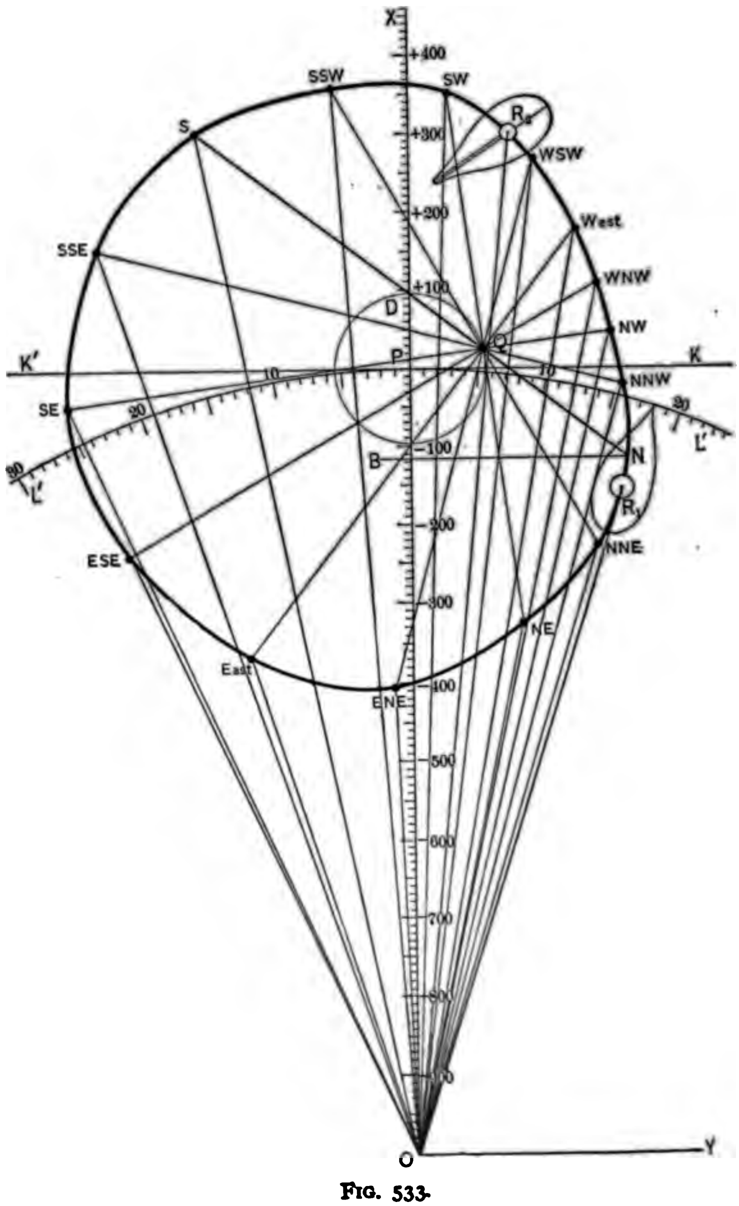
nating from D and the other from D'—we can find their united action by transferring one of each to either D or D'; that is, $\overline{DB'}$ to D', and then it acts jointly with $\overline{D'C'}$; an outline of the ship is to be drawn around D', headed as at R_2: or, transfer $\overline{DB}$ to D', and then $\overline{DB}$ and $\overline{D'C}$ act together; an outline of the ship is to be drawn around D', headed as at R_1: or, leaving the ships in their places at R_1 and R_2, we may transfer the forces in their proper directions so that the point of application shall be at the compass, as indicated by the dotted lines $\overline{R_1B_1}$, $\overline{R_1C_1}$, and $\overline{R_2B_2}$, $\overline{R_2C_2}$.

The direction of these lines—to bow, to starboard, to stern, or to port—indicates their algebraic signs; see Fig. 532.

Measuring the various lines, we have: $\overline{OP'} = \lambda = 1.014$; $\overline{P'D} = \overline{P'D'} = +\lambda.\mathfrak{D} = +.097$, $\therefore \mathfrak{D} = +.096$; $\overline{DB} = \overline{DB'} = -\lambda.\mathfrak{B} = -.215$, $\therefore \mathfrak{B} = -.212$; $\overline{D'C} = \overline{D'C'} = +\lambda.\mathfrak{C} = +.280$, $\therefore \mathfrak{C} = +.276$.

Laying off $\overline{R_1C_1} = +\mathfrak{C}$ and $\overline{R_1B_1} = \overline{C_1Z} = -\mathfrak{B}$, their resultant is $\overline{R_1Z} = \sqrt{\mathfrak{B}^2 + \mathfrak{C}^2}$, the total semicircular force; and UR_1Z is its direction, the starboard angle α, which is expressed by $\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{+\mathfrak{C}}{-\mathfrak{B}}$; the sign of $\mathfrak{B}$ indicates that the angle is in the second quadrant.

Now having the coefficients, except $\mathfrak{A}$ and $\mathfrak{E}$, which are supposed unknown, proceed to construct the dygogram as in Fig. 533: lay off $\mathfrak{D} = +.096$ from P toward the north and mark the end D; $\mathfrak{D}$ is the radius of the generating circle, which is to be drawn; from D lay off $\mathfrak{B} = -.212$ toward the south and mark the terminus B; from this point lay off $\mathfrak{C} = +.276$ toward the east and mark its end N—it is the north point of the dygogram. Draw $\overline{ND}$: it cuts the circle in Q, which is the pole: make $\overline{DS} = \overline{ND}$, and then S is the south point of the dygogram. Through Q draw a perpendicular to $\overline{NS}$, and we have the east-and-west line. Lay a paper circle, graduated to degrees, with


X
+400
SSW
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S
R2
+300
WSW
+200
West
SSE
+100
WNW
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10
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SE
20
20
L'
-100
B
N
30
L'
R1
-200
NNE
ESE
-300
NE
East
-400
ENE
-500
-600
-700
O
Y

FIG. 533.

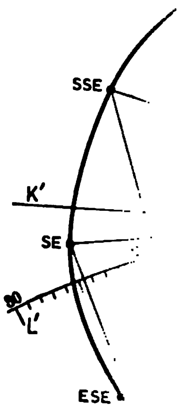
its center on Q, the $0° \ldots 180°$-diameter on $\overline{NS}$ and the $90° \ldots 270°$-diameter on the E.-W. line; make a dot at every $22° 30'$ of its circumference (or at every $11° 15'$): in reality, this need be done for only one side of $\overline{NS}$, but it is well to do it on both; then lines through the dots on one side and the point Q should pass through those on the other side—it is a test of accuracy. The rest of the construction is described in Art. 335: by it, we obtain the sixteen magnetic points of Fig. 533.

Drawing a line from O to each point, its length is (by the scale on $\overline{OX}$) the directive force on the compass when the ship is on that magnetic heading, and the intersection of each line with the arc $L'PL'$ indicates the deviation proper to that heading.

These deviations are to be plotted on Napier's Diagram—the curve traced—and the deviations on the compass points thence derived in accordance with the rules of Arts. 332 and 333. Having done this, the results are given in Table 83.

TABLE 83.

Ship's Head by Compass.	Deviations Observed.	Deviations Obtained from Dygogram, Fig. 533.	Difference.	Computed from Deviation Table.	Obtained from Dygogram, Figs. 532 and 533.	Difference.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
NORTH	18° 30′ E.	16° 0′ E.	2° 30′	Coefficients.		
N. NE.	15 0 E.	15 0 E.	0 0			
NE.	8 0 E.	8 0 E.	0 0			
E. NE.	2 0 W.	2 0 W.	0 0	$\mathfrak{A} = -.0082$		
EAST	12 0 W.	12 0 W.	0 0			
E. SE.	21 30 W.	20 0 W.	1 30	$\mathfrak{B} = -.223$	$= -.212$	.011
SE.	27 15 W	25 0 W.	2 15			
S. SE.	27 0 W.	25 30 W.	1 30	$\mathfrak{C} = +.284$	$= +.276$	.008
SOUTH	19 0 W.	16 30 W.	2 30			
S. SW.	7 0 W.	6 0 W.	1 00	$\mathfrak{D} = +.103$	$= +.096$	.007
SW.	3 0 E.	3 0 E.	0 0			
W. SW.	8 30 E.	9 0 E.	0 30	$\mathfrak{E} = +.0005$		
WEST	11 15 E.	11 30 E.	0 15			
W. NW.	13 0 E.	13 0 E.	0 0	$\lambda = 1.019$	1.014	.005
NW.	14 30 E.	14 30 E.	0 0			
N. NW.	16 30 E.	17 0 E.	0 30			



different

and

finally, for approximately determining the deviations and compensating the compass.

After the site for the compass has been chosen, all subsequent observations for this problem are to be taken in the exact place the needles will occupy; and these observations should be made with the utmost care and only when the ship is in dock; they are: direction of the ship's head by compass, ζ'; a time-azimuth of the sun by same, or a reciprocal bearing upon another compass on shore in a spot free from magnetic disturbance, by which the magnetic heading, ζ, may be deduced, and thence the deviation, δ, proper to that heading; the time of ten oscillations of a small horizontal needle in the spot ashore, and also in the place of the compass, by which we obtain $\frac{T^2}{T'^2}=\frac{H'}{H}$.

The coefficients $\mathfrak{A}$ and $\mathfrak{E}$ are considered zero, and $\mathfrak{D}$ and λ are either known or *assumed.*

The dygogram Form and the following data relative to the U. S. S. ATLANTA will be used for constructing the problem: $\zeta'=$S. 16° 30′ W. $=196°\ 30'$; $\zeta=$S. 35° 0′ W. $=215°$; $\frac{T^2}{T'^2}=\frac{H'}{H}=1.130$; $\mathfrak{D}=+.125$ and $\lambda=.945$ (both assumed); then $\frac{1}{\lambda}.\frac{H'}{H}=\left(\frac{1}{.945}\right)(1.130)=1.19$; $\delta=18°\ 30'$ E.

Referring to Fig. 534, $\overline{OX}$ and $\overline{OY}$ are the axes to magnetic north and east respectively; O is the center of coördinates and also of the arc $\overline{L'P}$; $\overline{KK'}$ is parallel to $\overline{OY}$; $\overline{OP}=1$, the scale of lengths divided into 100 parts.

To construct the data: with P as a center, and radius $\mathfrak{D}=+.125$ of scale, describe the generating circle, cutting $\overline{OX}$ in D and D'. Lay a paper circle graduated to degrees with its cardinal diameters on $\overline{OX}$ and $\overline{KK'}$, and then its center will be on the point P; count to the right from $\overline{OX}$ the magnetic heading ζ of the ship—in this case it is $RVF=215°$ —and mark it Z; draw $\overline{ZP}$, and through D' draw a parallel

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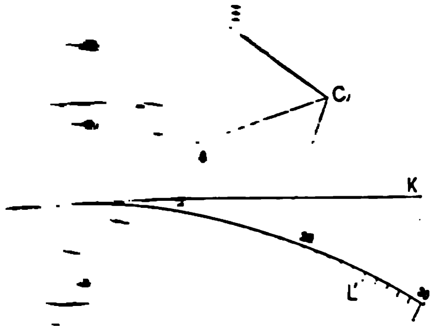

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$\overline{FLB}$ to $\overline{ZP}$—it is the keel line of the ship—her magnetic heading: this line will always cut the generating circle in another point L; where it does, draw an outline of the ship, heading in the direction in which the observations were made.

By means of the arc $\overline{PL'}$, lay off the angle $POC = \delta = 18° 30'$ E.: if west, it should be laid off to the left of $\overline{OX}$; make $\overline{OC} = \frac{1}{\lambda} \cdot \frac{H'}{H} = 1.19$; thus we have the direction and amount of the force acting on the compass for the heading of the ship.

From C let fall a perpendicular $\overline{CB}$ on the keel line $\overline{FB}$: then $\overline{LB} = -\mathfrak{B} = -.220$ of the scale $\overline{OP}$, and is minus because directed toward the stern; if toward the bow it would be plus; and $\overline{BC} = -\mathfrak{C} = -.160$, and is minus because directed to port; if to starboard it would be plus. The starboard angle is $HTM = \alpha$, and is measured by a protractor—laying its zero diameter on the *keel line* $\overline{FB}$, and counting around from the bow toward the right: it is here in the third quadrant because both $\mathfrak{B}$ and $\mathfrak{C}$ are minus, or $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$; that is, $\alpha = \tan^{-1} \frac{-\mathfrak{C}}{-\mathfrak{B}}$.

Having $\mathfrak{B}$ and $\mathfrak{C}$, take a blank dygogram Form, and from the point P lay off the assumed value of $\mathfrak{D} = +.125$ toward the north, and mark the end D; from D, lay off $\mathfrak{B} = -.220$ toward the south and mark the end B; from B lay off $\mathfrak{C} = -.160$ toward the west and mark the end N—it is the north point of the dygogram. With P as a center and $\mathfrak{D}$ as a radius draw the generating circle. From N draw $\overline{ND}$ and prolong it, making $\overline{DS} = \overline{ND}$; then S is the south point of the curve: where this line cuts the circle a second time, as it will, is the pole Q of the dygogram.

A figure is not supplied for the construction just de scribed, because the dygogram form of Fig. 533 will illustrate it. The remainder of the construction is identical with

that described in Arts. 335 and 336; and the procedure of obtaining the deviations and force on each magnetic point and of ascertaining the equivalents for compass points is given in Art. 337.

While in dry dock, observations of the kind stated in this article may be made in different parts of the ship, $\mathfrak{B}$ and $\mathfrak{C}$ thence deduced for each, and a comparison of all the values thus found, will indicate the spot most suitable for the compass.

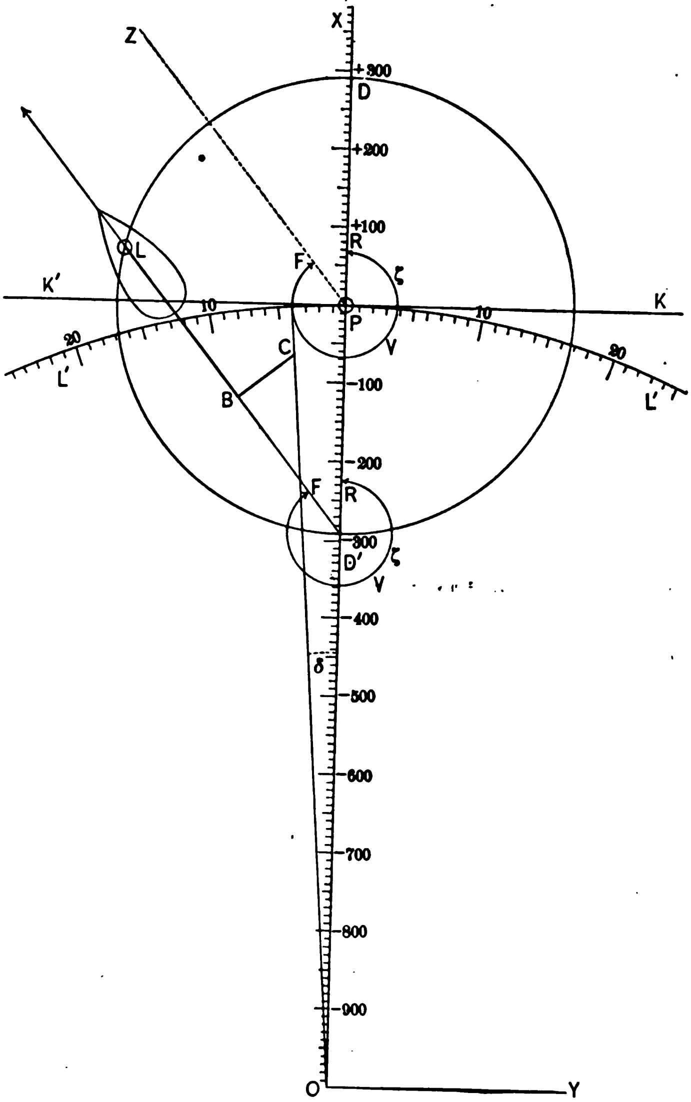
The relative vertical force at each place can be determined by oscillating a Dip circle needle both in the spot ashore and on board: and such sets should be used jointly with those of the horizontal needle to determine the site for the compass.

340. To find λ by construction.—This is the geometrical solution of formula (258), p. 944; and to some extent it is the converse of the procedure of the last article: there, λ was either known or assumed, and the coefficients deduced—here, the latter are given, and the former found.

The following data relative to the steering compass of the U. S. S. SAN FRANCISCO will be constructed on the dygogram Form: $\mathfrak{B} = -.233$; $\mathfrak{C} = +.085$; $\mathfrak{D} = +.290$; $\mathfrak{A} = -.007$; $\mathfrak{E} = +.005$; $\zeta =$ N. 38° 30′ W. $= 321°$ 30′; $\delta = 4°$ 0′ W.; and $\frac{T^2}{T'^2} = \frac{H'}{H} = .728$.

$\mathfrak{A}$ and $\mathfrak{E}$ have opposite signs and almost equal value—they would therefore cancel in any dygogram construction: $\mathfrak{A}$ does not enter the formula of this problem, but $\mathfrak{E}$ does; yet it is so small that in any event it would scarcely affect the result.

The dygogram Form has been explained in Art. 339; and the construction of the data on it is as follows—Fig. 535: with P as a center and $\mathfrak{D} = +.290$ as radius, describe the generating circle, cutting the magnetic meridian $\overline{OX}$ in D and D'. By means of a protractor lay down the


X
Z
+300
D
+200
+100
R
F
ζ
L
K'
K
10
10
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20
20
L'
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C
V
B
-100
-200
F
R
-300
D'
ζ
V
-400
δ
-500
-600
-700
-800
-900
O
Y

FIG. 535.

given magnetic course ζ—it is the angle RVF around each of the points P and D': from D' draw $\overline{D'B}$ in the direction of the ship's keel and where it cuts the circle at L, draw an outline of the ship, headed on the given course N. 38° 30′ W. $= 321° 30' = \zeta = RVF$; L is the location of the steering compass; from this point lay off $\overline{LB} = -\mathfrak{B} = -.233$, according to the scale $\overline{OP}$, toward the ship's stern, because $\mathfrak{B}$ is minus; if plus it would be laid off from L toward the bow: from the point B lay off $\overline{BC} = +\mathfrak{C} = +.085$ to starboard, because $\mathfrak{C}$ is plus; if minus it would be laid off from B to port. Draw $\overline{OC}$—it is the total force acting on the compass for the heading N. 38° 30′ W., and its direction is seen to be 4° W., that is, the deviation on the given course: the length $\overline{OC}$ is .94 by the scale $\overline{OP}$; and since $\overline{OC} = \frac{1}{\lambda} \cdot \frac{H'}{H}$, therefore, after substituting the above values in this equation, we have $\lambda = \frac{H'}{H} \cdot \frac{1}{\overline{OC}} = (.728)\left(\frac{1}{.94}\right) = .774$; by computation, using formula (258), λ was found on p. 945 to be .772, differing by only .002 from that here.

341. To compute the position of the pole of the dygogram, and deduce the equation of the curve.—Reproducing such portion of Fig. 531 as is needed for the present purpose, we have it in Fig. 536, with the points that are identical in both figures designated by the same letters. From eqs. (77) and (83), pages 871–'72, we have

$$\tan \alpha = \frac{\mathfrak{C}}{\mathfrak{B}}, \quad \text{or} \quad \alpha = \tan^{-1} \frac{\mathfrak{C}}{\mathfrak{B}}. \quad . \quad . \quad . \quad (367)$$

$$\tan \beta = \frac{\mathfrak{E}}{\mathfrak{D}}, \quad \text{or} \quad \beta = \tan^{-1} \frac{\mathfrak{E}}{\mathfrak{D}}. \quad . \quad . \quad . \quad (368)$$

In Fig. 536, $\overline{DB} = \mathfrak{B}$, and $\overline{BN} = \mathfrak{C}$, hence

$$BDN = LDQ = \alpha, \quad . \quad . \quad . \quad . \quad . \quad (369)$$

and $\overline{AE} = \mathfrak{E}$, and $\overline{ED} = \mathfrak{D}$, hence

$$ADE = ALE = MAL = \beta. \quad . \quad . \quad . \quad . \quad (370)$$

By the figure, $$MAQ = LAQ - MAL. \quad . \quad . \quad . \quad (371)$$

By Geom., $$LAQ = 2LDQ = 2\alpha. \quad . \quad . \quad . \quad . \quad (372)$$

Whence, $$MAQ = 2\alpha - \beta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (373)$$

Therefore in constructing the dygogram, after drawing the generating circle, draw a parallel to $\overline{OX}$ through the

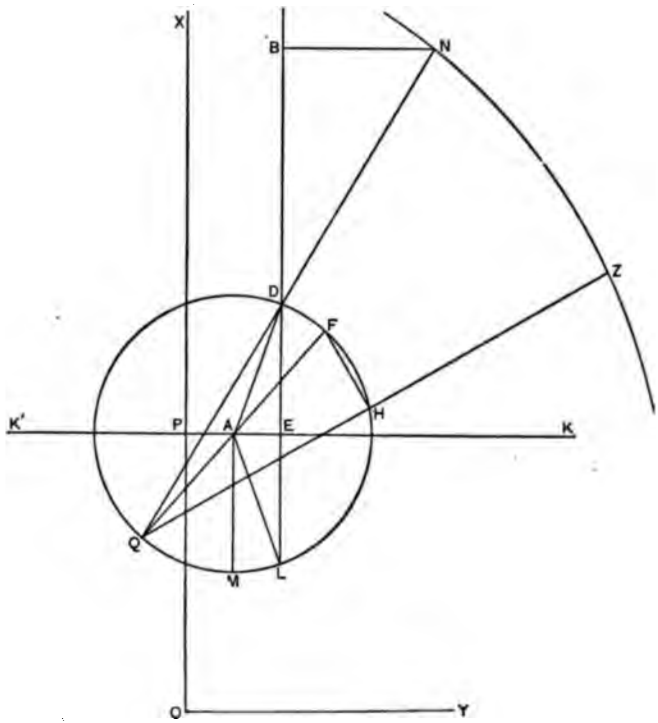

FIG. 536.

point A; from the point M where the parallel cuts the circle, lay off the angle MAQ equal to the second member of (373), and we have the pole Q. When $\mathfrak{A}$ and $\mathfrak{E}$ cancel,

or so small as to be negligible, then, by (368),
73) becomes

$$MAQ = 2\alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad (374)$$

ce the equation of the dygogram, let

$$p = \overline{DN} = \sqrt{\overline{DB}^2 + \overline{BN}^2} = \sqrt{\mathfrak{B}^2 + \mathfrak{C}^2}. \quad . \quad . \quad (375)$$

$$q = \overline{AD} = \sqrt{\overline{ED}^2 + \overline{AE}^2} = \sqrt{\mathfrak{D}^2 + \mathfrak{E}^2}. \quad . \quad . \quad (376)$$

ıce QAF is a diameter of the circle, QHF is, by Geom., angle, whence

$$\cos FQH = \frac{\overline{QH}}{\overline{QF}}; \quad \therefore \quad QH = \overline{QF}.\cos FQH. \quad . \quad (377)$$

By the Fig.: $\overline{AD} = \overline{AF}$, and $\overline{QF} = 2.\overline{AF} = 2q$; . . (378)

$\overline{QZ} = \overline{HZ} + \overline{QH}$; . . (379); $FQH = DQH - FQD$; (380)

and $\quad ADQ = AQD = FQD = EDQ - EDA = \alpha - \beta.$ (381)

By the principles of the dygogram, $\overline{DN} = \overline{HZ} = p$, . (382)

and $\quad DQH = \zeta$; . . . . (383)

whence, substituting these several quantities in (379), we have

$$\overline{QZ} = p + 2q.\cos\ (\zeta - \alpha + \beta), \quad . \quad . \quad . \quad (384)$$

which is the polar equation of the dygogram: the quantities of the second member are known, except ζ; and by assigning different values to this, that is, successive headings for the ship, corresponding values of $\overline{QZ}$ may be computed and from them the dygogram traced.

CHAPTER XXVI.

THE HEELING ERROR.

342. The downward pull on the Compass—Ship on an even keel.—In a former chapter the total magnetic force acting on the compass was conceived to be resolved in three directions as regards the ship—longitudinally, transversely, and vertically: analytical expressions were found for each, and those to bow and starboard—in the horizontal plane—have been treated; that toward the keel will now be investigated. Its fundamental formula is (96) of Art. 301, which is reproduced here and given number (385) of the regular sequence:

$$\frac{Z'}{Z}=\frac{g}{\tan\theta}.\cos\zeta-\frac{h}{\tan\theta}.\sin\zeta+1+k+\frac{R}{Z}. \quad . \quad (385)$$

The symbols of this have been defined in Arts. 297 and 298. In any locality the value of $\frac{Z'}{Z}$ will vary with the heading of the ship: if she be put on two diametrically opposite magnetic courses, or on 4, 8, 16, or 32 equidistant magnetic points, the terms of (385) containing the functions sin ζ and cos ζ will disappear in the summation of the observations made on such points, as explained in Art. 296, par. [6]; and thus (385) will reduce to

$$\frac{1}{n}\left\{\Sigma\left(\frac{Z'}{Z}\right)\right\}=\frac{1}{n}\left(1+k+\frac{R}{Z}\right). \quad . \quad . \quad . \quad (386)$$

or ar ... [illegible] vertical force of

$\beta = 0$ ... [illegible] component.

... [illegible] we have

T ... [illegible] (387)

... [illegible] the mean

... [illegible] mean hori-

... [illegible] composed

Si ... [illegible] Earth

a rigl ... [illegible] to have

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By ... [illegible] (388)

$\overline{QZ} = l$ ...

and ... [illegible] both hard

By th ... [illegible] (389)

and

whence ...

... [illegible] (390)

have

which ... [illegible] terms of the

of the s ...

differen ... [illegible] when the ship

ship, ... [illegible] force that is

from th ... [illegible] whatever the

... [illegible] heels, other

... [illegible] action.

... [illegible] rolling

... [illegible]—heeling,

... [illegible] period of the

compass, will render it very unsteady, with no remedy but compensation—the other may cause large deviations, but no oscillation; and these deviations can be observed and tabulated, as well as entirely avoided by compensation. On both accounts, therefore, it is almost essential to counteract the effects of the nether pole of the ship as expressed by eqs. (388) and (390): we thereby avoid the oscillation due to rolling and the deviations due to heeling.

A ship heels or is permanently listed under press of canvas, or from unequal stowage of the weights on board; she rolls from motion of the sea: it is only the effect of a steady list that will be considered in this chapter.

Two terms—the HEELING ERROR and the HEELING COEFFICIENT—will now be defined. When a ship on an even keel is completely swung, a table of deviations is obtained for every compass point; if, without changing anything on board—otherwise than listing her to a permanent angle to either side by the movement of *non*-magnetic material—she be completely swung again and observations made on the same compass points, another series of deviations will be obtained: the difference between the two series—listed and upright—constitutes the *Heeling Error* for each compass point for that particular angle of heel; and this error varies *with* the angle of heel.

Let δ_u denote the deviation on any compass point ζ' when the ship is upright; δ_i the deviation on the same compass point when the ship is inclined to one side by an angle i; and let J be the *Heeling Coefficient:* then

$$J = \frac{\delta_i - \delta_u}{i \,.\cos \zeta'} \quad \cdots \cdots \quad (391)$$

That is, J expresses the heeling error for *one degree* of inclination on the course ζ' in (391); therefore the product $J.i$ is the heeling error for *any* angle of heel (i) on that course.

In Fig. 444 the sources of disturbance of the compass are represented by soft-iron rods and magnets with the ship on an even keel: in Fig. 537 she is heeled to the angle i, and all the rods and magnets are inclined by the same amount. With the ship upright, the Earth's horizontal and vertical components induce transient magnetism in the soft iron lying in their directions; but heeled—and the iron correspondingly inclined—the value of the Earth's components in the direction of the iron must be found in order to ascertain their magnetizing power: then, as the compass-card is constrained to move in a horizontal plane by reason of its suspension, the forces that act upon it—the effects of the rods and magnets—must be resolved into that plane from the inclined position of the deck, and into the vertical from the line toward the keel.

The following symbols that enter the investigation will be defined—they are illustrated by solid and dotted lines in Fig. 537: X and Y represent the magnetic force of the Earth alone in the horizontal plane, X toward the bow and Y toward the side; Z is the corresponding vertical force; Y_i and Z_i represent the force of the Earth alone in the inclined directions of the deck and of the line toward the keel; there is no distinctive value of X corresponding to these, for the ship heels around X as an axis, so that X_i and X are identical; Y_i' and Z_i' are the combined forces of Ship and Earth, the first along the inclined direction of the deck toward the side, and the last toward the keel: X' and Y' represent the combined force of Ship and Earth in the *horizontal* plane, X' toward the bow and Y' toward the side; Z' is the corresponding vertical force.

To resolve the horizontal and vertical components of the Earth alone into the inclined directions of the deck and toward the keel, consider Fig. 538: $\overline{OL} = Y$ and $\overline{OF} = Z$, and we have to determine their values Y_i and Z_i.

From the figure $\quad \overline{OT} = \overline{OL}.\cos i; \quad . \quad . \quad . \quad . \quad . \quad (392)$

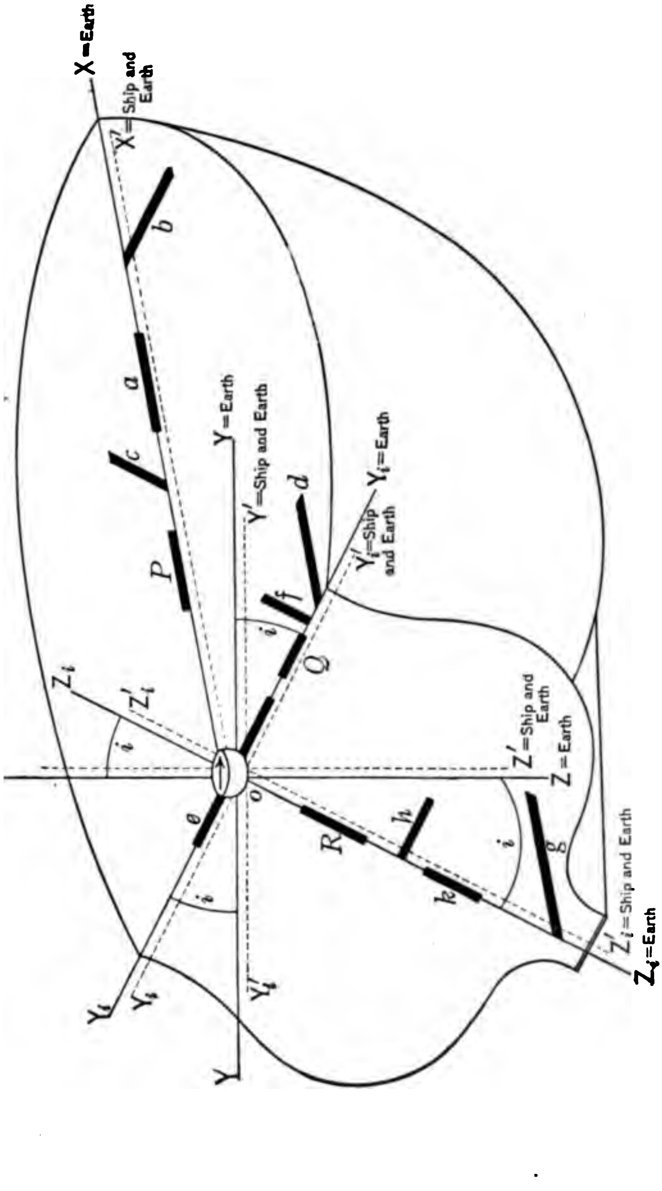


FIG. 537.—Physical Representation of Ship's Magnetism—Vessel Heeled to Starboard.

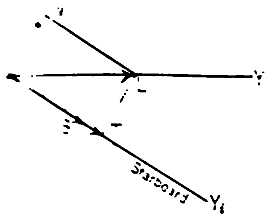
(393)

$= \overline{OS} =$ (395)

$-Z.\sin$ (396)

$-Y.\sin$ (397)

Earth alone in the direc-
eqs. (395) and (397).
is a, d, g are parallel to



by the Earth's
the contrary, are
will now be mag-
by eq. (396);
and will be
by eq. (397).
representatives
axes X, Y_i, Z_i,
Earth, that is, all
in these axes:
ically.

Recalling the definitions above given of Y_i' and Z_i', let the former be represented by $\overline{Oq}$ and the latter by $\overline{Or}$, Fig. 539. Then

$$\overline{Oq'} = Y_i'.\cos i \quad . \quad (398); \text{ and } \overline{Oq''} = qq' = Y_i'.\sin i. \quad (399)$$

$$\overline{Or'} = \overline{rr''} = Z_i'.\sin i \quad (400); \text{ and } \overline{Or''} = Z_i'.\cos i; \quad . \quad (401)$$

whence $$Y' = \overline{Oq'} - \overline{Or'} = Y_i'.\cos i - Z_i'.\sin i. \quad . \quad . \quad (402)$$

$$Z' = \overline{Oq''} + \overline{Or''} = Y_i'.\sin i + Z_i'.\cos i. \quad . \quad (403)$$

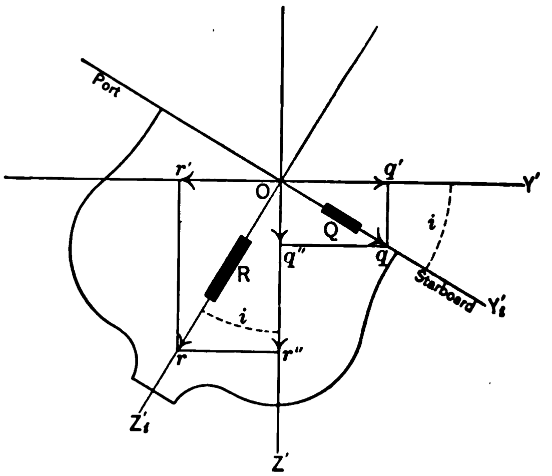


FIG 539.

As Y_i' and Z_i' embrace the effects of both hard and soft iron, their joint resolution, just performed, will be equally applicable to the effects of each resolved separately; and by reference to Art. 296 it will be seen that the process by which eqs. (396), (397), (402), and (403) have been obtained, may be either purely mathematical—the transformation of coördinates, or according to the principles of mechanics—the resolution of forces: in either case, certain values are given in one system of axes and their equivalents in another system are required.

With the ship upright the combined magnetic force of Ship and Earth acting on the compass was expressed by the following equations, reproduced from Art. 300, with their numbers there:

$$X' = X + a.X + b.Y + c.Z + P. \quad \text{To bow.} \quad . \quad . \quad (86)$$

$$Y' = Y + d.X + e.Y + f.Z + Q. \quad \text{To side.} \quad . \quad . \quad (87)$$

$$Z' = Z + g.X + h.Y + k.Z + R. \quad \text{To keel.} \quad . \quad . \quad (88)$$

But with the ship heeled—Fig. 537—the new values of the inducing force of the Earth, expressed in eqs. (396) and (397), must be introduced, instead of Y and Z, in (86), (87), and (88), whence these become

$$X' = X + a.X + b.Y_i + c.Z_i + P. \quad \text{To bow} . \quad . \quad . \quad (404)$$

$$Y_i' = Y_i + d.X + e.Y_i + f.Z_i + Q. \quad \text{To side, listed.} \quad (405)$$

$$Z_i' = Z_i + g.X + h.Y_i + k.Z_i + R. \quad \text{To keel, listed.} \quad (406)$$

Multiplying (405) throughout by *cos i* and (406) by *sin i*, and then subtracting latter from former, we have by means of (402)

$$Y' = [Y_i + d.X + e.Y_i + f.Z_i + Q] \cos i$$
$$- [Z_i + g.X + h.Y_i + k.Z_i + R] \sin i. \quad (407)$$

Similarly, multiplying (405) by *sin i* and (406) by *cos i*, and adding the results, we have by means of (403)

$$Z' = [Z_i + g.X + h.Y_i + k.Z_i + R] \cos i$$
$$+ [Y_i + d.X + e.Y_i + f.Z_i + Q] \sin i. \quad (408)$$

Substituting in (404), (407), and (408) the values of Y_i and Z_i from (396) and (397), and then, since $\sin^2 i + \cos^2 i = 1$, replacing $\cos^2 i$ wherever it occurs, by $(1 - \sin^2 i)$ we have

$$X' = X + a.X + (b.\cos i - c.\sin i) Y$$
$$+ (c.\cos i + b.\sin i) Z + P. \quad (409)$$

$$\left.\begin{aligned} Y' = Y &+ \{d.\cos i - g.\sin i\}\,X \\ &+ \{e - (f+h)(\cos i.\sin i) - (e-k)(\sin^2 i)\}\,Y \\ &+ \{f + (e-k)(\cos i.\sin i) - (f+h)(\sin^2 i)\}\,Z \\ &+ \{Q.\cos i - R.\sin i\}. \end{aligned}\right\} . \quad (410)$$

$$\left.\begin{aligned} Z' = Z &+ \{g.\cos i + d.\sin i\}\,X \\ &+ \{h + (e-k)(\cos i \sin i) - (f+h)(\sin^2 i)\}\,Y \\ &+ \{k + (f+h)(\cos i \sin i) + (e-k)(\sin^2 i)\}\,Z \\ &+ \{R.\cos i + Q.\sin i\}. \end{aligned}\right\} . \quad (411)$$

These three equations express the forces acting on the compass *with the ship heeled*—(409) and (410) in the *horizontal* plane, the first toward the bow, the second toward the side: (411) gives the vertical force; as this last is rendered ineffective on the compass by its suspension, it is therefore only (409) and (410) that produce deviation when the ship is heeled, and of these, (410) is the principal, (409) acting but a minor part.

If we designate by the subscript $_i$ the various rods and magnets in their *inclined* positions in Fig. 537, the equivalent of each, represented by rods and magnets set *horizontally* and *vertically* will be the quantities within parentheses and small brackets in (409), (410), and (411); that is,

$$a_i = a, \quad \ldots \quad (412)$$

since it is in axis of X, and does not become inclined.

$$b_i = b.\cos i - c.\sin i, \quad \ldots \quad (413)$$

$$c_i = c.\cos i + b.\sin i, \quad \ldots \quad (414)$$

$$d_i = d.\cos i - g.\sin i, \quad \ldots \quad (415)$$

$$e_i = e - (f+h)(\cos i \sin i) - (e-k)(\sin^2 i), \quad . \quad (416)$$

$$f_i = f + (e-k)(\cos i \sin i) - (f+h)(\sin^2 i), \quad . \quad (417)$$

$$g_i = g.\cos i + d.\sin i, \quad \ldots \quad (418)$$

$$h_i = h + (e-k)(\cos i \sin i) - (f+h)(\sin^2 i), \quad . \quad (419)$$

$$k_i = k + (f+h)(\cos i \sin i) + (e-k)(\sin^2 i), \quad . \quad (420)$$

$$P_i = P, \quad \ldots \quad (421)$$

since it is in axis of X, and does not become inclined.

$$Q_i = Q.\cos i - R.\sin i. \quad . \quad . \quad . \quad . \quad (422)$$

$$R_i = R.\cos i + Q.\sin i. \quad . \quad . \quad . \quad . \quad (423)$$

Substituting the first members of (412) to (423) in eqs. (409), (410), (411) these become

$$X' = X + a_i.X + b_i.Y + c_i.Z + P_i. \quad . \quad . \quad (424)$$

$$Y' = Y + d_i.X + e_i.Y + f_i.Z + Q_i. \quad . \quad . \quad (425)$$

$$Z' = Z + g_i.X + h_i.Y + k_i.Z + R_i. \quad . \quad . \quad (426)$$

And they are of the same *form* as eqs. (86), (87), and (88).

As the magnetic coefficients are directly dependent on the soft-iron rods and magnets, they will vary with any change in these: the coefficients representing the conditions with the ship heeled will now be deduced by substituting the new values of (412) to (423) for the old coefficients in eqs. (110) to (117), Art. 301. From eq. (110), p. 879, in connection with (412) and (416) above, we have

$$\lambda_i = 1 + \frac{a_i + e_i}{2} = 1 + \frac{a}{2} + \frac{e}{2} - \frac{(f+h)\cos i \sin i}{2} - \frac{(e-k)\sin^2 i}{2}$$

$$= \lambda - \frac{f+h}{2}.\cos i \sin i - \frac{e-k}{2}.\sin^2 i. \quad . \quad (427)$$

From (113), p. 879, and (413) and (415) above,

$$\lambda_i.\mathfrak{A}_i = \frac{d_i - b_i}{2} = \left(\frac{d-b}{2}\right)\cos i + \left(\frac{c-g}{2}\right)\sin i$$

$$= \lambda.\mathfrak{A}.\cos i + \left(\frac{c-g}{2}\right)\sin i. \quad . \quad (428)$$

From (116), p. 879, and (414) above, and since $\cos i = 1 - \text{versin } i$,

$$\lambda_i.\mathfrak{B}_i = c_i.\tan\theta + \frac{P_i}{H} = (c.\cos i + b.\sin i)\tan\theta + \frac{P}{H}, \text{ or}$$

$$\lambda_i.\mathfrak{B}_i = b.\sin i.\tan\theta + c.\tan\theta - c.\text{versin } i.\tan\theta + \frac{P}{H}$$

$$= \lambda.\mathfrak{B} + \{b.\sin i - c.\text{versin } i\}\tan\theta. \quad (429)$$

From (117), p. 879, and (417) above, and since $\tan\theta = \frac{Z}{H}$, we have

$$\lambda_i . \mathfrak{C}_i = f_i . \tan\theta + \frac{Q_i}{H}$$

$$= \left[\begin{array}{l} f . \tan\theta + \{(e-k)(\cos i \sin i) \\ \qquad - (f+h)(\sin^2 i)\} \tan\theta + \frac{Q}{H}.\cos i - \frac{R}{H}.\sin i \end{array} \right],$$

or

$$\lambda_i . \mathfrak{C}_i = \lambda . \mathfrak{C} + \left\{ \begin{array}{l} (e-k)(\cos i \sin i) - (f+h)(\sin^2 i) \\ \qquad - \frac{R}{Z}.\sin i \end{array} \right\} \tan\theta - \frac{Q}{H} \text{ versin } i. \quad (430)$$

From (114), p, 879, and (412) and (416) above, we have

$$\lambda_i . \mathfrak{D}_i = \frac{a_i - e_i}{2} = \frac{a-e}{2} + \left(\frac{f+h}{2}\right) \cos i \sin i + \left(\frac{e-k}{2}\right) \sin^2 i, \text{ or}$$

$$\lambda_i . \mathfrak{D}_i = \lambda . \mathfrak{D} + \left(\frac{f+h}{2}\right) \cos i \sin i + \left(\frac{e-k}{2}\right) \sin^2 i. \quad (431)$$

From (115), p. 879, and (413) and (415) above

$$\lambda_i . \mathfrak{E}_i = \frac{d_i + b_i}{2} = \frac{b.\cos i - c.\sin i + d.\cos i - g.\sin i}{2}$$

$$= \lambda . \mathfrak{E}.\cos i - \left(\frac{c+g}{2}\right) \sin i. \quad . \quad . \quad (432)$$

From (387), p. 1008, and (420) and (423) above, we have

$$\mu_i = 1 + k_i + \frac{R_i}{Z} = \left\{ \begin{array}{l} 1 + k + (f+h)(\cos i \sin i) \\ + (e-k)(\sin^2 i) + \frac{R}{Z}.\cos i + \frac{Q}{Z}.\sin i \end{array} \right\}, \text{ or}$$

$$\mu_i = \mu + (f+h) \cos i \sin i + (e-k) \sin^2 i$$

$$- \frac{R}{Z}.\text{versin } i + \frac{Q.\sin i}{H.\tan\theta}. \quad (433)$$

Eqs. (427) to (433) give the altered values of the magnetic coefficients due to heeling, *in their complete exactness.*

pproximate values of the Coefficients relating to ıeeled.—The standard compass is usually placed midship line—axis of X, Fig. 444: as a rule, the ..on is symmetrical with reference to this, so that its one side will wholly or nearly counterbalance that er when the ship is on an even keel; this reduces ıd h to zero, since they represent preponderance of ..n on one side or its *un*symmetrical disposition.

Moreover, as $\mathfrak{A}$ and $\mathfrak{E}$ depend directly on b and d—eqs. (113) and (115), p. 879—these coefficients also reduce to zero; and hence the terms of (427) to (433) containing them disappear. This simplifies the equations, but the process may be carried further.

A ship would seldom have a steady list of more than 12°: the value of this in parts of radius is .218; its sine is .208; sine², .043; and cosine, .978; therefore, without committing very much error, we may replace sin i by i; make $\sin^2 i = 0$; versin $i = 0$; and regard $\cos i = 1$. Introducing these conditions into (427) to (433) they become

$$\lambda_i = \lambda. \quad (434)$$

$$\mathfrak{A}_i = +\left(\frac{c-g}{2\lambda}\right)(i). \quad (435)$$

$$\mathfrak{B}_i = \mathfrak{B}. \quad (436)$$

$$\mathfrak{C}_i = \mathfrak{C} + \frac{1}{\lambda}\left(e - k - \frac{R}{Z}\right)(i)(\tan\theta). \quad (437)$$

$$\mathfrak{D}_i = \mathfrak{D}. \quad (438)$$

$$\mathfrak{E}_i = -\left(\frac{c+g}{2\lambda}\right)(i). \quad (439)$$

$$\mu_i = \mu + \left(\frac{Q}{Z}\right)\left(\frac{1}{\lambda}\right)(i). \quad (440)$$

Let
$$J = \frac{1}{\lambda}\left(e - k - \frac{R}{Z}\right)(\tan\theta); \quad (441)$$

then eq. 437 becomes

$$\mathfrak{E}_i = \mathfrak{E} + (J)(i). \quad . \quad . \quad . \quad . \quad . \quad (442)$$

These are the approximate coefficients relating to the ship heeled: if the list is more than 12°, larger errors will result from their use; if less than 12°, smaller errors.

In (434) to (442) the angle i is expressed in parts of radius, not degrees.

345. Computation of the deviations due to heeling, and analysis of the formula therefor.—The object of Arts. 343 and 344 was to provide formulas by which the deviation due to heeling could be computed: they are supplied in two sets—eqs. (427) to (433), which are exact; and (434) to (442), which are approximate. Both sets afford values of the coefficients with the ship heeled, and the substitution of these in any of the formulas for computing the deviations will give the deviations due to heeling, just as the substitution of the coefficients applicable to an even keel would give the deviations proper to that condition.

For instance, they might be used in eq. (120), p. 880, and the heeling deviation thence computed by means of its *tangent;* or in (328), p. 965, and it obtained by means of its *sine.* The latter will be employed, and is reproduced here, with, however, the angle itself (δ) substituted for its sine.

Let δ_u denote the deviation with the ship upright, and δ_i when heeled, on the same compass course, ζ': then from eq. (328) we have for each condition

$$\delta_u\,(1 - \mathfrak{D}.\cos 2\zeta') = \mathfrak{A} + \mathfrak{B}.\sin \zeta' + \mathfrak{C}.\cos \zeta' + \mathfrak{D}.\sin 2\zeta' + \mathfrak{E}.\cos 2\zeta'. \quad (443)$$

$$\delta_i\,(1 - \mathfrak{D}_i.\cos 2\zeta') = \mathfrak{A}_i + \mathfrak{B}_i.\sin \zeta' + \mathfrak{C}_i.\cos \zeta' + \mathfrak{D}_i.\sin 2\zeta' + \mathfrak{E}_i.\cos 2\zeta'. \quad (444)$$

Substituting in (444) the values of $\mathfrak{A}_i$, $\mathfrak{B}_i$, etc., from (434) to (442), there will occur in (444) some terms identical

with others in (443); subtracting the latter from the former will cause all such terms to disappear: of the remaining terms, those containing $\mathfrak{A}$ and $\mathfrak{E}$ will reduce to zero because of the hypothesis in Art. 344, and the terms $-\delta_i.\mathfrak{D}.\cos 2\zeta'$ and $+\delta_u.\mathfrak{D}.\cos 2\zeta'$ may be considered close enough in value to cancel; then the following is the final result:

$$\delta_i - \delta_u = +\left(\frac{c-g}{2}\right)\left(\frac{i}{\lambda}\right) + \frac{1}{\lambda}\left(e - k - \frac{R}{Z}\right)(\tan\theta)(i)\cos\zeta' - \left(\frac{c+g}{2}\right)\left(\frac{i}{\lambda}\right)\cos 2\zeta'. \quad (445)$$

By means of (441) this becomes

$$\delta_i = \delta_u + \left(\frac{c-g}{2\lambda}\right)(i) + J.i.\cos\zeta' - \left(\frac{c+g}{2\lambda}\right)(i)(\cos 2\zeta'). \quad (446)$$

Substituting for $\cos 2\zeta'$ its equivalent $(\cos^2\zeta' - \sin^2\zeta')$, and multiplying the term $\left\{\left(\frac{c-g}{2\lambda}\right)(i)\right\}$ by $(\sin^2\zeta' + \cos^2\zeta' = 1)$, eq. (446) becomes, after rearranging its terms and cancelling,

$$\delta_i = \delta_u + J.i.\cos\zeta' + \frac{c}{\lambda}.i.\sin^2\zeta' - \frac{g}{\lambda}.i.\cos^2\zeta'. \quad (447)$$

And this is the complete expression for calculating the heeling deviation, using the approximate values of the coefficients: it is composed of the deviation when upright, (δ_u), and three other terms whose nature will now be stated.

In the term $\left(-\frac{g}{\lambda}.i.\cos^2\zeta'\right)$ the factor g alone requires consideration. Fig. 537 shows that it represents soft iron parallel to the axis of X, above or below the compass—generally below, such as the keel or propeller shaft; it will therefore have greatest effect when the compass is near the bow or stern and when the ship's head is north or south; for with the first condition, the compass will be near the

pole of g, and with the second the pull will be at right angles to the needle; on the other hand, g will have little effect when the compass is placed near the middle of the ship, and none at all when she heads east or west, for then opposite polarities extend along the sides of the iron—equal in amount at each end—and therefore ineffective. The influence of g is seldom *very* great; but even at its utmost—ship's head north—it rapidly diminishes by change of course, on account of the factor $\cos^2 \zeta'$.

In the term $\left(+\frac{c}{\lambda} . i . \sin^2 \zeta'\right)$ only the effect of c need be examined. It represents vertical soft iron—such as the smoke-stack—in the midship line. By Fig. 537, when the ship heads north or south, the pull of c is in the direction of the needle, and without avail; but heading east or west, it is at right angles to it, and has a maximum effect; the degree of this depends on the proximity of the compass to the pole of the iron c represents: of course, it may be placed so near the upper end of a smoke-stack as to be completely dominated by it; but such sites would naturally be avoided.

The term $(+J . i . \cos \zeta')$ is the most prolific source of the heeling error; for a fuller view of its composition, its equivalent, (441), will be put under the following form by replacing the angle i by its *sine* and substituting $\frac{Z}{H}$ for $\tan \theta$:

$$\left\{\frac{e . Z . \sin i}{\lambda . H} - \frac{k . Z . \sin i}{\lambda . H} - \frac{R . \sin i}{\lambda . H}\right\} \cos \zeta' . \quad . \qquad (448)$$

Referring to Fig. 537, the part $(e . Z . \sin i)$ of eq. (448) represents the effect of transverse soft iron (e)—such as beams, a superstructure deck or bridge; the part $(-k . Z . \sin i)$ represents the influence of soft iron (k) directly above or below the compass—such as stanchions on the gun-deck; the part $(R . \sin i)$ stands for the heeling force of permanent magnetism in the hull when this is listed to the angle i: all

three terms represent action against a directive force $\lambda . H$, and by observing the location of their sources in Fig. 537, it will easily be seen that their effect is a maximum with the ship's head north or south, and zero when it is east or west; and this is otherwise apparent by the fact that $\cos \zeta'$ is a factor of *all* three terms *in* eq. (448), for $\cos 0°$ and $180° = 1$; and $\cos 90°$ and $270° = 0$.

346. Various forms of the equation for J, and special use of each.—Eqs. (445) to (448) are useful chiefly for exhibiting the separate sources of the heeling error: they are varied forms of the same equation for computing δ_i, but up to this stage of the subject are not available for numerical calculation, because some of the quantities are not known.

Considering for the present that c and g are known (as the means of making them so will be supplied in the next article), there remains only the coefficient J to be determined, and then eq. (447) becomes available for practical use.

Reproducing (441), changing all the signs, and rearranging the terms, we have

$$-J = \left\{ \frac{1}{\lambda}(-e)(\tan \theta) \right\} + \left\{ \frac{1}{\lambda}\left(+k + \frac{R}{Z} \right)(\tan \theta) \right\}. \quad (449)$$

The minus sign indicates that the force J is exerted in a direction opposite to the heeling of the ship—that the north point of the compass is drawn toward the high side: what its actual sign will be in any given circumstances depends on the signs of the quantities in the second member of (449). From eq. (308), p. 957, we have

$$-e = 1 - \lambda + \lambda . \mathfrak{D}, \quad . \quad . \quad . \quad . \quad . \quad (450)$$

and from (387), p. 1008,

$$\mu - 1 = +k + \frac{R}{Z}. \quad . \quad . \quad . \quad . \quad . \quad (451)$$

Substituting these values in (449), it becomes

$$-J=\left(\mathfrak{D}+\frac{1}{\lambda}-1\right)\tan\theta+\left(\frac{\mu-1}{\lambda}\right)\tan\theta. \quad . \quad (452)$$

The coefficient $\mathfrak{D}$ is deduced from a table of deviations; λ and μ are obtained by oscillation experiments with horizontal and vertical needles respectively, and θ is found on a magnetic chart: thus all the quantities of the second member become known with the ship upright, and the *principal term* of the heeling deviation may thence be computed by (452) *without actually listing the vessel.*

The second members of (449) and (452) are made up of two distinct parts; replacing tan θ by $\frac{Z}{H}$ in the former, and equating like terms of both the second members, we have

$$\left(-\frac{e.Z}{\lambda.H}\right)=\left(\mathfrak{D}+\frac{1}{\lambda}-1\right)\tan\theta. \quad . \quad . \quad (453)$$

$$\left(+\frac{k.Z+R}{\lambda.H}\right)=\left(\frac{\mu-1}{\lambda}\right)\tan\theta. \quad . \quad . \quad . \quad (454)$$

By (453) the horizontal induction in soft iron (e) may be computed; and by (454) the vertical induction in soft iron and permanent magnetism (that is, $k+R$) may be obtained.

347. Formulas for computing c, e, g, μ, k, **and** R.

c] There are two methods for determining c—with the ship upright, but the observations must be made in different latitudes; and heeled, when they may be made in the same locality. For the first, from (341), p. 970,

$$c=\lambda\left(\frac{H_1.B_1-H_2.B_2}{H_1.\tan\theta_1-H_2.\tan\theta_2}\right). \quad . \quad . \quad . \quad (455)$$

In this, H_1 and θ_1, H_2 and θ_2 are the Earth's Intensity and Dip in each latitude; B_1 and B_2 the coefficients deduced from swingings at both places; and λ is computed.

For the second, (447), p. 1020, is used: if the ship be headed east or west by compass, since $\cos 90° = \cos 270° = 0$, the terms containing $\cos \zeta'$ will disappear; and then, as $\sin 90° = \sin 270° = 1$, eq. (447) becomes

$$\delta_i = \delta_u + \left(\frac{c}{\lambda}\right)i; \quad \text{whence} \quad c = \frac{\lambda(\delta_i - \delta_u)}{i}. \quad . \quad (456)$$

Therefore, with the ship upright and east or west by compass, an observation for deviation is to be made; then she must be heeled to the angle i—brought to head the *same* course by compass, *while heeled*—and another observation for deviation made: δ_u and δ_i are thus obtained; λ is computed; and i observed.

e] From (252), p. 939,

$$e = \lambda(1 - \mathfrak{D}) - 1. \quad . \quad . \quad . \quad . \quad . \quad (457)$$

The coefficient $\mathfrak{D}$ is deduced from a table of deviations, and λ computed.

g] If the ship head north or south, $\sin \zeta' = 0$, $\cos \zeta' = 1$, and (447), p. 1020, becomes

$$\delta_i = \delta_u + J.i - \frac{g}{\lambda}.i; \quad \text{whence} \quad g = \lambda\left\{J + \left(\frac{\delta_u - \delta_i}{i}\right)\right\}. \quad (458)$$

That is: head the ship north or south, and while first upright and next heeled—both on the same compass course—observe for deviation; δ_u and δ_i are thence obtained; λ is computed; i observed; and J determined by (452).

As c and g are obtained by similar observations, it is well to determine them in order to know exactly how their omission would affect equations in which they enter. Many ships can be heeled without much labor: the requisite list of 6° to 8° ought to be attainable by trimming coal, filling boilers, lowering boats, massing the crew, training guns, and otherwise disposing heavy weights—all on one side. If g be known, μ can be computed from an observation on *one* magnetic heading, as will soon be seen; and

as g does not change with geographical position, it should be determined with the utmost accuracy once for all.

μ] Values of μ and g can be obtained from (96), p. 875: the equation is reproduced as follows, after substituting in it the value of μ from (387), p. 1008, and transposing

$$\mu = \frac{Z'}{Z} + \frac{h}{\tan\theta} \cdot \sin\zeta - \frac{g}{\tan\theta} \cdot \cos\zeta. \quad . \quad . \quad (459)$$

In this Z is the vertical force of the Earth alone, Z' that of Earth and Ship combined—both determined by oscillation of a vertical needle; θ the Dip, taken from a magnetic chart; and ζ the magnetic course on which the observation is made: this leaves only g and h to be considered.

It was explained in Art. 342 how the terms containing g and h disappear when observations of $\frac{Z'}{Z}$ are made on certain equidistant divisions of the circle, not compass points, thus giving μ.

When the observations Z_1' and Z_2' are made, respectively, on *two opposite* headings, there will result two equations like (459), in which the terms having $\sin\zeta$ and $\cos\zeta$ as factors will have opposite signs, and therefore cancel; then by addition

$$2\mu = \frac{Z_1'}{Z} + \frac{Z_2'}{Z}; \quad \text{whence} \quad \mu = \frac{1}{2}\left(\frac{Z_1'}{Z} + \frac{Z_2'}{Z}\right) . . \quad (460)$$

There are other ways in which (459) may be used to furnish a value of μ.

FIRST. If the soft iron that h represents be symmetrical with respect to the axis of X, h reduces to zero; or the want of symmetry may be so slight that h may be omitted: in either case a single observation with the ship's head east or west will cause the term in g to vanish, and then *μ is simply the ratio* $\frac{Z'}{Z}$.

Regarding h as negligible, observations on
netic headings ζ_1 and ζ_2 will give both μ and
n (459), omitting the term in h, we have,

$$\frac{g}{\tan\theta}.\cos\zeta_1,\ (461);\ \text{and}\ \mu=\frac{Z_2'}{Z}-\frac{g}{\tan\theta}.\cos\zeta_2.\quad(462)$$

›lying (461) by cos ζ_2 and (462) by cos ζ_1 and sub-
g latter from former, we thence deduce for μ

$$\mu=\frac{\left(\frac{Z_1'}{Z}\right)\cos\zeta_2-\left(\frac{Z_2'}{Z}\right)\cos\zeta_1}{\cos\zeta_2-\cos\zeta_1}.\quad .\quad .\quad .\quad(463)$$

nd for g, equating the second members of (461) and (462),
nd solving

$$g=(\tan\theta)\frac{\left(\frac{Z_1'}{Z}-\frac{Z_2'}{Z}\right)}{(\cos\zeta_1-\cos\zeta_2)}.\quad .\quad .\quad .\quad(464)$$

Third. If h be either zero or so small as to warrant omitting it, and at the same time g be known, then a single observation on *any* magnetic heading will make eq. (459) as follows:

$$\mu=\frac{Z'}{Z}-g.\cos\zeta.\cot\theta.\quad .\quad .\quad .\quad .\quad(465)$$

k] Let observations denoted respectively by Z_1, Z_1', μ_1, and Z_2, Z_2', μ_2, be made in two places: then from (387), p. 1008, we have for both places

$$1+k+\frac{R}{Z_1}=\mu_1\quad .\quad(466);\quad \text{and}\quad 1+k+\frac{R}{Z_2}=\mu_2.\quad .\quad .\quad(467)$$

Subtracting (467) from (466) after clearing of fractions, then

$$k=\frac{Z_1(\mu_1-1)-Z_2(\mu_2-1)}{(Z_1-Z_2)}.\quad .\quad .\quad .\quad(468)$$

R] Subtracting (467) from (466) as they stand, and solving, we have

$$R=\frac{Z_1 . Z_2(\mu_1-\mu_2)}{(Z_2-Z_1)}. \quad . \quad . \quad . \quad . \quad . \quad (469)$$

The separation of k and R is essential to proper compensation of the heeling error; for k represents transient magnetism in vertical soft iron, while R represents permanent magnetism; and it is evident that both cannot be corrected by the same means.

To render the data for (465), (468), and (469) the more accurate, observations should be made where Z_1 and Z_2 have the greatest possible values, preferably in different hemispheres; then the Dip changes sign, and the sum of Z_1 and Z_2 replaces their difference in the denominators of (468) and (469), which conduces to accuracy of the results.

348. Graphical method of determining μ and g.—The substance of this article is taken from the excellent work of Captain A. Collet of the French Navy, to which the present writer is otherwise indebted for many useful points.

This method is the geometrical solution of (459), Art. 347, if we consider h zero or negligible on account of its smallness. Omitting the term containing h, then, the equation is

$$\mu=\frac{Z'}{Z}-\frac{g}{\tan\theta}.\cos\zeta. \quad . \quad . \quad . \quad . \quad . \quad (470)$$

In this, Z is to be observed ashore, and Z' on board; the latter may be done with the ship successively on various headings, thus affording a series of values of $\frac{Z'}{Z}$ for a number of magnetic courses (ζ) that need not necessarily be equidistant: two very important quantities are thence

deduced from few or many observations; if these are carefully made, and the construction accurately done on a large scale, reliable results should be obtained.

The following observations will be constructed to illustrate the method: $\zeta_1 = \text{N. } 3° \text{ E.}$, or $3°$, and $\frac{Z_1'}{Z} = 1.172$; $\zeta_2 = \text{S. } 26° \text{ W.}$, or $206°$, and $\frac{Z_2'}{Z} = 1.141$; $\zeta_3 = \text{N. } 54° \text{ W.}$, or $306°$, and $\frac{Z_3'}{Z} = 1.196$. With a radius assumed equal to unity, the Earth's vertical force (Z) at the place, describe a circle—Fig. 540: this radius, divided into one hundred

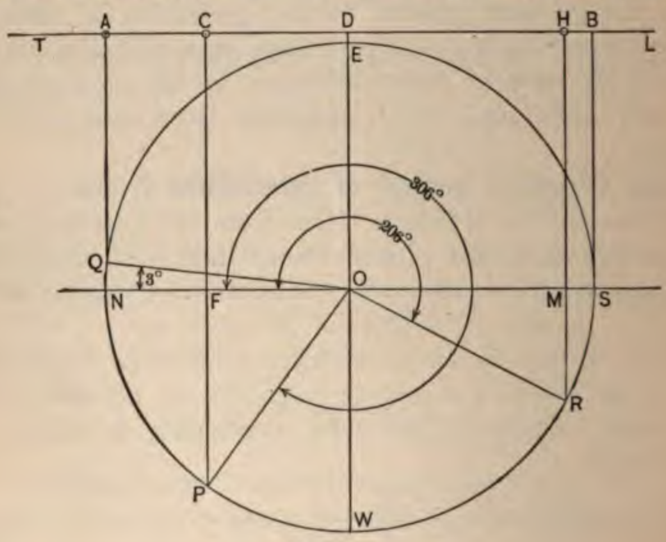


FIG. 540.

parts is then the scale for lengths. Draw two diameters at right angles to each other and mark their extremities the cardinal points—that to the left north, to the right south, top east, bottom west. Count from north to the right the successive magnetic courses: $NOQ = \zeta_1 = 3°$; $NOR = \zeta_2 = 206°$; and $NOP = \zeta_3 = 306°$; and plot them

accordingly. From P draw $\overline{PF}$ perpendicular to $\overline{NS}$; then $\overline{PF}=\cos\zeta_3$, since radius is unity, and similarly, $\overline{RM}=\cos\zeta_2$.

From the points N, F, M, that is, where the various cosines cut the meridian line, lay off the values of $\frac{Z'}{Z}$ corresponding to the magnetic courses ζ_1, ζ_2, ζ_3, toward the top if positive, toward the bottom if negative; thus, $FC=\frac{Z_3'}{Z}$. Negative values occur when the *lower* pole of the dipping needle used in observing Z' on board is of opposite name to the lower pole when Z is observed ashore. The *ends* A, C, H of the lines representing $\frac{Z_1'}{Z}$, $\frac{Z_2'}{Z}$, $\frac{Z_3'}{Z}$ should abut on a straight line $\overline{TL}$, which is to be drawn; but this will occur only when the observations are rigorously exact—which is scarcely ever; therefore the line $\overline{TL}$ should pass equally distant from all the *ends*.

Eq. (470) being thus constructed, we have μ equal to the length $\overline{OD}$, which corresponds to $\zeta=90°$, or $\zeta=270°$; in both these cases $\cos\zeta=0$, whence the term of eq. (470) in which it occurs vanishes, leaving

$$\mu=\frac{Z'}{Z}=\overline{OD}. \quad . \quad . \quad . \quad . \quad . \quad (471)$$

Or, again: $\overline{NA}$ is the value of $\frac{Z'}{Z}$ corresponding to $\zeta=0°$, and $\overline{SB}$ that for $\zeta=180°$; and $\cos 0°=+1$; $\cos 180°=-1$; substituting these successively in (470) we have

$$\mu=\overline{NA}-\frac{g}{\tan\theta}(+1),\ (472);\ \text{and}\ \mu=\overline{SB}-\frac{g}{\tan\theta}(-1). \quad (473)$$

Adding, and deducing value of μ, it is

$$\mu=\tfrac{1}{2}(\overline{NA}+\overline{SB}). \quad . \quad . \quad . \quad . \quad . \quad (474)$$

Subtracting (472) from (473), and deducing value of g, it is

$$g=\tfrac{1}{2}(\overline{SB}-\overline{NA})\tan\theta. \quad . \quad . \quad . \quad . \quad (475)$$

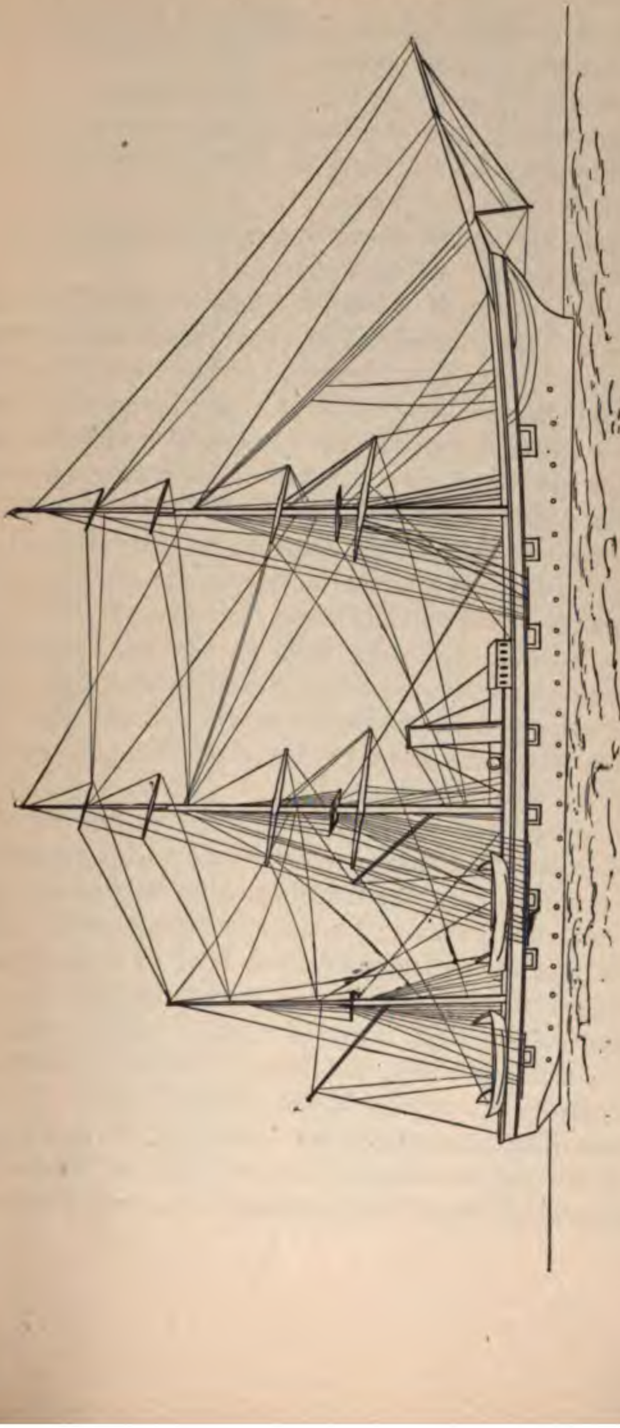
Thus, by measuring $\overline{OD}$, $\overline{NA}$, and $\overline{SB}$ by the scale of the radius, and taking θ from a magnetic chart, we obtain values of μ and g from eqs. (471), (474), and (475).

349. The heeling error ascertained by listing the ship.—The foregoing articles contain various formulas for computing the deviations due to heeling, without actually listing the vessel: the formulas that would generally be used are the approximate ones, based on certain assumptions regarding the soft iron in the structure, and the smallness of some of the quantities entering the equations, which warrants their omission.

While it is probable that the simplified formulas would in most cases give results adequate to the requirements of navigation, still this cannot be positively known: THE ONLY WAY TO DETERMINE THE ACTUAL CONDITION OF AFFAIRS, IS TO HEEL THE SHIP AND SWING HER. The means suggested on page 1024 for heeling, may suffice to produce a list of 6°, and this done first to one side and then to the other and the ship swung in each condition as well as upright—*all on the same compass-points*—at the same time and place, would certainly be a more satisfactory and complete procedure than any computations. The conditions in their entirety and as they really exist would then be known, instead of dealing partially with conditions that to some extent are *assumed;* and the matter is altogether too important to be treated otherwise than in the most thorough manner.

Once the observations completely made, the analysis by formulas can be applied in any desired way, the details of the condition fully investigated, and the compass intelligently and completely compensated.

In order that this part of the subject may be better



U. S. S. RANGER.

appreciated, the examination of the U. S. S. RANGER (iron), both heeled and upright, made in 1883 by Lieut. W. P. Ray, U.S.N., at San Francisco, will be reproduced here.

The analysis of the observations was made by Comdr. John Hubbard, U.S.N., then on duty in the Office of Superintendent of Compasses.

To make the matter complete, a short account of the career of the vessel will be given.

The RANGER was built at Wilmington, Delaware; launched in 1874, completed in 1877; displacement when ready for sea, 1,020 tons; length, 175 feet; beam, 32 feet; depth from rail to keel, 23 feet; iron beams and knees; wooden masts and yards; four transverse iron bulkheads.

On being commissioned she made a few passages between Norfolk and New York, and then proceeded to the Asiatic Station *via* the Suez Canal, stopping at various ports en route. While on the station during three years, she visited the principal ports of China and Japan, and then sailed for San Francisco. During 1880–81 she was repaired at the Mare Island Navy-yard, California, and was subsequently employed on surveying service on the west coast of Mexico, where she was up to the time of the observations given hereafter.

Fig. 541 shows the results of a magnetic survey of the ship made shortly before the observations for heeling: it will be perceived that the distribution of magnetism in the hull is clear and striking; on the starboard side the line of neutrality extends diagonally from bow to stern; on the port side, similarly, only that it meets the keel at about two-thirds of its length; on both sides, above this line, we have south polarity, and below it north polarity—the characteristics of a ship built head northward.

The standard compass is in the midst of the south polarity, whose focus lies *abaft* it, and not very far distant. As the ship swings out of the meridian to the right, this pole

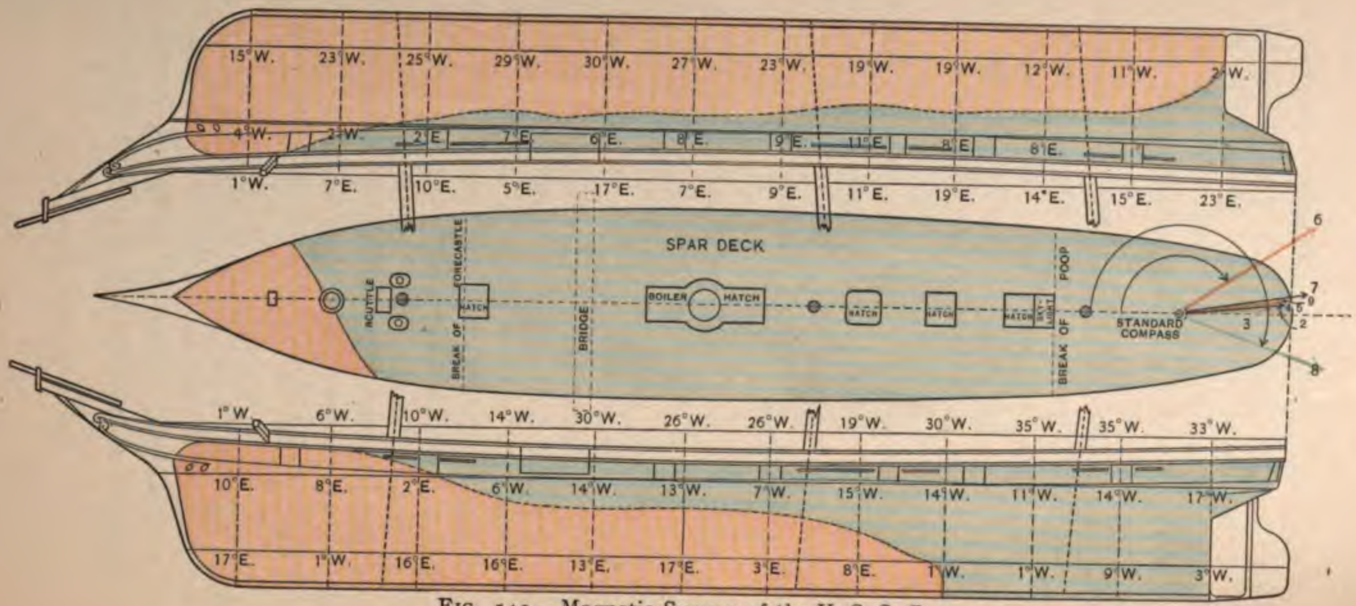


FIG. 541.—Magnetic Survey of the U. S. S. RANGER.

(To face p. 1032.)

turns to the left and attracts the north end of the needle, or, which is equivalent, repels its south end to the right, so that we have the curve of deviations traced first to the westward of the central line, as shown by the tables hereafter.

The arrows numbered 2 to 9, both inclusive, except 6 and 8, represent the direction and also (relatively among themselves) the amount of the polar force acting upon the compass; those numbered 6 and 8 represent this force with the ship heeled respectively to port and to starboard.

The two different methods of determining the magnetism acting upon the compass—namely, that by survey of the hull in dock, and from tables of deviation by swinging the ship—are thus seen to be, as they should be, in accord.

Now directing our attention to the tables, those numbered 84, 85, 86, and 87 relate to observations with the ship upright, made a year previous to the series heeled: they are inserted as part of the magnetic history of the ship.

Table 84 contains the results of separate swingings at different places as indicated at the heads of the columns: the method of time-azimuths of the sun was used while steaming in a circle, and the observations were made on every point—there are no interpolations.

Tables 85, 86, and 87 contain the analyses of Table 84, and the numbers of the columns and office index letter correspond throughout all the tables; that is, the letter "*A*" and col. (2) of Tables 85, 86, and 87 contain the analysis of col. (2), letter "*A*," Table 84; and similarly for the other columns.

Cols. (3) and (4), Table 84, are observations made to ascertain the effect of elevation on the compass: for col. (3) it was at the extreme height of screw and ratchet, 9½ feet above the deck, and for col. (4) it was at the usual elevation of 4¾ feet. That the increased elevation was

Table 84.

)TAL DEVIATIONS, U. S. S. RANGER.

	Upright.	Upright.	Upright.	Upright.
a-..	February 1 to Mar. 18 1877.	March 16, 1882.	March 16, 1882.	Nov. 18, 1882.
serva-.....	Hampton Roads and New York.	Isla Grande, Mexico.	Isla Grande, Mexico.	Acapulco, Mexico.
Index Let-...........	"*A.*"	"*C.*"	"*C*" bis.	"*D.*"
(1)	(2)	(3)	(4)	(5)
Ship's Head by Standard Compass.	Deviation.	Deviation.	Deviation.	Deviation.
NORTH	0° 25′ East.	0° 23′ West.	3° 16′ East.	2° 16′ East.
N. by E.	1 45 West.	1 06	0 41	0 06 West.
N. NE.	2 0	2 15	2 53 West.	1 23
NE. by N.	4 45	3 49	3 58	6 17
NE.	8 45	4 52	6 34	7 14
NE. by E.	10 45	6 57	9 09	9 49
E. NE.	13 15	8 0	11 43	11 40
E. by N.	12 55	9 04	12 47	12 24
EAST	14 45	9 39	13 52	14 09
E. by S.	15 15	10 13	14 58	15 25
E. SE.	15 30	10 18	14 03	16 17
SE. by E.	15 35	9 20	14 09	16 19
SE.	13 45	8 56	14 15	14 52
SE. by S.	12 20	7 30	11 22	13 16
S. SE.	9 15	5 04	8 29	10 08
S. by E.	3 25	2 10	4 09	5 55
SOUTH	0 37 East.	0 14	0 48	2 40
S. by W.	4 15	2 41 East.	4 04 East.	1 59 East.
S. SW.	6 05	5 36	7 56	6 11
SW. by S.	9 45	7 31	11 48	9 55
SW.	12 45	8 26	13 40	12 36
SW. by W.	13 30	9 21	15 03	13 47
W. SW.	14 15	10 17	15 28	15 23
W. by S.	14 15	9 43	15 51	16 05
WEST	13 15	9 07	14 43	13 47
W. by N.	10 15	8 32	14 17	13 59
W. NW.	9 50	7 28	13 13	13 35
NW. by W.	10 40	6 55	11 08	11 13
NW.	9 20	4 51	9 04	9 27
NW. by N.	8 10	3 18	7 30	7 39
N. NW.	4 50	1 44	5 55	5 58
N. by W.	2 40	0 41	3 51	3 11

TABLE 85.

SEMICIRCULAR COMPONENTS, U. S. S. RANGER.

Office Index Letter.	"A."	"C."	"C" bis.	"D."
(1)	(2)	(3)	(4)	(5)
NORTH	− 0° 06′	−0° 05′	+ 2° 02′	+ 2° 28′
N. by E.	− 3 0	−1 53	− 1 42	− 1 03
N. NE.	− 4 02	−3 55	− 5 24	− 3 47
NE. by N.	− 7 15	−5 40	− 7 53	− 8 06
NE.	−10 45	−6 39	−10 07	− 9 55
NE. by E.	−12 07	−8 09	−12 06	−11 48
E. NE.	−13 45	−9 08	−13 36	−13 32
E. by N.	−13 35	−9 23	−14 19	−14 14
EAST	−14 0	−9 23	−14 22	−13 58
E. by S.	−12 45	−9 23	−14 37	−14 42
E. SE.	−12 40	−8 53	−13 38	−14 56
SE. by E.	−13 07	−8 08	−12 39	−13 46
SE.	−11 33	−6 53	−11 39	−12 03
SE. by S.	−10 15	−5 24	− 9 26	−10 27
S. SE.	− 7 03	−3 24	− 7 12	− 8 03
S. by E.	− 3 02	−1 26	− 4 0	− 4 33

TABLE 86.

QUADRANTAL COMPONENTS, U. S. S. RANGER.

Office Index Letter.	"A."	"C."	"C."	"D."
(1)	(2)	(3)	(4)	(5)
NORTH	+0° 38′	−0° 01′	+0° 22′	−0° 01′
N. by E.	+1 52	+0 49	+1 22	+0 50
N. NE.	+2 26	+1 33	+1 28	+1 52
NE. by N.	+2 43	+1 32	+2 43	+2 11
NE.	+2 06	+1 55	+3 04	+2 42
NE. by E.	+1 44	+1 39	+2 37	+2 24
E. NE.	+1 21	+1 24	+1 35	+1 58
E. by N.	+0 32	+0 31	+0 51	+1 36

TABLE 87.

MAGNETIC COEFFICIENTS, U. S. S. RANGER.

	° ′		° ′		° ′		° ′	
A; 𝔄.....	− 0 19	−.0055	−0 07	−.0020	+ 0 46	+.0133	− 0 05	−.0014
B; 𝔅.....	−14 30	−.2562	−9 38	−.1698	−14 56	−.2644	−15 10	−.2683
C; ℭ.....	+ 0 43	+.0105	−0 21	−.0058	+ 1 17	+.0212	+ 1 52	+.0216
D; 𝔇.....	+ 2 27	+.0427	+1 52	+.0322	+ 2 47	+.0485	+ 2 38	+.0459
E; 𝔈.....	+ 0 45	+.0132	+0 05	+.0014	+ 0 13	+.0031	− 0 06	−.0017
Ship's force and "Starboard Angle."	177°	0.250	182°	.167	175°	.258	173°	.263

beneficial is apparent from the reduced deviations of col. (3), the maximum *reduction* due to height alone being 6° 08′.

The unchanging character of the quadrantal deviation in different parts of the globe will be remarked: it should be so, and attention is directed to it only to point out the coincidence of theory with observation.

On examining cols. (2), (4), and (5) of Table 84, in connection with col. (7) of Table 88, we observe how little the deviations on the same point differ, although determined in such widely separated places as New York, Isla Grande (near Acapulco), and San Francisco, and although they cover a period of six years, beginning with the completion of the vessel and extending through a cruise nearly around the globe, in which the ship was strained and shaken by many a rough sea and periodic target practice with heavy guns. Truly, a hard-steel magnet could scarcely show greater permanency of power.

The compass was *not* compensated at any time.

The quadrantal components being very small (see Tables 86 and 90), the deviation is chiefly semicircular, caused by hard iron, and vertical soft iron; and, as found by further analysis, the effect of the vertical soft iron was to that of the hard iron in the ratio of 1 to 2.3: it follows that the magnetic retentive power of the iron of which the RANGER is built is almost that of tempered steel.

Col. (9) of Table 88 may be designated the "characteristic deviations" of the RANGER; for, being the mean of four separate determinations of those quantities at different times and places, but under similar conditions, the errors incidental to each series may be considered partly corrected or reduced. Fig. 542 is the representation of these "characteristic deviations": it forcibly illustrates a remark made in a previous part of this Treatise—that no matter how irregular the curve of total deviations may be, still its component parts are symmetrical. The curve of

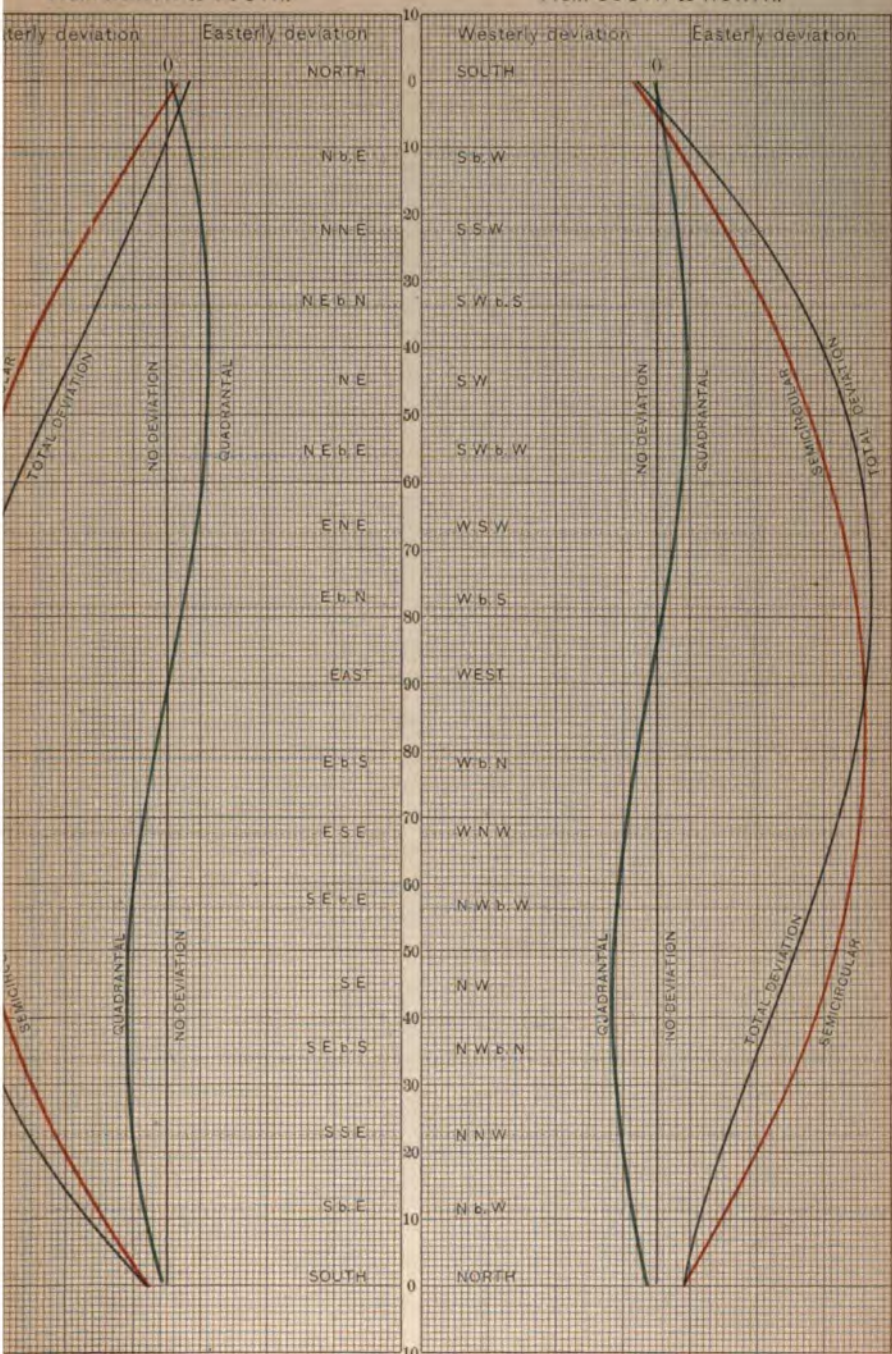


FIG. 542.—Illustrating col. (9) of Tables 88, 89, and 90.

(*To face p.* 1036.)

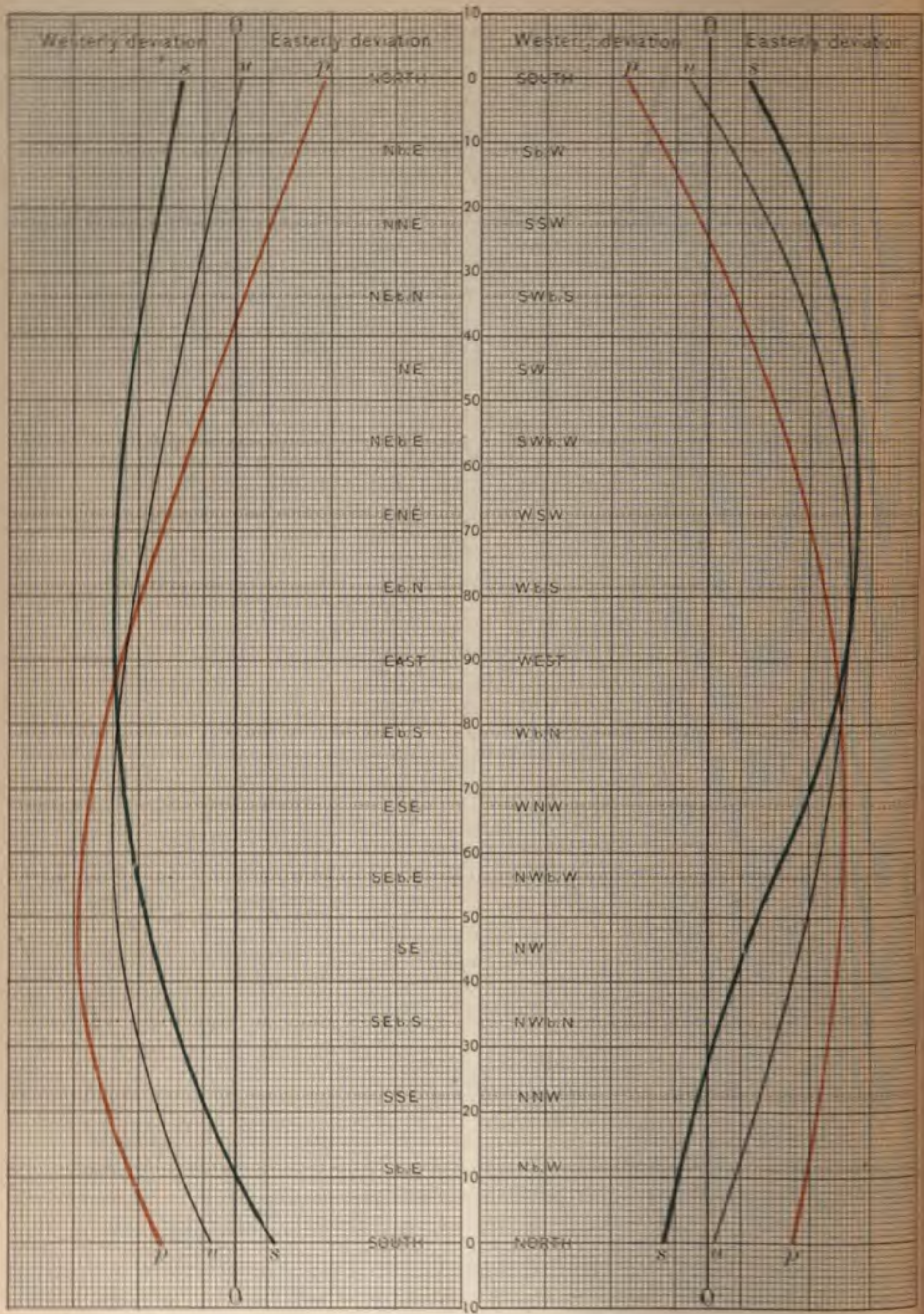


FIG. 543.—Deviations of the U. S. S. RANGER. Illustrating cols. (6), (7), and (8) of Table 88. u = ship upright; s = heeled to starboard; p = heeled to port. Curves natural size.

(To face p. 1037.)

total deviation is full and rounded in both the SE. and SW. quadrants, whereas it is flat and has little curvature in the other two. The semicircular component is almost perfectly regular, and has its maxima at east and west. The quadrantal component, likewise, is almost perfectly symmetrical, with its maxima where they should be, at the intercardinal points. Whence, then, the irregularity of their combination? A glance at the figure gives the answer. In the NE. and NW. quarters the two components are on opposite sides of the central line, hence their resultant is their difference—a flattened curve on the side of the greater; in the SE. and SW. quarters, on the other hand, the components are on the same side of the central line, and so we have a full rounded curve—their sum.

All the curves of Fig. 542 are increased to double their natural size, according to the central vertical scale of degrees

Tables 88, 89, 90, 91, and 92 (except col. 9 wherever it occurs) relate to a connected series of observations made specially to determine the heeling error: the method of reciprocal bearings with a theodolite on shore was used, and an actual observation was made on every point—there are no interpolated values.

Cols. (6), (7), and (8) of Table 88, and the illustration of the total deviations of that table in Fig. 543 again beautifully show how theoretical deductions are borne out by experience. The mathematical consideration of the subject requires that the heeling error should be a maximum at north and south and a minimum near east and west. The wide separation of the curves, Fig. 543, due to heeling, at north and south, and their approximate intersection at east and west, could scarcely be more perfect, considering the difficulties and conditions of this class of observations.

Cols. (10), (11), (12), and (13) of Table 92 need no special explanation; they show that the RANGER has a very large heeling coefficient, and this might readily be

TABLE 88.

TOTAL DEVIATIONS, U. S. S. RANGER

Condition of Ship.	Heeled 6° to Port.	Upright.	Heeled 6° to Starboard.	Upright.
Date of Observation..........	August 4, 1883.	August 1, 1883.	August 2 and 3, 1883.	Mean of Total Deviations from Reports Whose Office Index Letters are "*A*," "*C*," "*D*," and "*F*," "Ship's Characteristic Dev."
Place of Observation..........	San Francisco, Cal.	San Francisco, Cal.	San Francisco, Cal.	
Office Index Letter............	"*H*."	"*F*."	"*G*."	
(1)	(6)	(7)	(8)	(9)
Ship's Head by Standard Compass.	Deviation.	Deviation	Deviation.	Deviation.
NORTH	13° 07′ East.	1° 09′ East.	8° 06′ West.	1° 46′ East.
N. by E.	10 49	0 48 West.	10 33	0 29 West.
N. NE.	6 10	3 15	12 26	2 23
NE. by N.	2 27	5 02	14 57	5 00
NE.	1 33 West.	8 11	15 26	7 41
NE. by E.	4 56	10 54	17 14	10 09
E. NE.	10 12	13 03	18 01	12 25
E. by N.	12 54	14 38	18 27	13 11
EAST	17 00	16 10	17 53	14 44
E. by S.	20 07	18 18	17 57	15 59
E. SE.	22 24	19 04	16 41	16 14
SE. by E.	24 02	18 16	13 45	16 05
SE.	24 01	16 31	12 39	14 51
SE. by S.	23 08	14 46	8 56	12 56
S. SE.	18 31	13 00	4 55	10 13
S. by E.	16 45	8 43	0 40 East.	5 33
SOUTH	11 40	3 20	6 08	1 33
S. by W.	6 00	1 26 East.	11 02	2 56 East.
S. SW.	1 03	7 03	15 58	6 49
SW. by S.	3 39 East.	11 09	19 01	10 39
SW.	8 03	17 01	21 10	14 00
SW. by W.	11 33	18 02	22 10	15 05
W. SW.	14 16	18 31	22 09	15 54
W. by S.	17 05	21 02	20 35	16 48
WEST	18 47	19 26	20 01	15 18
W. by N.	19 08	19 02	16 12	14 23
W. NW.	20 03	16 50	11 59	13 22
NW. by W.	20 20	15 28	9 27	12 07
NW.	19 54	13 21	5 29	10 18
NW. by N.	18 44	11 23	2 24	8 40
N. NW.	16 54	9 05	0 16 West.	6 27
N. by W.	15 11	4 07	4 58	3 27

TABLE 89.

SEMICIRCULAR COMPONENTS, U. S. S. RANGER.

Office Index Letter.	"*H.*"	"*F.*"	"*G.*"	Mean from Reports "*A*," "*C*," "*D*," and "*F*."
(1)	(6)	(7)	(8)	(9)
NORTH	+12° 24′	+ 2° 14′	− 7° 07′	+ 1° 39′
N. by E.	+ 8 34	− 1 07	−10 47	− 1 43
N. NE.	+ 3 37	− 5 09	−14 12	− 4 35
NE. by N.	− 0 36	− 8 05	−16 59	− 7 50
NE.	− 4 48	−12 36	−18 18	−10 36
NE. by E.	− 8 15	−14 28	−19 42	−12 37
E. NE.	−12 14	−15 47	−20 05	−14 10
E. by N.	−14 59	−17 50	−19 31	−14 59
EAST	−17 53	−17 48	−18 57	−15 02
E. by S.	−19 37	−18 40	−17 04	−15 11
E. SE.	−21 24	−17 57	−14 20	−14 48
SE. by E.	−22 11	−16 52	−11 36	−14 06
SE.	−21 57	−14 56	− 9 04	−12 33
SE. by S.	−20 56	−13 04	− 5 40	−10 48
S. SE.	−17 43	−11 03	− 2 19	− 8 20
S. by E.	−15 58	− 6 25	+ 2 49	− 4 30

TABLE 90.

QUADRANTAL COMPONENTS, U. S. S. RANGER.

Office Index Letter.	"*H.*"	"*F.*"	"*G.*"	"*A*," "*C*," "*D*," and "*F*."
(1)	(6)	(7)	(8)	(9)
NORTH	−0° 05′	−1° 22′	−1° 01′	−0° 06′
N. by E.	+1 28	−0 01	+0 34	+0 46
N. NE.	+1 56	+1 31	+2 03	+1 49
NE. by N.	+2 27	+2 14	+2 05	+2 28
NE.	+2 39	+3 00	+3 14	+2 43
NE. by E.	+2 46	+2 38	+2 52	+2 21
E. NE.	+1 25	+2 21	+2 20	+1 49
E. by N.	+1 26	+2 45	+1 37	+1 26

TABLE 91.

MAGNETIC COEFFICIENTS, U. S. S. RANGER.

	° ′		° ′		° ′		° ′	
A; $\mathfrak{A}$....	+ 0 40	+.0116	+ 0 37	+.0107	− 0 16	−.0046	+ 0 15	+.0043
B; $\mathfrak{B}$....	−18 15	−.3199	−18 42	−.3274	−19 03	−.3376	−15 49	−.2796
C; $\mathfrak{C}$.....	+12 35	+.2103	+ 2 35	+.0458	− 6 49	−.1111	+ 1 37	+.0248
D; $\mathfrak{D}$....	+ 2 44	+.1476	+ 2 49	+.0491	+ 3 20	+.0581	+ 2 40	+.0466
E; $\mathfrak{E}$....	− 0 03	−.0008	− 0 55	−.0165	− 0 53	−.0154	− 0 01	−.0005
Ship's force and "Starboard Angle."	145°	0.380	172°	0.320	200°	0.350	174°	0.273

PART FIFTH.

SWINGING SHIP AND COMPENSATION OF THE DEVIATIONS.

CHAPTER XXVII.

VARIOUS METHODS OF SWINGING SHIP.

350. General remarks on the procedure.—To *swing ship* is to head her successively on different courses either at equal or unequal intervals all round the compass. This may be done in many ways: by steaming in a circle; by warping at buoys; by kedging, when anchored, the kedges being laid out from each quarter; or, if the ship be moored in still water, a tug might pull or push the stern around; or, if moored in a river like the Hudson, each tide will turn her through a semicircle by suitably working the helm.

Whatever the process, the bearing by standard compass of some object whose true or magnetic bearing may be determined, is to be observed while on each heading: as, for instance, the sun; or a distant peak; or a pole erected above another compass set up on the shore near by.

Before beginning the observations, the ship should make a rapid circle with each helm, to shake out any transient magnetism that may have acquired lodgement from steering the same course or lying in the same direction for some time.

Experience shows that a difference of 2° to 3° will often result from swingings in opposite directions; therefore, when accuracy is required, two series should be made—one with each helm—and the mean taken: or, if only one set with either helm, the ship should rest nearly five minutes on each heading before taking the bearing that is to be

recorded; and during this time it is well to look frequently at the bearing of the object in order that its exact direction be determined. The stop on each heading is both to allow the Earth's induction its full effect and the compass card to come to rest: and further to contribute to this end, the change from one heading to another should be made with a slow movement of both helm and propeller.

The observations would of course be undertaken only under favorable conditions of sea and weather.

About the points of maximum and minimum deviation there is greatest liability to inaccuracy, because the change in the deviation is then slowest and least for the same angular motion of the ship: if, therefore, for any special purpose, observations are required on two or more headings, it would be best to select those that fall about midway between the maximum and minimum.

When using the sun, the lowest practicable altitude is the best: a series on 16 points occupying an hour and a half, with a low altitude for all, would be preferable to one on 32 points covering nearly three hours, with a consequent greater change of altitude.

351. Time-azimuths—steaming in a circle.—This method is the quickest and most easily carried out.

Fig. 544 illustrates the astronomical basis of the method: SHN is the plane of the horizon, and QMR that of the equator; P is the elevated pole; PZQ the meridian; Z the zenith; PCM a declination circle, and ZCH a vertical circle—both through the sun C. Arcs of these three circles form the astronomical triangle CPZ, any three of whose parts being known, the others may be computed by formulas of spherical trigonometry.

In the figure, $\overline{QZ}=L$, the latitude of the place of observation; $\overline{MC}=D$, the declination of the sun at the apparent time T, which latter is equal to CPZ, the hour angle; NOH or $PZC=A$, is the true azimuth of the sun, reckoned from

the north or south point of the horizon, according to the latitude of the same name. The parts, then, that enter into the present problem are:

$\overline{PZ} = 90° - L.$ (1); $\overline{PC} = 90° - D.$ (2); and $CPZ = T$, (3)

all known, and $PZC = A$, required. . . . . . (4)

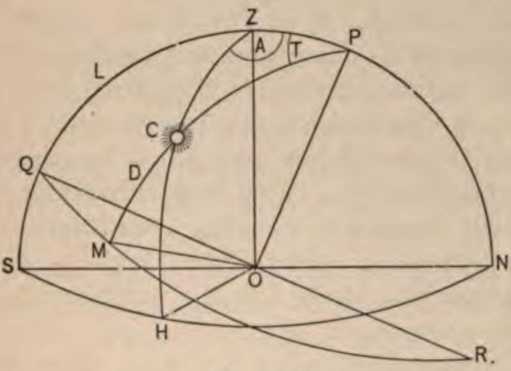


FIG. 544.

It is the case of two sides and the included angle being given, to compute one of the remaining angles.

Let ϕ be an auxiliary angle such that

$$\tan \phi = \cot D . \cos T, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

then $$\cot A = \frac{\cos(\phi + L) \cot T}{\sin \phi} . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This affords the means of calculating the true azimuth A for various hour angles T, different latitudes L, and every degree of the sun's declination D; and in this way *Azimuth Tables* are formed.

When the navigator observes a series of magnetic bearings of the sun for col. (5), Form 7, *noting the time of each by chronometer*, he thereby, in effect, determines a number

of hour angles for col. (4); the latitude of the place and the declination of the sun for the mean period of observation are computed, and with these and the series of hour angles, any Azimuth Tables will afford the data for col. (6)—not directly, perhaps, but by interpolation. Experience will readily suggest to the navigator short methods of interpolation for the tabulated quantities.

Comparison of cols. (5) and (6), Form 7, gives col. (7)—that is, values of the Variation and Deviation combined; the means of separating these is indicated in col. (8). The Variation thus obtained, should be corrected by applying the constant A when it has been found by analysis of the deviations on Form 10. This will necessitate using the new (correct) Variation with the quantities in col. (7), which will give other values of the deviation for col. (9) than those first obtained; but the work should be done, as exactness is essential.

A few time-azimuths can easily be taken in a short period, and it would be a good precaution to do it on every occasion of closing in with the land, at least for those courses that the ship is likely to use in running into port.

352. Observation of a distant object.—When moored, so that the radius of swing is practically the distance of the compass from the bow, and a sharply defined object—peak, light-house, or tower—is clearly in view at a considerable distance, its compass bearing on a series of headings can be taken.

When the compass is in line with bow or stern *and the object*, there is no parallactic error; but when the ship swings 90° from this direction, it is greatest.

In Prof. Greene's work, "Finding the Compass Error," Tables LII and LIII give information regarding the related quantities—distance of object, radius of swing, and parallactic correction; as the book is in ships' libraries, it

will be necessary to state here only a few of the usual cases:

Distance of Object, in Nautical Miles.	Radius of Swing, in Feet.	Maximum Parallactic Error, in Arc.
4.9	200	0° 24′
7.4	225	0 18
11.5	200	0 12

The error increases with the radius of swing and decreases with the distance of the object.

The magnetic bearing of the object from the ship must be determined and one of the following methods will suffice: 1st, the mean of two bearings with the compass in line with the object, first, bow toward it, secondly, stern; 2d, if the bearings by compass in a complete swing be equidistant, their mean will be the magnetic bearing; 3d, if the compass be landed in a spot free from magnetic material and in exact line with the object and the ship (head-on or stern-to), it will be the magnetic bearing. Comparison of the bearing on each heading with its magnetic bearing will give the deviation.

The true bearing of the object may also be used, and Prof. Greene's book contains methods for computing it. If the compass be not landed in the way stated, it will be necessary to compute the true bearing when the compass bearings on the several headings are *un*equally distributed; for then we cannot take their mean as the magnetic bearing. When the true bearing is used, the compass must be landed in a spot free from magnetic material, and the Variation determined by a careful series of time-azimuths; for comparison of the individual bearings with the true bearing will give only a series of Compass Errors, and we need the Variation to separate the parts of these and get the Deviations.

353. **Reciprocal bearings.**—Whoever wishes to use this method will find it explained in Prof. Greene's book.

When the compass buoys were planted in Newport Harbor for swinging ship, two very large tripods were made to be used with this method: when opened out they admitted an observer standing comfortably beneath, and it was intended that one tripod should be placed over the standard compass on board and the other over that ashore; a strong pole about ten feet long, having on the lower end a brass ring on which a thread was cut, was provided for each tripod; it was screwed into a socket in the block to which the legs were attached, and formed a good mark directly above the center of each compass for the observers to sight upon.

The distinctive feature of reciprocal bearings is, that they are to be taken simultaneously upon a preconcerted signal: this requires alertness and attention on the part of the observers, *additional* to that of noting the bearing itself; and the method otherwise has inherent sources of trouble and error that render it less practicable than time-azimuths.

354. **Known lines of magnetic bearing.**—In some ports of Europe advantage has been taken of a tall chimney or tower in the vicinity, to make it a mark for observation from different points of view—the center of a series of magnetic directions that, like radii, spread out at intervals of one degree. Where they strike the walls of a wharf or dock, large numbers indicating the amount of the bearing are painted, and these are clearly seen from the water.

A ship moored off the wharf may be swung by warps, and as she crosses any of the radii, the bearing of the tower is taken at the instant the standard compass is upon the line—its number is noted—and comparison of the two bearings gives the deviation directly. This is the principle—it may be variously carried out in different ports. The method is illustrated on page 71 of the Liverpool Compass Report.

355. The best means of determining the deviations. To swing ship at a compass station, the bow fast to the central buoy and a line from each quarter to the corner buoys for warping the stern around and steadying the ship on each point; the water smooth; the weather clear and calm; in the early morning or late afternoon; with a rest of about five minutes on each heading; and observing a very careful series of time-azimuths:—these are probably the conditions most conducive to accuracy. They are preferable to time-azimuths under way.

CHAPTER XXVIII.

INSTRUMENTS AND MECHANICAL APPLIANCES USED IN COMPENSATING.

356. The theory of compensation.—The *principle* of neutralizing the source of the deviations is so simple that it will be well to illustrate it apart from any particular case.

In Art. 173 the field of a magnet is fully described, and in compensating compasses it is with magnetic fields that we have to deal.

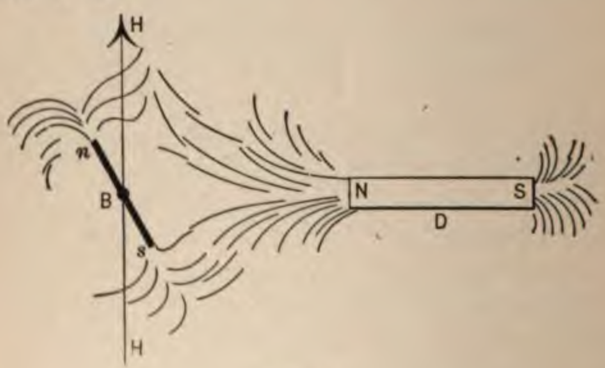


Fig. 545.

Consider Fig. 545: a small pivoted needle B is deflected from the meridian by a steel magnet D, and this is conceived to be effected by the lines of force surrounding N coming in conflict with those about n as well as uniting with others around s. The field of the Earth tends to turn the needle parallel to H.

The needle B may be brought to the meridian by neutralizing the field of D; and as the influence of magnets extend in space all round them, this neutralization can be effected in several ways: by placing another magnet sym-

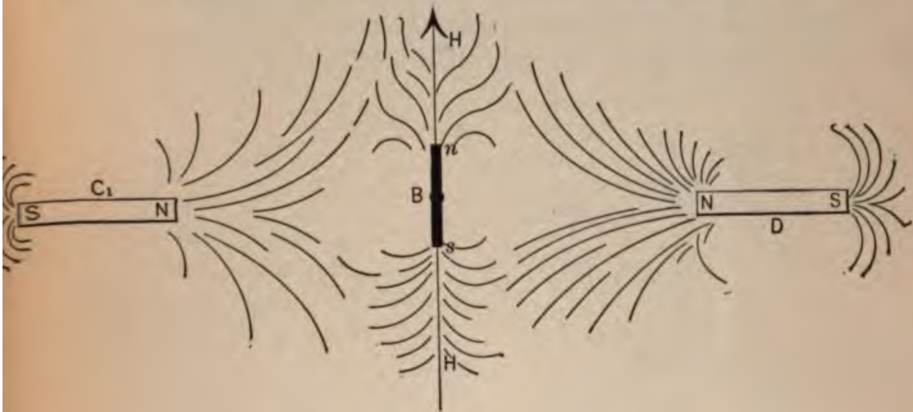


Fig. 546.

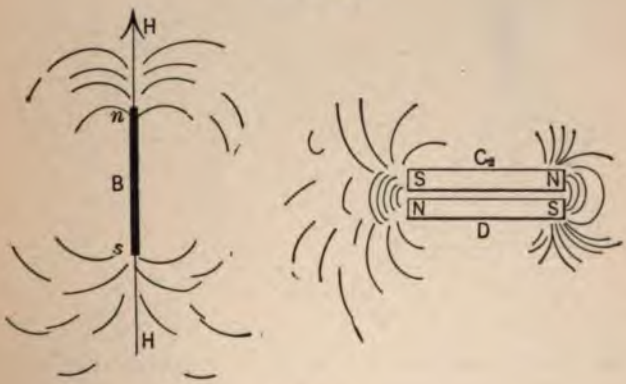


Fig. 547.

metrically opposite, as C_1, Fig. 546; or beside D, as C_2, Fig. 547; or at right angles to B, either north or south of it, as C_3 and A, Fig. 548; or directly above or below it, as A' and C_4, Fig. 549.

So far, the effect of *D* has been considered at right angles to the meridian: to neutralize it, the magnet *C* had to be similarly placed also at right angles to the meridian.

If the disturbance is *in* the meridian or parallel to it, the counteracting influence must also be—alongside the disturbance; or symmetrically placed on the opposite side of *B*; or parallel to the meridian on the right or left, above

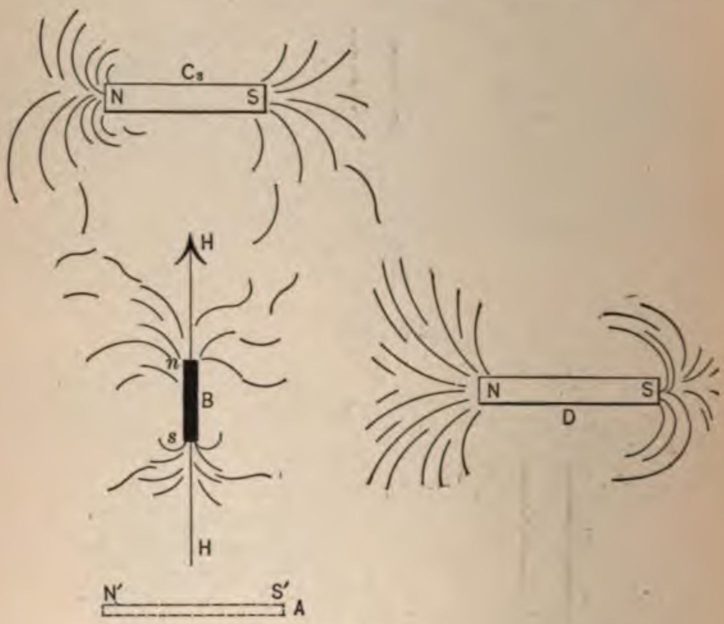


Fig. 548.

or below—that is, in the space about *B*, but always either in the meridian or parallel to it.

The two positions—in the meridian and across it—are therefore analogous, and the disturbing force in each is counteracted in a similar way.

Their composition—a single disturbing force in any other direction—will be overcome by a combination of the

compensating magnets used to neutralize the components of the disturbing force; this is shown in Fig. 550.

When the ship heads N. magnetic, the needle is deflected into the position B' by that component of D which may be resolved across the meridian; this is therefore neutralized

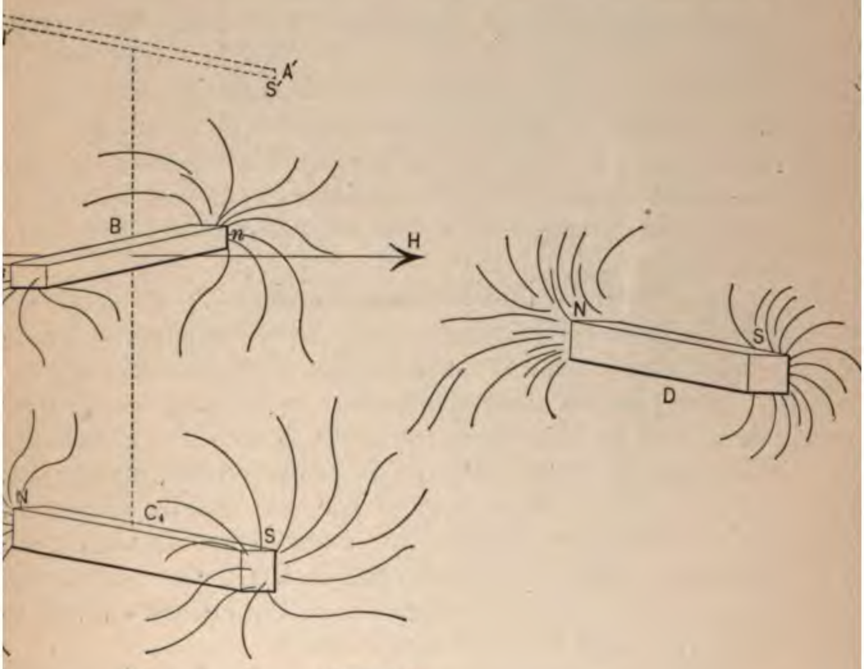


Fig. 549.

and the needle brought into the meridian as at B by a magnet placed as C_5: when the ship swings to east, the part of D resolved parallel to the meridian will now deflect the needle into the position B'', and it must be brought back by a magnet C_6. The influence of D is now neutralized for *every* heading.

Looking at Figs. 545 to 549, it would appear that—theoretically—the influence of the Ship, when simple, can

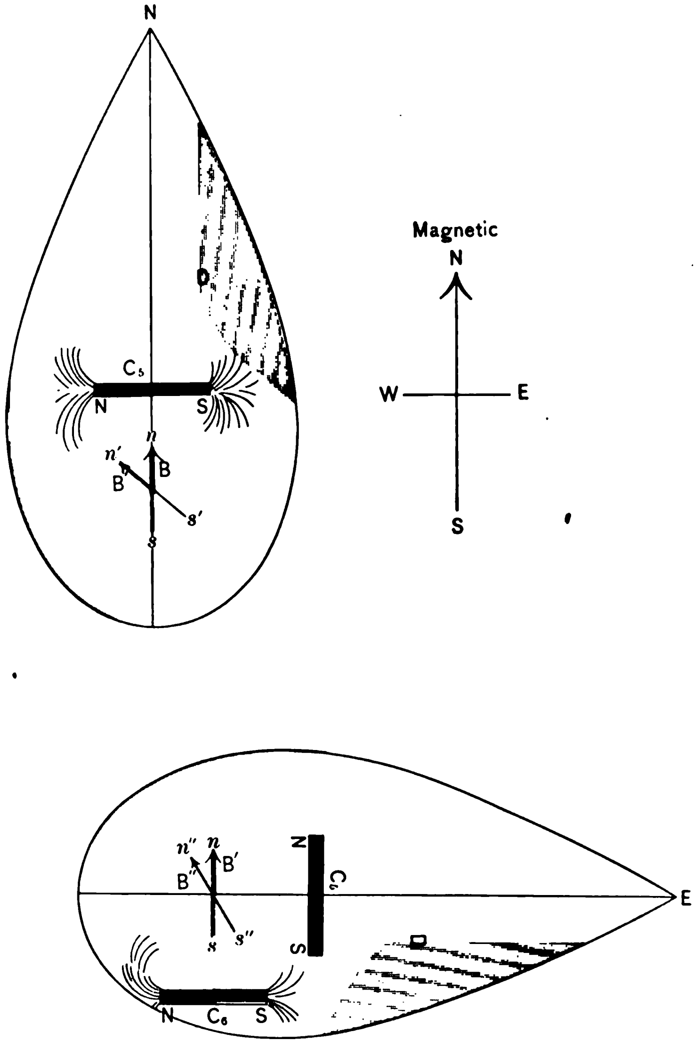


FIG. 550.—Compensation by the Component Method.

(*To face p.* 1056.)

be neutralized by a single magnet suitably placed; and that this once effected, it holds good for every azimuth and all regions of the Earth, *provided* the disturbing and compensating forces retain their relative strength intact. But these conditions do not exist. As has been shown in *Part Third*, the resultant disturbing force of the ship is most complex in its composition and varies with time, place, and azimuth; the compass-card is constrained to move in a horizontal plane; all disturbing and compensating forces must be resolved into this plane and the vertical; and as the disturbances are both permanent and transient in each plane, the appliances to correct them must be in both planes and possess magnetism both permanent and transitory.

357. **Material used for compensating.**—To no condition of things does the phrase *similia similibus curantur* apply more exactly than to compensation of the deviations.

For permanent magnets, steel rods about one-quarter inch thick and of lengths from 6 to 12 inches will probably be found the best: they can be tempered and magnetized more uniformly than bars of the size usually employed, and—weight for weight—are more powerful than solid masses. Besides, their strength is likely to endure, and their number can be increased or diminished at will to overcome large or small deviations.

The rods should be tempered "glass-hard," magnetized by a powerful electric current, and laid away six months before using—the longer the better—to season them.

They are used in two ways: 1st, according to the RESULTANT METHOD—in a single group, horizontally, beneath the compass, their axes in the direction (starboard angle) of the horizontal resultant of the ship's permanent m netism; 2d, in two groups (Fig. 550), the COM METHOD—one group fore-and-aft, the other ath

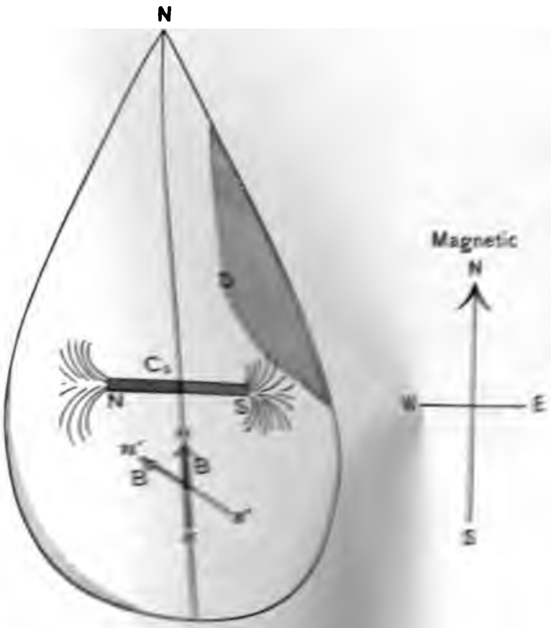


FIG. 550.—[illegible]

these neutralize P and Q (Figs. 434 and 444). They are also used to neutralize R (Figs. 459 and 537), the vertical part of the ship's permanent magnetism.

To counteract the effect of transient magnetism we need material that gives the freest entrance and exit to magnetism, and acquires it in the highest possible ratio to the inducing force: and cast and wrought iron fulfill these requirements.

The iron is generally required in two forms: tubes about three inches external diameter with walls one-quarter inch thick, and of various lengths from three inches to twelve, so that, screwing several together, we may form one tube of about six feet; and hollow spheres whose walls are one-quarter inch thick, and of diameters varying between five and ten inches.

Before using, both tubes and spheres should be soaked in a blood-red fire for some hours and then cooled very slowly by burying in the ashes or allowing the fire to go out; subsequent work upon them, such as hammering, polishing, or filing, should be avoided, as it hardens the iron, makes it less susceptible to magnetism, and causes a trace of what enters, to remain.

The transient magnetism induced in the spheres counteracts that part of the transient magnetism in the soft iron of the ship which is conceived to be resolved into the horizontal plane and which produces quadrantal deviation; they overcome the effect of the rods a, b, d, and e, Figs. 435 to 439.

A few words relative to the magnetization of the spheres will be proper here. Let (4), Fig. 551, represent one in a northern latitude where the Dip is $\overline{DD'}$, and magnetic meridian $\overline{HH'}$—a horizontal line through the center; the sphere is viewed from the west. It becomes magnetic by terrestrial induction—all above the line $\overline{AA'}$ (perpendicular to $\overline{DD'}$) acquires blue polarity, and all below red, which repels

the north end of the needle. If the sphere be spun about any diameter as an axis, this will not affect the indicated polarity: the metallic mass turns, but the induction takes place in each new part of the sphere coming under its influence, and this however rapidly it be revolved, so that its magnetic condition is always as in (4), viewed from the west.

In (1), (2), (3), Fig. 551, spheres are represented attached to a binnacle, an outline of a ship being drawn to indicate the heading; the spheres are variously turned in azimuth, but this does not change the distribution of their magnetism—each, viewed from the west, is still as in (4).

Now let F represent a disturbing soft-iron mass on the ship: heading north, obviously it will not cause deviation, nor will the spheres affect the compass. Heading east, (3), the magnetism of F is distributed as in the sphere at (4)—*both* polarities, equal in amount, at the ends of F and therefore inoperative: the spheres being in line with the compass, produce only the effect of greater directive force; for in the port sphere the blue region H' (4) is toward the north end of the needle and in the starboard sphere the red region H is toward the south end—all mutually attracting. Heading NE., (2), the blue pole of F tends to draw the needle to the east as at n', while at the same time the blue region of the port sphere attracts the north end of the needle, and the red region of the starboard sphere the south end, and the needle rests in the meridian as at n. With obvious modifications similar conditions will prevail as the ship swings round the circle.

Wrought-iron cylinders or tubes are used to compensate the effect of transient magnetism in vertical soft iron represented by c and f, Figs. 441, 442, and 444, such as a smoke-stack, rudder-post, ventilator, stanchion, or davit: if it is the upper pole of the disturbing mass that affects the com-

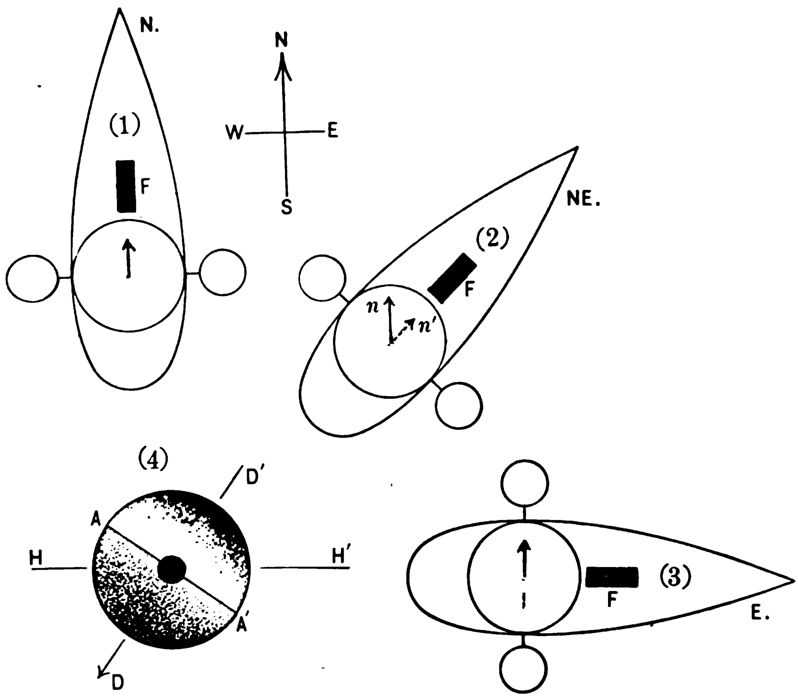


FIG. 551.—Magnetic Induction by the Earth in Soft Iron.

(To face p. 1058.)

pass, the compensating cylinders must be placed so that their lower poles will counteract the effect—that is, vertically at a suitable distance from the compass and diametrically opposite to the resultant of the disturbance. Conversely, if it is the lower pole of this that acts, the upper pole of the cylinder must be used.

For compensating the effects of transient magnetism in soft iron, represented by *g*, *h*, *k*, Figs. 437, 440, 443, 459, and 537, which comes into action when the vessel heels, wrought-iron cylinders are also used.

358. What is effected by compensation.—With the horizontal and vertical components of the Ship's permanent magnetism (*P*, *Q*, and *R*) neutralized by steel magnets; the transient magnetism of horizontal soft iron (*a*, *b*, *d*, and *e*) overcome by cast-iron spheres; the transient magnetism of vertical soft iron (*c* and *f*) outside the axis of the compass counteracted by wrought-iron cylinders placed vertically and symmetrically to the disturbing force; and the transient magnetism of the soft iron (*e*, *g*, *h*, and *k*) that acts when the ship heels met by wrought-iron tubes suitably placed:—with all this accurately done after careful analysis and computation of the disturbing forces, the compass may be considered completely compensated for the ship, both upright and heeled, not only for one locality, but for all regions of the globe, *provided the conditions remain unchanged.* But they will not: neither the permanent magnetism of the ship nor that of the compensating magnets will continue invariable; transient magnetism that cannot be compensated will periodically arise from steering the same course or lying in the same direction for some time—it is temporary, but dangerous; the shock of waves, vibration from the screw, firing of the battery, violent strain of rolling and pitching, and numerous other causes arise to alter the conditions from what they were when

number always large but varying with the deviations to be overcome.

This ensured quite a uniform field in the region of the compass, and the source of this field (at a considerable distance) being like the compass itself, provided with gimbals, always swung parallel to the card and never acquired the motion of the ship.

360. Regarding the instruments used in compensating. A central force may be defined as one whose power is concentrated at the center of a sphere, and which decreases regularly as the surface of this sphere enlarges, the observer being always on the enlarging surface.

Sound, Heat, Light, Gravity, Electricity, and Magnetism are all central forces: the intensity of sound, heat, and light, and the force of gravity, electricity, and magnetism, vary inversely as the square of the distance from the point of concentration, and the formulas applicable to one are equally so to the others; hence the propriety of using the oscillation of a magnetic needle to determine the intensity of magnetism in the same way that we do the swinging of a pendulum to ascertain the force of gravity.

Small, short needles are preferable to long heavy ones: their store of moving energy is less, and they are affected by small degrees of magnetic force which the mere momentum of a heavy needle would pass over.

To obtain the time of one oscillation from a series of fifty swings without stopping is more accurate than from five sets of ten oscillations each; for we thereby incur the error incident to noting only one start and one stop, instead of the errors of five such pairs; it would therefore be most desirable to have a small pivoted horizontal needle that, in an ordinary magnetic field, would make fifty oscillations between an arc of 20° at beginning and 5° at the 50th; but

this is not easy of attainment, even in the natural field of the Earth.

The binnacle, pelorus, azimuth circle, horizontal needle, and balanced needle, instruments used in the Navy, are all inspected by the Superintendent of Compasses to ensure the issue of only reliable ones to the Service.

CHAPTER XXIX.

COMPENSATION OF THE COMPASS.

Section One: Illustration of Compensation by Means of the Scoresby.

361. The observations. — The experiments with the Scoresby have the advantage of exposing to view the location and nature of the material that is the seat of the disturbing forces, the deviations produced by these, and the means taken to neutralize them: the observations used in this chapter were made in 1898; they exemplify what, in substance, would be done on a ship.

The same compass, No. 24025, was used throughout: it was a four-needle liquid compass of the type described in Part Second—identical with those used in the Navy for standard compasses.

The following was the condition of the Scoresby during the experiments—Fig. 553. The vessel is pivoted at P, the axis of rotation being the prolongation of the compass-pivot as nearly as it could be attained; the wheel W rolls on a graduated brass circle screwed to the floor, and by means of the handle K the vessel may be swung; the *disturbing* magnetic material is colored red and purple, the COMPENSATING appliances, green; the compass is represented by a single red arrow at N and the compensating magnets in the starboard angle by a single bar H, but the

compass was the four-needle one specified above and there were four cylindrical magnets at *H*.

For convenience of reference, the various stages of the procedure are numbered within parentheses in this article.

(1.) Removed all iron and magnets from the SCORESBY and its vicinity to the corners of the room, so that only the natural field of the Earth (as it existed in the room) should affect the compass.

(2.) With conditions of (1), swung the vessel on 16 points and observed the bearing of the "electric sun": on different points it varied between N. 33° W. and N. 34° 30′ W. and included the parallactic angle; to indicate the magnetic direction of each heading, radii to the brass circle on the floor were drawn with chalk—keel lines of the vessel while on each heading.

The procedure thus far would correspond to determining the magnetic bearing of a distant object; it is likewise analogous to finding the bearing of the Sun from Azimuth Tables. In *this* case the sun was fixed.

(3.) Removed Compass from binnacle, and with all else as in (1), placed a small Horizontal Needle in the exact site of the compass and observed ten sets of ten oscillations each, noting the time with a stop-watch marking quarter seconds. The needle was about three inches long, in a brass case covered with glass. Amplitude of swing about 20° at beginning. Mean (T) of the ten sets is given in col. (3), Table 94: it was 17.92 sec.

(4.) Removed horizontal needle and substituted a Dip Circle, Fig. 168: magnetic conditions otherwise as in (1), observed the Dip (67° 30′) and ten sets of oscillations, the plane of the moving needle being transverse to the meridian. Amplitude about 20° at beginning. Mean (T) of the ten sets of oscillations is given in col. (4), Table 94.

(5.) Removed dip circle, and substituted Balanced Needle in plane of compass-card, and conditions being

otherwise as in (1), noted point of bead on scale (1.7), col. (5), Table 94, when the needle was horizontal—see Fig. 552.

(6.) Observations (3), (4), and (5) correspond to the determination of same quantities on shore in a spot free from magnetic disturbance.

(7.) Replaced the balanced needle by compass No. 24025, thus restoring, the conditions of (1).

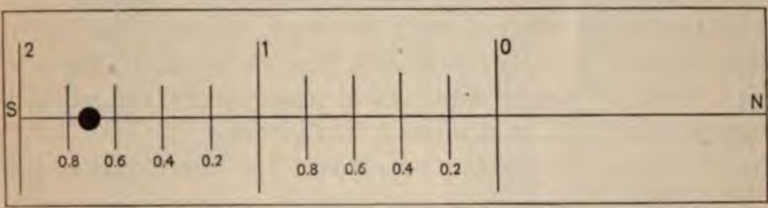


FIG. 552.

(8.) Now placed the disturbing material as follows on the vessel, Fig. 553:

The ship's head being NE. *magnetic* both by compass and radius on floor, placed three thick soft-iron rods at A, to produce both semicircular deviation and heeling error: the compass now indicated a heading of N. 38° 30′ E., thus showing the effect of the rods to be 6° 30′ easterly deviation. These rods were 42 inches long and inclined at an angle with the vertical; their nearest ends were 12 inches from pivot of the compass.

(9.) Next put a hollow soft-iron sphere B, 9 inches in diameter, forward and below plane of compass-card, with the nearest part of surface 18 inches from pivot: its effect was to cause 1° 30′ easterly (quadrantal) deviation, for the ship's head by compass was now N. 37° E.

(10.) Then placed a large thick plate C of soft boiler iron in the horizontal plane of the needles to produce quadrantal deviation: its nearest end was 14 inches forward

of the compass, which now indicated N. 32° E., showing the effect of the plate to be 5° easterly deviation.

(11.) Finally, put six powerful bar-magnets D on spar-deck, horizontally, forward and below plane of needles and inclined to the fore-and-aft line, with their south poles nearest the compass and at a distance of 32 inches from the pivot: these magnets were 19 inches long. The compass indicated a heading of N. 10° 30′ E., thus showing their effect to be 21° 30′ easterly deviation. During the procedure of (8) to (11), both inclusive, the ship had been kept blocked on NE. *magnetic*. The (*red*-colored) disturbing materials E and F were added subsequently for another purpose and will be described further on.

(12.) The foregoing completed the combination of (*purple*-colored) disturbing material, and with it in position as shown in Fig. 553, but *without* any compensating appliances on the binnacle, and *also without* E and F, swung ship and observed the bearing of the electric sun on each of the 32 *compass* headings, resting three to four minutes on each point. Comparison of these bearings with those of the electric sun when no disturbing matter was on the vessels, gave the deviations—Table 93. All this procedure is clearly analogous to steaming in a circle or swinging at compass-buoys, and observing azimuths of the sun.

(13.) The compass was now removed from the vessel; the disturbing material A, B, C, D was still in place just as when the ship was swung for Table 93; *no* compensating appliances (H, V, or S) were on the binnacle, neither was E nor F: and under these conditions placed the same horizontal needle employed in (3) where it had been used there, and observed the times of five sets of ten oscillations each, with the ship headed successively on the eight principal *magnetic* points, as shown by radii on the floor. The mean values (T_0', T_4', T_8', etc.) of these observations are

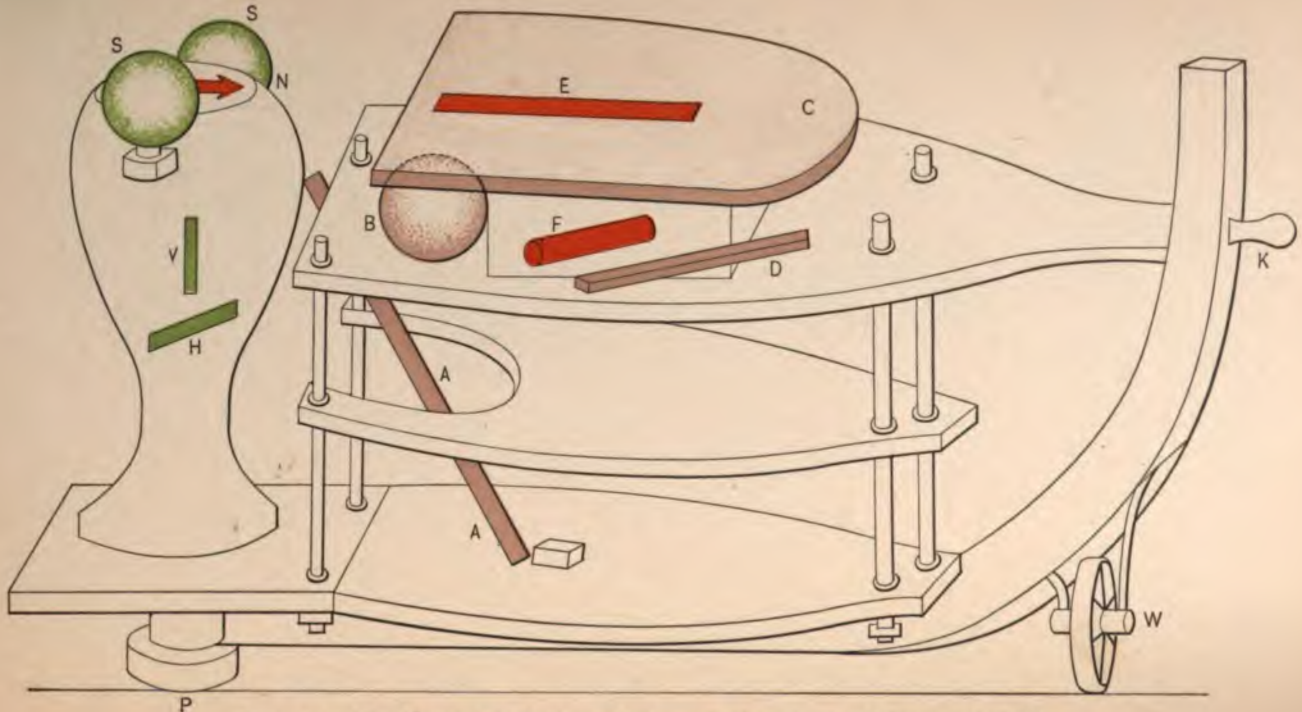


FIG. 553.—The SCORESBY during experiments described in Chapter XXIX.

(To face p. 1066.)

given in col. (3), Table 94. The amplitude of oscillation began at about 20°.

TABLE 93.

SCORESBY'S Head by Compass No. 24025.	Deviations of Compass No. 24025.	Condition of Compass No. 24025.	Quadrantal Deviation from Analysis of Column (2).
(1)	(2)	(3)	(4)
N.	29° 30′ E.	Steady.	+0° 15′
N. by E.	36 30 E.	Steady.	+4 30
N. NE.	40 00 E.	Steady.	+6 22
NE. by N.	40 30 E.	Steady.	+7 30
NE.	38 30 E.	Steady.	+8 15
NE. by E.	34 00 E.	Unsteady.	+7 45
E. NE.	28 30 E.	Very unsteady.	+5 38
E. by N.	21 30 E.	Very unsteady.	+3 00
E.	15 00 E.	Very unsteady.	
E. by S.	3 30 E.	Very unsteady.	
E. SE.	0 30 W.	Very unsteady.	
SE. by E.	7 00 W.	Very unsteady.	
SE.	14 00 W.	Very unsteady.	
SE. by S.	20 00 W.	Very unsteady.	
S. SE.	24 00 W.	Unsteady.	
S. by E.	27 00 W.	Unsteady.	
S.	29 00 W.	Steady.	
S. by W.	28 30 W.	Steady.	
S. SW.	26 30 W.	Steady.	
SW. by S.	24 30 W.	Steady.	
SW.	22 00 W.	Steady.	
SW. by W.	19 30 W.	Steady.	
W. SW.	18 00 W.	Steady.	
W. by S.	16 00 W.	Steady.	
W.	15 30 W.	Steady.	
W. by N.	13 30 W.	Steady.	
W. NW.	11 00 W.	Steady.	
NW. by W.	7 00 W.	Steady.	
NW.	2 30 W.	Steady.	
NW. by N.	3 30 E.	Steady.	
N. NW.	12 00 E.	Steady.	
N. by W.	20 30 E.	Steady.	

(14.) Removed the horizontal needle and substituted the dip circle as in (4), and with everything else as described in (13), observed the times of five sets of ten oscillations each, the needle swinging in the vertical plane transverse to the lines of horizontal force while the ship headed as stated in (13): amplitude about 20°. The mean

values (T_0'', T_4'', etc.) are given in col. (4), Table 94; and since all are less than the value due to the Earth's field alone, they show a downward pull by the ship.

TABLE 94.

(1) Magnetic Headings of SCORESBY.	(2) Deviations Proper to Them.	(3) Times of Ten Oscillations of Horizontal Needle.	(4) Times of Ten Oscillations of Vertical Needle.	(5) Position of Bead on Balanced Needle.	(6) Values of λ on Different Headings.
FIELD OF EARTH ALONE		$T=17.92$ s.	$T=16.89$ s.	1.7	1.0000
FIELD OF EARTH AND SHIP TOGETHER, FOR MAGNETIC HEADINGS OF LATTER.					
N. $=\zeta_0$	$\delta_0=17°$ E.	$T_0'=14.60$ s.	$T_0''=15.19$ s.	2.1	1.4340
NE. $=\zeta_4$	$\delta_4=36$ E.	$T_4'=17.00$ s.	$T_4''=14.87$ s.	2.1	0.8899
E. $=\zeta_8$	$\delta_8=34$ E.	$T_8'=24.37$ s.	$T_8''=15.32$ s.	2.1	0.4476
SE. $=\zeta_{12}$	$\delta_{12}=25$ W.	$T_{12}'=23.47$ s.	$T_{12}''=15.02$ s.	2.1	0.5255
S. $=\zeta_{16}$	$\delta_{16}=26$ W.	$T_{16}'=18.27$ s.	$T_{16}''=15.29$ s.	2.1	0.8531
SW. $=\zeta_{20}$	$\delta_{20}=19$ W.	$T_{20}'=16.52$ s.	$T_{20}''=15.44$ s.	2.1	1.1151
W. $=\zeta_{24}$	$\delta_{24}=13$ W.	$T_{24}'=15.25$ s.	$T_{24}''=15.35$ s.	2.1	1.3539
NW. $=\zeta_{28}$	$\delta_{28}=1$ W.	$T_{28}'=14.22$ s.	$T_{28}''=14.35$ s.	2.1	1.5784
					8)8.1975
			Mean value of λ		$=1.0247$

(15.) Removed the dip circle and put the balanced needle in the place it occupied in (5). Condition of ship same as in (13) and (14). The needle—pointing north—now dipped, so that the bead had to be moved to 2.1 of the scale in order to make it level—thus showing a downward pull by the Ship, and agreeing (as it should) with the oscillations of the vertical needle.

362. The Ship's vertical force exerts a pull or a thrust on the compass.—Consider Fig. 555: it represents a ship whose forward body is pervaded by red polarity, and after body by blue; these are injected into the natural field of the Earth represented by black arrows E: as the ship swings

round, the resultant vertical force acting on the north end of the needle *C* will vary—as may be inferred from Fig. 555.

If the compass at *C* be near the neutral line and the ship head north, there will be a thrust from both polarities of the ship on the respective ends of the needle: in northern latitudes the red polarity tends to neutralize the Earth's field, while the blue adds to it, and the thrust is unequal on both ends. The arrows of the blue polarity are pointing downward, to represent a *pull* on the *north* end of the needle and consequently a thrust upon its south end.

Heading south, there would be a pull on both ends, and, for a reason similar to the foregoing, it would be unequal on both ends of the needle.

Between these north and south headings many degrees of pull and thrust may occur.

If the compass be at *L* entirely over the red region, this will exert a continuous thrust on the north end of the needle—that is, the resultant directive force of Ship and Earth will be less than the latter alone; on the other hand, if the compass be at *M* entirely over the blue region, this will exert a continuous pull on the north end of the needle—that is, the resultant directive force of Ship and Earth will be greater than the Earth alone: in these two cases either thrust or pull will be nearly uniform in all azimuths because of the small area of the field acting on a short needle. This fact was experimentally established as follows:

When the bead was placed at 2.1 of the scale, as stated in (15), its weight at the increased leverage (from 1.7) counteracted the magnetic pull of Earth and Ship when latter headed north. Then without disturbing the bead, and while the ship continued blocked on N., turned the instrument round a central vertical axis so that its needle pointed successively in all azimuths—it remained level on all. Secondly, still without disturbing the bead, and with

the needle constantly pointing toward the *bow* of the vessel, swung this through a circle—and the needle still remained level on every heading.

In the first case, the *relative directions* of needle and ship's force changed at every azimuth, while in the second the relative directions of both remained the same: the observations are given in col. (5), Table 94. Fig. 555 is greatly exaggerated to render the matter clear.

363. The steady and unsteady arcs of an uncompensated compass.—In col. (3), Table 93, the motion of the compass while swinging ship is described: it will be considered in connection with Fig. 554. From E.NE. to SE. by S. the card scarcely came to rest, and had to be closely watched for some time to get an average bearing of any accuracy—the least disturbance made it oscillate through several degrees. In this unsteady arc the directive force was greatly diminished as will be seen by cols. (3) and (6), Table 94. Furthermore, by Fig. 554, in this part of the circle, seven *compass*-points, E. by N. to SE. by S., are crowded into the space of three *magnetic* points—the compass *indicated* a change of heading of 79°, but the arc passed over was only 34°. Throughout what is marked the *steady* arc on Fig. 554, the card came quickly to rest, observations were speedy and accurate, and the change of heading by compass corresponded fairly with the absolute change in space. The inner radii are magnetic points; the outer, compass-points. Cols. (3) and (6), Table 94, show that in this steady arc the directive force was increased. The conditions here described are typical of what will be experienced on every ship, in varied degree, according to her magnetic character and the location of the compass.

The foregoing completed the *observations* for Deviation and Force.

364. Computations based on the observations.—The deviations of Table 93 were now transcribed to Form 10

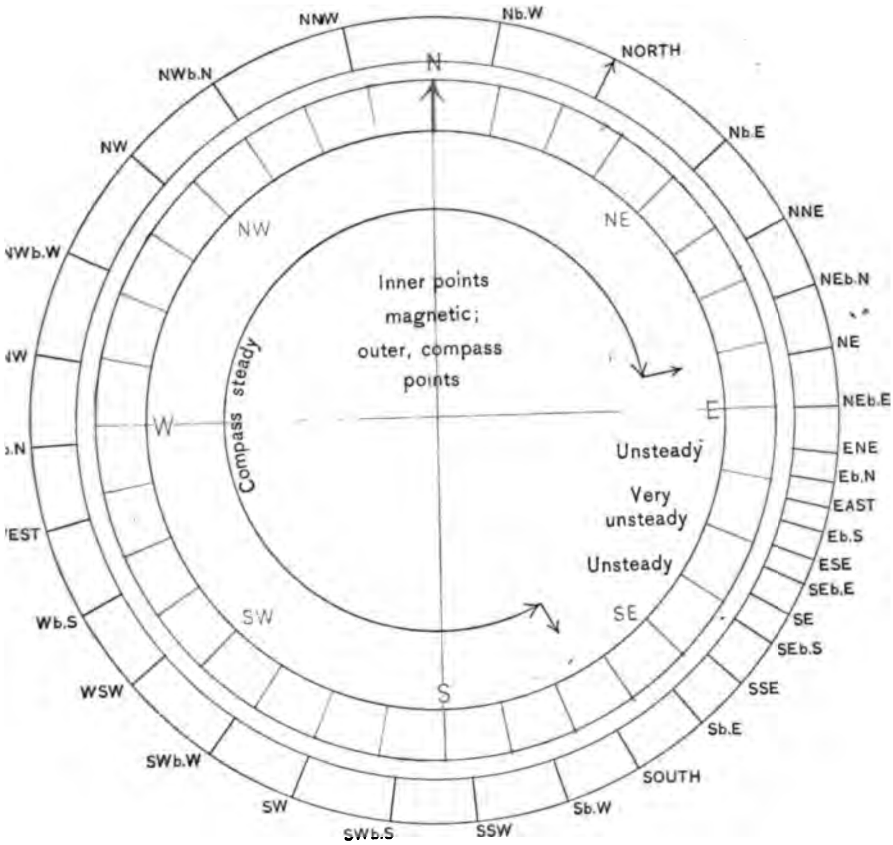


Fig. 554.

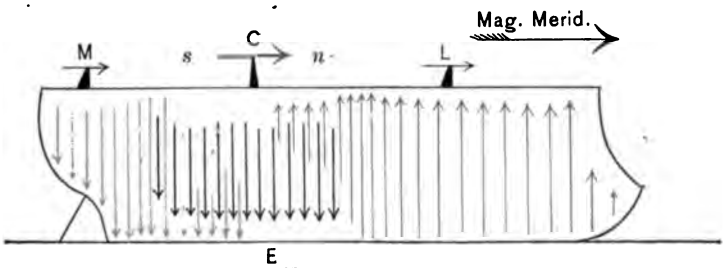


Fig. 555.

(To face p. 1070.)

and analyzed; as the process has been already illustrated, only the results in the present case need be given; they are:

$\mathfrak{A} = -.0017$ (7)	$\mathfrak{B} = +.2854$ (8)	$\mathfrak{C} = +.4246$ (9)
$\mathfrak{D} = +.1467$ (10)	$\mathfrak{E} = +.0092$ (11)	$A = -0° 06'$ (12)
$B = +15° 54'$ (13)	$C = +27° 12'$ (14)	$D = +8° 26'$ (15)
$E = +0° 31'$ (16)		$\alpha = 56° 6'$ (17)

Fig. 556 illustrates col. (2), Table 93.

For calculating λ three methods offer: FIRST, by construction—using the observations of cols. (2) and (3), Table 94, for any two diametrically opposite headings; SECOND, by computation—using the coefficients (7) to (11) above, together with the observations in cols. (2) and (3), Table 94, for any one heading of col. (1), and employing formula (258), p. 944; and THIRD, also by computation—using all the data of cols. (1), (2), (3), Table 94, and employing formula (259), page 945.

The first and second methods have been illustrated numerically elsewhere; and the third will now be used. It will be recalled that

$$\frac{T^2}{T'^2} = \frac{H'}{H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The deviations of col. (2), Table 94, are taken from Fig. 556.

Substituting, then, the requisite quantities in formula (259), p. 945, we find $\lambda = 1.0247$. It was also determined by construction, using the observations on NW. and SE., and found to be 1.054—a value less accurate.

On whatever compass-point it may be decided to head the ship while compensating by the Component method, the Quadrantal Deviation on that point must be calculated; and formula (120), p. 880, affords the means of doing it, by using only those terms that express the quadrantal deviation: representing this by δ_q, and reproducing the necessary part of (120), we have

$$\tan \delta_q = \frac{\mathfrak{D}.\sin 2\zeta + \mathfrak{E}.\cos 2\zeta}{1 + \mathfrak{D}.\cos 2\zeta - \mathfrak{E}.\sin 2\zeta}. \quad . \quad . \quad (19)$$

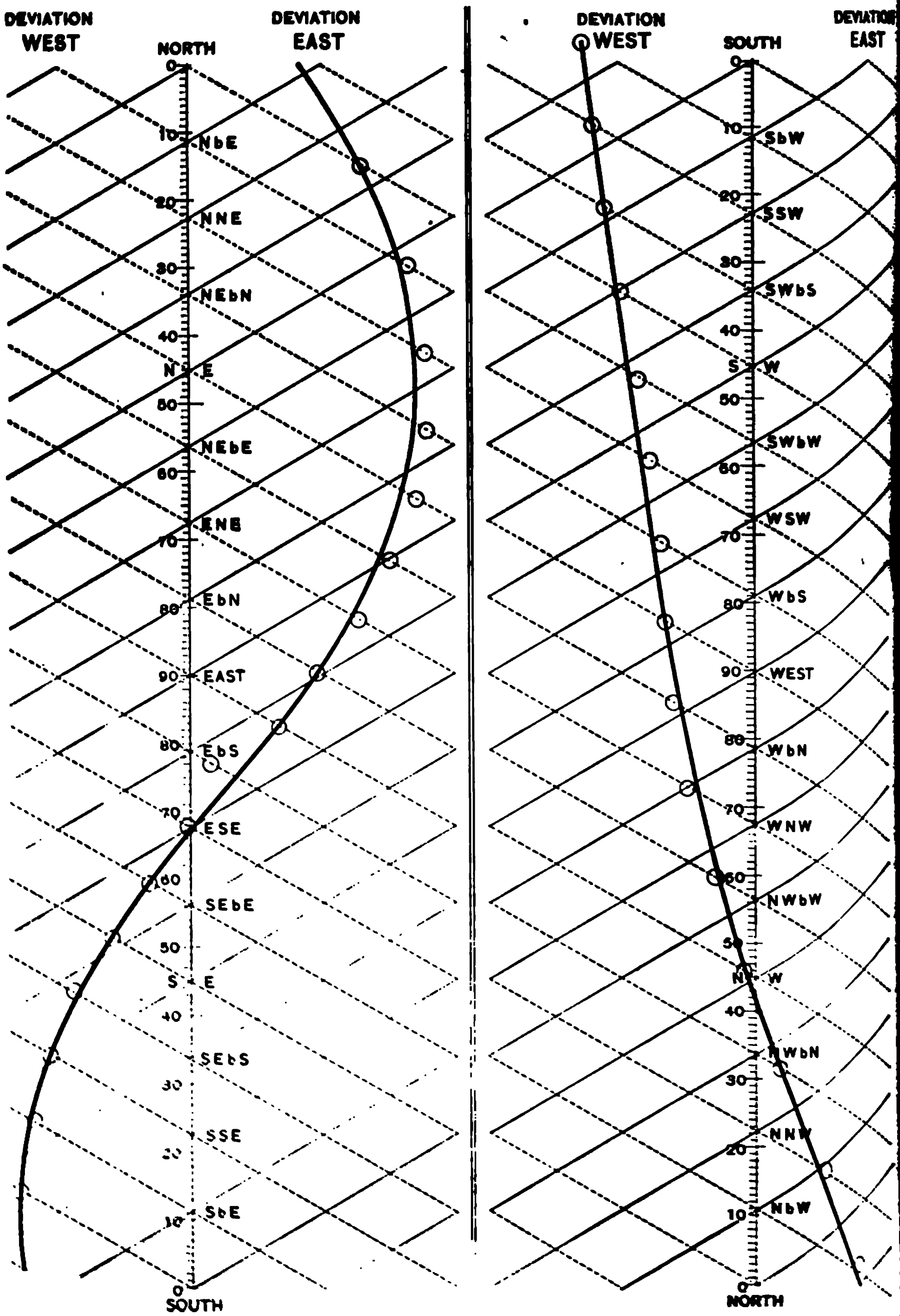


FIG. 556.—Illustrating Col. (2), Table 93.

... it is customary to omit the terms containing 𝔈, ... affect the result only slightly; but they will be used. To illustrate the computation, δ_q will be found for ...-point (ζ') S. by W.: by Table 93, the total deviation ... this is 28° 30′ W.; hence the corresponding magnetic course (ζ) is S. 17° 15′ E., or $\zeta = 162^\circ 45'$ and $2\zeta =$ ...30′; and its required functions are:

$$\ldots 5^\circ 30' = \sin(360^\circ - 34^\circ 30') = -\sin 34^\circ 30' = -.5664; \quad (20)$$

$$\ldots 5^\circ 30' = \cos(360^\circ - 34^\circ 30') = \cos 34^\circ 30' = +.8241. \quad (21)$$

...stituting in (19) the values from (10), (11), (20), and ...), it becomes

$$\tan \delta_q = \frac{\{(+.1467)(-.5664)\} + \{(+.0092)(+.8241)\}}{1 + \{(+.1467)(+.8241)\} - \{(+.0092)(-.5664)\}}$$

$$= \frac{(-.0841) + (+.0075)}{1 + (+.1195) - (-.0053)} = \frac{-0.0766}{+1.1248} = -0.0681;$$

Whence $\delta_q = -3^\circ 54'$; and this is the amount of the quadrantal deviation to be corrected by the spheres when the ship is headed S. by W. per compass for the purpose of compensating. When observations are made on 32 points and the results analyzed, the quadrantal deviation on each point becomes known—col. (12), Form 10—and then it is unnecessary to calculate it by eq. (19).

Section Two: Regarding Compensation in General.

365. The order in which to compensate the parts of the deviation.—The combination of disturbing forces on the SCORESBY, Fig. 553, produces a *definite* magnetic field around the compass, and the instant we inject into it a field from a compensating magnet, that instant we

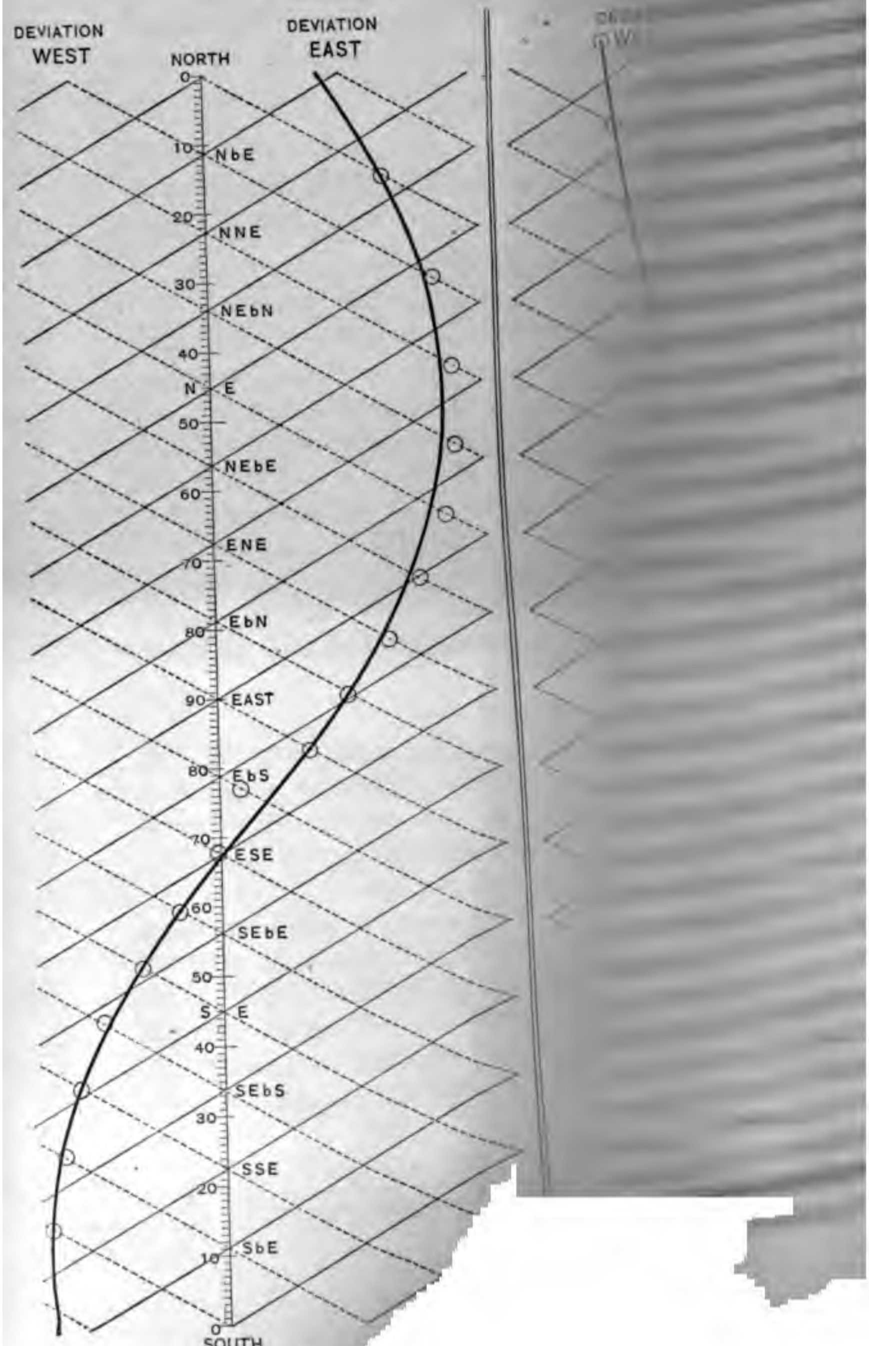

DEVIATION
WEST
NORTH
DEVIATION
EAST
NbE
NNE
NEbN
N E
NEbE
ENE
EbN
EAST
EbS
ESE
SEbE
S E
SEbS
SSE
SbE
SOUTH

FIG.

...one due to the disturbing forces of the ... the correctors; and it would be strange ...lance could be adjusted to a nicety; or, ... could continue under the varying mag... the ship encounters, or the shock, vibra... treatment she receives from wind and

... to compensate with exactitude on a *new* ... with disappointment; the surcharge is still ... it dissipates slowly—and during the ... the deviations change from day to day. ...hip several rapid circles should be made ... before compensating, or even observing ... in order to shake out loose magnetism and ...omewhat stable condition.

...al **rule as to size of magnets and their dis... the compass.**—The term of the semicircular ... to hard iron being appropriately neutralized ... these may be used either in variable number ...ance, or in definite number at a variable dis... it is decidedly best, and the distance should ...st the binnacle will allow. Then quite a uni...anating from them will surround the compass ... affect both alike—and not be liable to break ... of compensation. The same is true of the ...net: it should be thick and at a distance, rather ... thin, and close under the compass.

... **determine the strength of the magnets for com... by the resultant method.**—The correctors to be ... determined beforehand.

...hore a compass, azimuth circle, tripod, pelorus, ...tity of magnets, spheres, and heeling correctors. ... free from magnetic disturbance, and leave the ...nd spheres well outside its area.

... the compass in the spot and lay out a meridian

change it, and the next corrector has not quite the original conditions to deal with. If the semicircular deviation be corrected first, the magnets will produce induction in the spheres when put on, just as the Earth will: the spheres gather up the lines of force from the magnets and pass them on, much in the same way that a lens does light.

On the other hand, if the spheres be applied first—and this seems the proper course, for we thereby correct the real quadrantal deviation of the ship—then an error is liable to arise from the effect upon the spheres of the magnets when these are introduced.

Similarly, with the heeling magnet—it has an effect upon the spheres.

The matter is complicated, but experience will enable the navigator to determine the order in which best to apply the correctors.

After the spheres it would be natural to correct the effect of vertical soft iron by a Flinders' bar and a tube to neutralize k; but this is impracticable, as the ship must proceed to a port where the change in the magnetic elements affords data for estimating the amount of the correction required. In placing the magnets a vertical plane should pass through the compass-pivot and their middle points, whether they are laid in the starboard angle of the resultant method, or fore-and-aft and athwartships as in the component method. The axis of heeling correctors must coincide with the vertical line through the compass-pivot when the ship is on an even keel. The horizontal plane through the needles must pass through the centers of the spheres, and the acting pole of the Flinders' bar: the pole is about $\frac{1}{12}$ the length of the bar from its extremity. The constant error is seldom compensated, but remains as part of the residual deviations.

366. The magnetic condition surrounding a compensated compass.—At the location of the con

antagonistic fields—one due to the disturbing forces of the ship, the other to the correctors; and it would be strange indeed if their balance could be adjusted to a nicety; or, if once attained, could continue under the varying magnetic conditions the ship encounters, or the shock, vibration, and rough treatment she receives from wind and wave.

The endeavor to compensate with exactitude on a *new* ship will meet with disappointment; the surcharge is still in the structure—it dissipates slowly—and during the transition period the deviations change from day to day. With such a ship several rapid circles should be made with each helm before compensating, or even observing for deviations, in order to shake out loose magnetism and get down to a somewhat stable condition.

367. General rule as to size of magnets and their distance from the compass.—The term of the semicircular deviation due to hard iron being appropriately neutralized by magnets, these may be used either in variable number at a fixed distance, or in definite number at a variable distance: the first is decidedly best, and the distance should be the greatest the binnacle will allow. Then quite a uniform field emanating from them will surround the compass and spheres—affect both alike—and not be liable to break the balance of compensation. The same is true of the heeling magnet: it should be thick and at a distance, rather than long, thin, and close under the compass.

368. To determine the strength of the magnets for compensating by the resultant method.—The correctors to be used can be determined beforehand.

Take ashore a compass, azimuth circle, tripod, pelorus, and a quantity of magnets, spheres, and heeling correctors. Find a spot free from magnetic disturbance, and leave the magnets and spheres well outside its area.

Set up the compass in the spot and lay out a meridian

; right angles to it—both through the compass-
e the pelorus centrally beneath the compass,
nd, so that its cardinal points shall coincide
ridian and the E.-and-W. line.
ltant disturbing force (R) of the ship and the

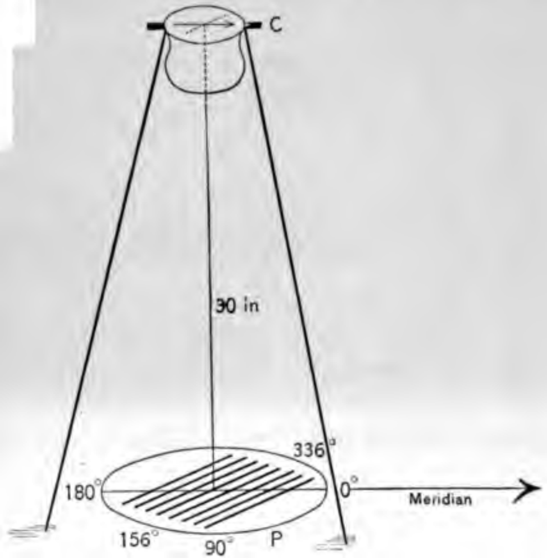


FIG. 557.

angle (α) of its direction are given by equations (77) and (78), page 871, as follows:

$$R=\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}. \quad . \quad . \quad . \quad (22); \quad \text{and} \quad \tan\alpha=\frac{\mathfrak{C}}{\mathfrak{B}}. \quad . \quad . \quad . \quad (23)$$

This force is exerted in the field around the compass, denoted by λ, and it produces a deflection equal to the angle whose tangent is $\lambda\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$, since (as shown in Part Second) the strength of a magnet may be represented by the natural tangent of the deflection it produces in a specific field: to neutralize R, therefore, we must find a certain number of magnets equal to it.

Let the distance at which they are to be placed from the needles be thirty inches. Then in Fig. 557, C is the compass and P the pelorus: upon its rim lay two magnets on the diameter through the angle α; they will deflect the card; lay two others beside them, and continue adding pairs, one on each side, until the deflection is equal to the angle whose tangent is $\lambda\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$; then these are the particular magnets to be used on board—to be placed centrally

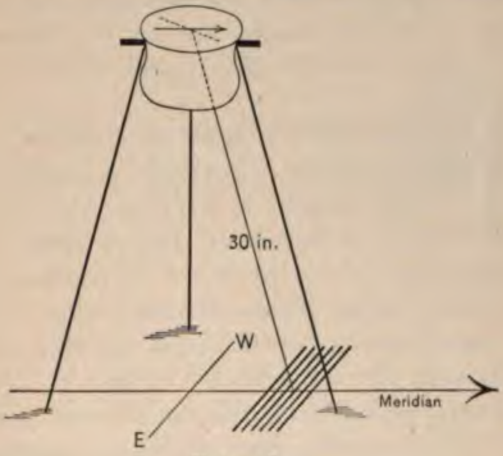


FIG. 558.

and symmetrically beneath the compass, in the starboard angle α, and thirty inches from the card: they will compensate the semicircular deviation on whatever course the ship be put.

The above is the mode of determining the number of magnets for a stated distance; that of ascertaining the distance for a definite number of magnets is obvious.

Analysis of a table of deviations affords the data for calculating the quantities R and α.

369. To determine the magnets for the component method.—This is merely to find two groups of magnets equal to the components $\mathfrak{B}$ and $\mathfrak{C}$ at right angles to each

other, into which the resultant $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$ of the preceding article is resolved: they are determined in a similar way.

In Fig. 558, lay two magnets across the meridian, centers on it, and thirty inches from the compass-pivot—they will deflect the card: continue to add magnets until the deflection is equal to the angle whose tangent is $\lambda.\mathfrak{B}$, and these are the magnets for neutralizing $\mathfrak{B}$. They are to be placed horizontally fore-and-aft, on either side of the compass, below or above it, centers on the transverse line in vertical plane through pivot, and thirty inches from this.

Remove these magnets to a distance, and in like manner find another group which will produce a deflection equal to the angle whose tangent is $\lambda.\mathfrak{C}$, and they are the ones for neutralizing $\mathfrak{C}$: to be placed athwartships, forward or abaft the compass, above or below it, centers on midship line, and thirty inches from compass-pivot.

When these correctors of $\mathfrak{B}$ and $\mathfrak{C}$ are thus placed, they will compensate the semicircular deviation, on whatever course the ship be headed—it is not necessary that she be put on North or South, East or West, magnetic for the purpose.

370. To determine strength of heeling magnet.—Usually, the whole amount of the Heeling Error is corrected *at first* by magnets.

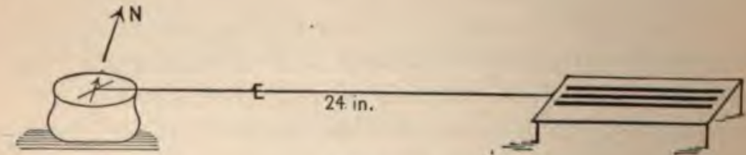


FIG. 559.

To determine the strength of these, place as in Fig. 559 one due East of compass in plane of needles, blue pole

toward them, and 24 inches from pivot, if that be the distance decided upon for the nearest extremity of the corrector when in the binnacle.

A deflection will occur; add magnet by magnet until it reaches the angle ϕ whose natural tangent is to be computed by eq. (26), p. 1084: then this is the *bundle* of magnets to be used.

371. To determine size of spheres and soft-iron rods for correctors.—Place two spheres, as in Fig. 560, equally dis-

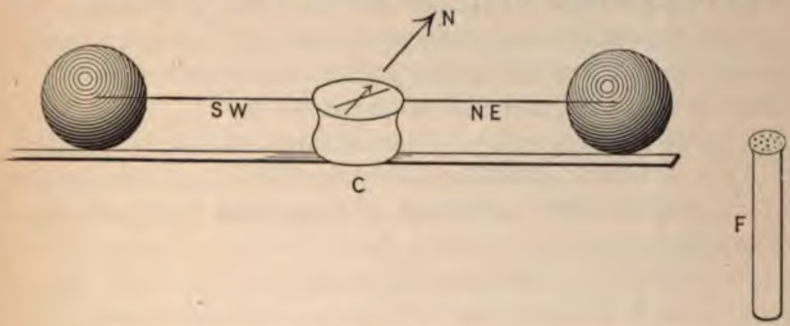


FIG. 560.

tant NE. and SW. of the compass; move both toward it until a deflection occurs equal to the maximum quadrantal deviation found by analysis of the deviations. It is better to have large spheres at a considerable distance than small ones close to.

The size and distance of the Flinders' bar *F*, and soft iron corrector for *k* may be determined in a similar way: place the tube or cylinder E. or W. of the compass at the distance it is decided to have it in on board—it should not be very close—and add other tubes until they produce the deflection previously calculated.

372. Destructive surroundings of compasses.—In the examples of experimental work with the SCORESBY described in this chapter, it will be seen that the disturbing combinations were devised for a crucial test; in one case

the maximum deviation was 40° 30′, and the least value of the directive force 0.4476; in another, the deviation reached 54°, and directive force fell to 0.255; it is to be hoped that few compasses are subject to such destructive influences, yet these were brought well under control and only small residuals remained.

This, however, is not a plea *for* compensation—quite the contrary—it is a plea against its *necessity*, and merely to illustrate how a bad situation is met. On a *ship* it would be far better not to create the situation—that naval architects would realize the importance of placing the compass midst such non-magnetic material as to leave it approximately in the natural field intended for it.

An actual example of vicious surroundings is afforded by the ATLANTA's steering compass, Table 74: the deviations contain both sextantal and octantal terms of considerable amount.

Even with a compass favorably located, such terms will arise from too close proximity of the magnets and spheres—consequent upon interaction between them and the needles: these must have *some* length—not be the "infinitely small" (and useless) particles of theory—and their value should not be destroyed by concentrated fields of correctors at close range.

373. Regarding heading of ship on which to compensate by the resultant method.—With the Resultant method, we may—theoretically—head the ship on *any* compass-point and perform the compensation: so—theoretically—the latitude, longitude, and azimuth can be determined at any time of day; but it is well known that great inaccuracy attends observations for the last two near the meridian, and for the first near the prime vertical.

An analogous condition exists with regard to compensation: it cannot be done equally well on all headings—not that there are (as with the navigation problem) *certain*

points that of themselves are liable to inaccuracy and others not; but because of the relative juxtaposition of the mechanical correctors used, and the direction of the needles with regard to them on some courses: this condition will vary somewhat with every ship, and the best and worst points will vary accordingly.

Generally it will be best to compensate on a heading having a moderate deviation and mean directive force and where the magnets and spheres make quite an angle with the direction of the needles—if such a combination can be found; the matter will be illustrated in subsequent articles.

374. **Advantages derived from compensation.**—It is seen by col. (6), Table 94, that the value of the directive force varies extremely on different headings; on some the card is apathetic and listless as to course—on others it is alive, nervous, and tenacious of direction; also, by Fig. 554, a large number of compass points are crowded into the space of a few magnetic courses: all this, compensation remedies—the directive force is equalized on all points, and when a change of course is made per compass, the ship's head actually swings over the arc indicated.

Section Three: Compensation of the Heeling Error.

375. **The balanced needle — its principle and use.** In paragraphs (4), (5), (14), and (15) of Art. 361, examples are given of the observations necessary to determine the vertical force that causes heeling error: the observations are to be made on the ship and on shore with one of two kinds of instrument—an oscillating, or a balanced, needle.

The latter has an axle through its center, and a small sliding weight or bead on one arm; it is enclosed in a brass box covered with glass having a horizontal scale by which the position of the bead may be noted—Fig. 552.

Placed in the Earth's field, in the meridian, Fig. 561, the needle dips on account of the vertical force Z, and the bead must be moved to 1.7, for example, to make it level: taken on board and placed in the meridian in the exact spot of the compass (this having been removed), it will dip more, Fig. 562, because of the combined pull Z_1 of Ship

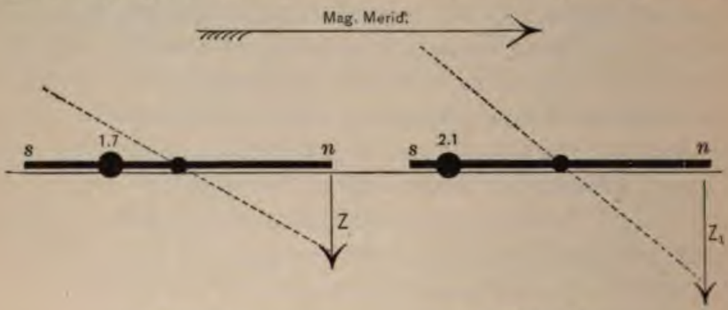


FIG. 561. FIG. 562.

and Earth; and the bead must be moved to, say, 2.1 of scale to restore level. The ship may, however, exert an upward thrust—then the dip will be less—and the bead must be moved toward the axle. The principle is therefore a balancing of gravity against magnetism.

This vertical force of the ship—either pull or thrust—is determined *en bloc* by the observations, and the separation of its two parts, and correction of that due to soft iron must be deferred until the ship has made such change of position as will afford data for estimating its amount: it is represented by the rod k, whose value is found by eq. (468), p. 1026, and its mechanical correction is entirely analogous to that of the other vertical soft iron by a Flinders' bar. With the view of doing this eventually, the oscillating needle should be used at first; the balanced needle is more of a temporary expedient.

When the whole of the semicircular deviation (ship upright) is corrected by *magnets*, the compensation is good

for the locality only where it was performed; so also with the heeling error if similarly dealt with: but if the parts of each be separated and accurately compensated—that due to hard iron by magnets, and that to soft iron by a Flinders' bar and a tube to neutralize k, then the compensation should hold good for all parts of the globe.

376. Connection between the heeling error and the quadrantal deviation.—From pages 879 and 1018 we have the following equations:

$$\mathfrak{D}=\frac{1}{\lambda}\left(\frac{a-e}{2}\right).\quad (24);\quad J=\frac{1}{\lambda}\left(e-k-\frac{R}{Z}\right)\tan\theta.\quad .\quad (25)$$

The first expresses the principal part of the quadrantal deviation, and the second that of the heeling error: both have e as a basis—in part: to ascertain the nature of their connection consider Figs. 563, 564, and 565.

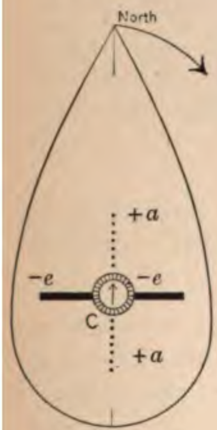


FIG. 563.

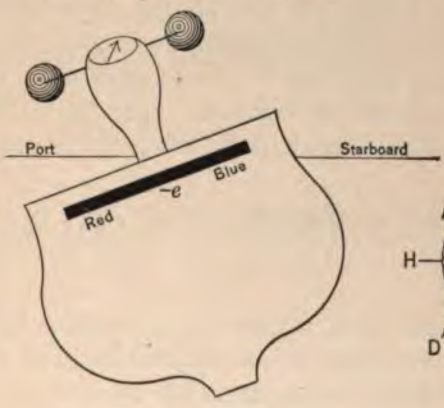


FIG. 564.

FIG. 565.

In 563, as the ship swings to the eastward, the starboard end of $-e$ becomes a blue pole, and the port, a red; and both conspire jointly with $+a$ to cause easterly quadrantal deviation, or $+\mathfrak{D}$. If the spheres be on the binnacle, this *horizontal* effect of $-e$ will be neutralized: now note

what occurs with the ship heeled to port and heading north Fig. 564; the upper end of $-e$ becomes a blue pole and the lower, a red; and both turn the card to the high side of the ship. But the spheres being in place, the lower forward half of the starboard one—a red region—repels the north end of the needle, and the upper after half of the port sphere—a blue region—repels its south end, and thus they tend to make it return to the meridian. Compensation of the quadrantal deviation, therefore, annuls some of the heeling error and hence should be performed first.

Fig. 565 is one of the spheres viewed from the west; its polarity is explained in connection with Fig. 551.

377. To compute the strength of the magnet required to correct the heeling error.—There are three methods of determining the strength of this magnet: 1st, by the deflection it produces; 2d, by balancing its force against gravity; and 3d, by the change it causes in the time of oscillation of a vertical needle.

Denote the power of the magnet at a specific distance by M; it must be equal to $-J$, the heeling coefficient; hence from eq. (452), page 1023, we have.

$$-J=\left(\mathfrak{D}+\frac{\mu}{\lambda}-1\right)\tan\theta=M=\tan\phi. \quad . \quad . \quad (26)$$

The strength of a magnet at a certain distance east of a needle being equal to the natural tangent of the deflection (ϕ) it produces, compute $\tan\phi$ by (26) and then proceed as in Art. 370; by this means we get M, the magnet required to neutralize $-J$.

In (26) θ is the dip on shore, either observed or taken from a chart; μ and λ are determined by observations similar to those described in paragraphs (3), (4), (13), and (14), Art. 361; and $\mathfrak{D}$ is derived from a table of deviations.

The magnet is to be placed in the vertical axis through the compass-pivot with its nearest end at the same distance below the needles that it was from the compass while deflecting: red pole up if the ship's force be a pull—blue pole up if a thrust.

378. To compensate from observations with balanced needle.—In the second and third methods, two cases may arise in each—spheres on, or off, the binnacle.

1. *Spheres off:* Let the scale division of the bead with needle level, on shore, be n; and on board, n': their ratio is that of $\frac{Z'}{Z}$, the vertical force in both places. The mean value of this has been denoted by μ, hence

$$\mu = \frac{Z'}{Z} = \frac{n'}{n} = \lambda(1 - \mathfrak{D}). \quad . \quad . \quad . \quad . \quad (27)$$

To derive the last member, if in (26) we make

$$\mathfrak{D} + \frac{\mu}{\lambda} - 1 = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

it will reduce $-J$ to zero, that is, compensate the heeling coefficient; hence from (28)

$$\mu = \lambda(1 - \mathfrak{D}), \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

and from (27)
$$n' = n \, . \, \lambda \, . \, (1 - \mathfrak{D}). \quad . \quad . \quad . \quad . \quad . \quad (30)$$

2. *Spheres on:* Fig. 563 shows that when the quadrantal deviation is compensated—reducing eq. (24) to zero—a portion of $-e$ equal to $+a$ is annulled, leaving only a part, *if any*, of $-e$ as a contributor to the heeling error; that is, with spheres on, $e = a$ . . . (31) and by (251) and (252), page 939, eq. (27) above becomes

$$\mu=\frac{Z'}{Z}=\frac{n'}{n}=\lambda\,(1+\mathfrak{D}). \quad . \quad . \quad . \quad . \quad (32)$$

The computation and procedure is the same as with the spheres off.

379. **To compensate from observations with oscillating needle.**—1. *Spheres off:* By observations similar to paragraphs (4) and (14), Art. 361, we obtain

$$\frac{T^2}{T''^2}=\frac{Z'}{Z}=\mu. \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

Substituting for $\frac{Z'}{Z}$ its value from eq. (459), p. 1025, we have

$$\frac{T^2}{T''^2}=\mu-\frac{h}{\tan\theta}.\sin\zeta+\frac{g}{\tan\theta}.\cos\zeta. \quad . \quad . \quad (34)$$

Omitting the term containing h as too small to affect the result materially, (34) becomes

$$T''=\frac{T}{\sqrt{\mu+g.\cot\theta.\cos\zeta}}. \quad . \quad . \quad . \quad . \quad (35)$$

Substituting the value of μ from (29), we have

$$T''=\frac{T}{\sqrt{\lambda\,(1-\mathfrak{D})+g.\cot\theta.\cos\zeta}} \quad . \quad . \quad . \quad (36)$$

Heading the ship E. or W. magnetic, cos ζ becomes zero,

and then
$$T''=\frac{T}{\sqrt{\lambda\,(1-\mathfrak{D})}}. \quad . \quad . \quad . \quad . \quad . \quad (37)$$

And this is the form in which the equation would generally be used, as the ship could nearly always be put E. or W., or within a few degrees of it, which would suffice. T'' is

the time in which the needle must make ten oscillations under following conditions: vertical needle in exact site of compass, and placed so that plane of oscillation is across meridian; ship heading east or west magnetic, or nearly so; magnet end-on centrally beneath compass, red pole up if oscillation is too fast—blue pole if too slow; move magnet up or down until exact time T'' is attained, when fix the magnet at that point, and the heeling error is compensated.

2. *Spheres on:* the only change occurs in eq. (37), which becomes

$$T'' = \frac{T}{\sqrt{\lambda\,(1+\mathfrak{D})}}. \quad . \quad . \quad . \quad . \quad . \quad (38)$$

If the ship cannot be headed east or west, for the method of this article, eq. (36) must be used, and then g must be known.

Section Four: Compensation of the Semicircular and Quadrantal Deviations.

380. Basis of semicircular and quadrantal compensation. Semicircular deviation results from the joint action of hard and soft iron: Art. 329 treats of the separation of their effects, and when it is accomplished the proper compensation of each can be performed by magnets and soft iron. The maximum resultant of both, R, and its direction α, are given by eqs. (77) and (78), page 871, as follows:

$$R = \sqrt{\mathfrak{B}^2 + \mathfrak{C}^2}. \quad . \quad . \quad . \quad (39); \quad \tan \alpha = \frac{\mathfrak{C}}{\mathfrak{B}}. \quad . \quad . \quad . \quad . \quad (40)$$

The effect of R, as the ship swings round, is to draw the compass-card away from the meridian by the amount of

the deviation on each heading: R must therefore be neutralized. As a rule, it is done at first entirely by magnets—either in one group, as an equal and opposite resultant force whose direction is the angle α; or in two groups, fore-and-aft and athwartships, to correspond to the components $\mathfrak{B}$ and $\mathfrak{C}$ of this resultant.

While compensating any one compass, the ship must be held steady by means of another compass on a particular point: for the resultant method, this point is the magnetic course corresponding to the compass course upon which it is decided to perform the work both for semicircular and quadrantal deviations; for the component method, it is magnetic north or south, to compensate for $\mathfrak{C}$, magnetic east or west for $\mathfrak{B}$, and magnetic NE., SE., NW., and SW. for the maximum quadrantal deviation.

The maximum value of the force R' that produces quadrantal deviation and the angle β of its direction are given by eqs. (83) and (85), page 872, as follows:

$$R' = \sqrt{\mathfrak{D}^2 + \mathfrak{E}^2}. \quad . \quad . \qquad (41); \qquad \tan\beta = \frac{\mathfrak{E}}{\mathfrak{D}}. \quad . \quad . \quad . \quad (42)$$

Being due to transient magnetism, it must be neutralized by soft-iron spheres which will acquire this magnetism to the degree denoted by R'; and they must be placed in the direction of the angle β. Analysis of a table of deviations affords the information required by eqs. (39) to (42) to perform the compensation.

The quadrantal deviation is alternately plus and minus in the successive quadrants: as a rule, it is plus in the NE. and SW. quarters, and minus in the SE. and NW.—and this order is designated as a positive quadrantal; the converse—that is, minus in the NE. and SW. and plus in the SE. and NW.—is called a negative quadrantal; it rarely occurs.

If we measure the angle α from the bow of the ship to starboard, to stern, and to port continuously through 360°, then we shall have from eq. (40):

$$\text{If } \mathfrak{C}=0, \tan\alpha=\frac{0}{\mathfrak{B}}=0, \text{ and hence } \alpha=0°. \quad . \quad .(43)$$

$$\text{If } \mathfrak{B}=0, \tan\alpha=\frac{\mathfrak{C}}{0}=\infty, \text{ and hence } \alpha=90°. \quad (44)$$

These results are derived from Algebra and Trigonometry.

Eq. (43) gives the direction of the corrector for $\mathfrak{B}$, that is, parallel to the keel; and (44) that for $\mathfrak{C}$, transverse thereto.

Similarly, for eq. (42), but on the basis that the angle β is reckoned from an origin transverse to the keel:

$$\text{If } \mathfrak{E}=0, \tan\beta=\frac{0}{\mathfrak{D}}=0, \text{ and hence } \beta=0°. \quad . \quad (45)$$

$$\text{If } \mathfrak{D}=0, \tan\beta=\frac{\mathfrak{E}}{0}=\infty, \text{ and hence } \beta=90°. \quad . \quad (46)$$

Eq. (45) shews that the spheres should be fixed on a line transverse to the keel to correct the effect of $\mathfrak{D}$, and this is their usual position; but it is correct only when $\mathfrak{E}$ is so small as to be negligible, or zero. When this is not the case, the proper direction for the line of spheres is given by eq. (42); and in this case, if $\mathfrak{E}$ be minus, the starboard sphere must be forward of the transverse line, and the port abaft it; but if $\mathfrak{E}$ be plus, the starboard sphere must be abaft the transverse line and the port sphere forward.

On the other hand, should $\mathfrak{E}$ be large and $\mathfrak{D}$ so small as to be negligible, eq. (46) shows that the line of spheres must be fore-and-aft: but this condition is unusual.

From eqs. (113) and (115), p. 879, it is seen that the

coefficients $\mathfrak{A}$ and $\mathfrak{E}$ have the same basis—soft iron represented by b and d: if $\mathfrak{E}$ should be large (2° to 3°), as sometimes occurs with steering compasses located outside the midship line, then compensation of $\mathfrak{E}$ will entail change in $\mathfrak{A}$, which should be determined by a swing for residuals and analysis of these.

381. Practical illustration of compensation by means of the Scoresby—resultant method.—After the observations described in Art. 361 had been completed, the resulting deviations—Table 93—were compensated with the vessel headed successively on different points, for the purpose of comparing the accuracy of the work on each.

The procedure on one of these will now be reproduced: it was compass heading N. 67° 30′ E. Fig. 553 represents the Scoresby during the work, except that the materials marked E and F were not on the vessel: they were added subsequently for another purpose. Fig. 554 exhibits a comparison of the compass and magnetic headings before compensation. Analysis of Table 93 furnished the information for compensating—it is given in Art. 364. Table 94 is a summary of the observations made for force, which are described in Art. 361. The quadrantal deviation on every point was obtained from analysis of Table 93: it was plotted in double size on Napier's diagram, and from this the amount on compass-point N. 67° 30′ E. was taken: it was 6°.

The data for compensating, then, were: compass heading N. 67° 30′ E., corresponding to magnetic heading N. 96° 30′ E.; total deviation on former, 29° E., of which 6° was quadrantal, leaving 23° semicircular; $\alpha = 56°$.

As D was + 8° 26′ and E only +0° 31′, the spheres, according to Art. 380, were properly set—transverse to the keel. The compass to be compensated was the same by which the deviations were observed—the four-needle liquid standard No. 24025. The binnacle in which it was

mounted on the SCORESBY was constructed for compensating with a specific number of magnets at a variable distance: they consisted of four equal bundles of wires, to be placed in tubes on a tray which had motion both vertically and in azimuth, with attached scales to indicate the amount of each motion. In Fig. 567, which represents the conditions when compensation was complete, the tray-magnets appear as broad dark bands and the compass needles as fine lines.

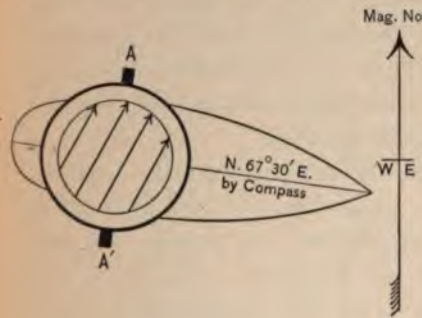


FIG. 566.

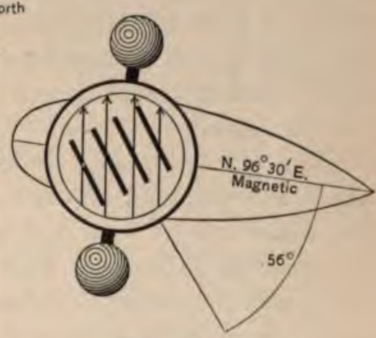


FIG. 567.

The vessel was headed N. 67° 30′ E. per compass and blocked there—Fig. 566, which exhibits the conditions before compensating: the brass circle on the floor indicated the equivalent magnetic heading, and this corresponded to holding a ship steady on the magnetic course by another compass.

The heeling error was compensated first, as described in Art. 378. In Fig. 566 it is seen from the relative directions of the needles and arms AA' for the spheres, that the latter can exert a good pull: they were therefore put in place and moved equally toward the compass—the card turning the while toward magnetic north—until 6° quadrantal deviation was corrected; the vessel's head by compass was then N. 73° 30′ E.; the spheres were locked on

the arms, and the center of each was then 12¼ inches from the compass-pivot.

The tray was now run well down, the tubes turned to the angle $\alpha = 56°$, and the four magnets put in them as indicated by the heavy bars of Fig. 567; red ends toward bow, as the relative positions of the poles of magnets and needles showed that that was requisite to turn the needles into the meridian: raised the tray slowly—the card turning meanwhile toward magnetic north—until 23° of semicircular deviation was corrected; the needles then coincided with the meridian, and the vessel's head per compass was N. 96° 30′ E., which was of course the magnetic heading also. Locked the tray, and then the plane of the magnets was ten inches from that of the needles.

Finally swung the vessel on sixteen points—resting about five minutes on each—and observed the bearing of the electric sun; the residual deviations thence deduced are given in Table 95. They are not large; and in view of the existing conditions—that both the disturbing material and the correctors were in close proximity to the needles—it may be said that the adjustment of the two antagonistic fields was quite accurate.

It is evident that the whole procedure might just as well have taken place on the largest battleship as on the SCORESBY: it matters little where the conditions portrayed by Tables 93 and 94 occur, they must be met in substantially the same way, and that way the one described in this article.

On a ship, however, no such concentrated disturbing field as that devised for the SCORESBY would exist in a location suitable for the standard compass, so that this phase of the matter was more unfavorable to the experimental work. Again, the point N. 67° 30′ E. was purposely chosen for some of its *un*favorable conditions: it was in the "very unsteady arc," with a directive force of

only 0.4476: in that arc the card is always restless and prone to swing at the least motion of the ship, and it would be difficult on a *ship* to estimate the amount of the deviation one was correcting.

TABLE 95.

SCORESBY'S Head by Compass No. 24025.	Residual Deviations After Compensation of Table 93.
(1)	(2)
NORTH	1° 30′ E.
N. NE.	2 00 E.
NE.	2 00 E.
E. NE.	2 00 E.
EAST	1 00 E.
E. SE.	1 00 W.
SE.	2 00 W.
S. SE.	1 30 W.
SOUTH	1 30 W.
S. SW.	1 30 W.
SW.	1 30 W.
W. SW.	1 30 W.
WEST	1 30 W.
W. NW.	1 00 W.
NW.	0 30 W.
N. NW.	0
NORTH	1 30 E.

TABLE 96.

SCORESBY'S Head by Compass No. 24025.	Deviations.
(1)	(2)
NORTH	46°.00′ W
N. NE.	42 00 W.
NE.	32 00 W.
E. NE.	22 00 W.
EAST	15 00 W.
E. SE.	6 00 W.
SE.	5 00 E.
S. SE.	25 00 E.
SOUTH	45 30 E.
S. SW.	54 00 E.
SW.	47 00 E.
W. SW.	31 30 E.
WEST	14 00 E.
W. NW.	4 00 W.
NW.	22 00 W.
N. NW.	37 00 W.

The heading was favorable as regards the directions of the needles, spheres, and magnets, as shown by Figs. 566 and 567. As illustrative of an *un*favorable relative direction of these, consider Fig. 568: the vessel is headed still in the "very unsteady arc," N. 124° E. per compass, corresponding to N. 117° E. magnetic, on which there is a total deviation of 7° W., with a quadrantal deviation of 8°, thus making an opposite semicircular deviation of 1°; directive force 0.5255.

The spheres can exert a good pull; but the magnets placed as indicated by the dotted arrow BB' in the starboard angle $\alpha = 56°$ are almost parallel to the needles; they have only 1° to correct, and may be moved ver-

tically through a long distance without any definite action upon the compass—the analogue of two lines drawn at a very acute angle—they nearly coincide for much of their length and the point of intersection cannot be distinguished.

The compass could not be compensated with the vessel heading on this point—the case is really indeterminate—and attention is directed to it merely for illustrative purposes. The whole unsteady arc should be avoided.

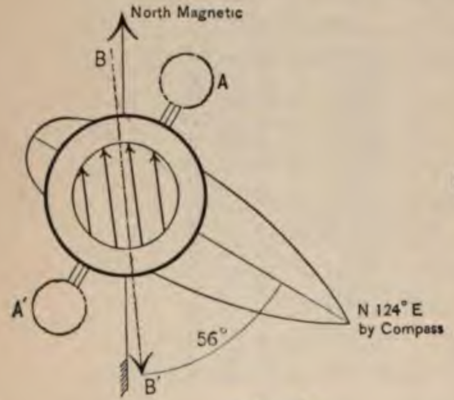


FIG. 568.

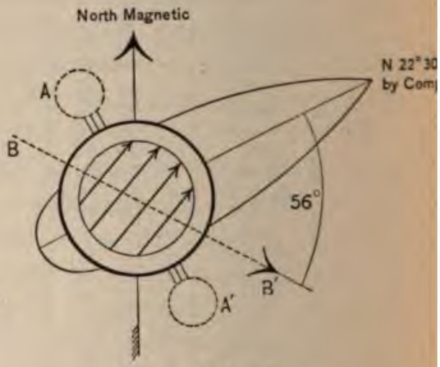


FIG. 569.

An inspection of Tables 93 and 94 in connection with Fig. 554 will enable one to select the point that holds out the best promise of easy work and accurate results. The entire SW. quarter seems favorable—also the arc from NE. to N.NW.; but the preference for any one point would probably fall on N.NE. per compass, or N. 22° 30′ E., corresponding to N. 62° 30′ E. magnetic, on which there is a directive force of about unity, a total deviation of 40° E., of which 6° 22′ are quadrantal, and 33° 38′ semicircular. The compass would be steady, so that on board a ship it would be easy to hold her by another compass on N. 62° 30′ E. magnetic while the compass to be compensated was turned by the spheres through 6° 22′, and by the magnets through 33° 38′.

The case is represented by Fig. 569; and it is seen that the pull of the spheres AA' would be strong, and that the magnets represented by BB' set in the starboard angle $\alpha = 56°$, red ends towards B', would exert a powerful thrust on the needles to turn them from their present deflection of 40° E. into the meridian.

382. Compensation by the component method — illustrated by the Scoresby.—The theory of this procedure is given in Art. 356—its practical performance will be described here, and there are three methods of carrying it out.

The Scoresby and its vicinity were first cleared of all magnetic material, and a series of observations made with the horizontal needle to obtain the time of ten oscillations: the mean was $T = 15.98$ seconds. Removed the needle and put the compass in the binnacle: all else being the same, observed the bearing of the electric sun during a swing on sixteen points.

First Method.—Then loaded the Scoresby with a powerful combination of magnets and soft iron and swung on the same sixteen points, observing the bearing of the electric sun on each: comparison of the two series of bearings gave the deviations of Table 96, of which Fig. 570 is the curve—natural size.

Analyzed this table on Form 10 and obtained the following: $A = -0° 15'$; $B = -16° 15'$; $C = -41° 31'$; $D = +7° 50'$; $E = +0° 6'$.

Table 97 is a ready reference for directions as to placing the magnets: *longitudinal* means parallel to the midship fore-and-aft line, and transverse at right angles to this.

The compensation was done on south and west magnetic; the corresponding headings by compass No. 24025 were found from Fig. 570 to be: South (magnetic) = S. 23° E.; West (magnetic) = S. 42° W.

To compensate on a *ship*, she should be headed successively S. 23° E. and S. 42° W. by compass No. 24025, if

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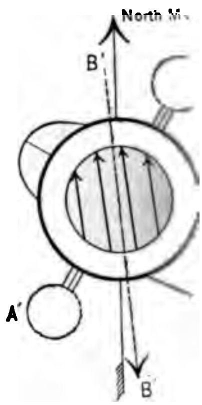

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that were to be compensated, and on each the heading by another compass noted and recorded, so that the ship could be held by this latter compass on its own headings corresponding to S. and W. magnetic while compensating compass No. 24025.

TABLE 97.

TO COMPENSATE THE COEFFICIENT *B*.

Ship's Head Magnetic.	The Deviation Being	Position and Direction of Magnets Used as Correctors.
(1)	(2)	(3)
East	Easterly	Longitudinal: red ends toward bow
East	Westerly	Longitudinal: red ends toward stern
West	Westerly	Longitudinal: red ends toward bow
West	Easterly	Longitudinal: red ends toward stern

TO COMPENSATE THE COEFFICIENT *C*.

North	Easterly	Transverse: red ends to starboard
North	Westerly	Transverse: red ends to port
South	Westerly	Transverse. red ends to starboard
South	Easterly	Transverse: red ends to port

For the SCORESBY, she was swung to head south magnetic and blocked; four transverse magnets, red ends to port, were placed in the tray, and it was raised until the heading was due south: then she was swung to head west magnetic and blocked; two longitudinal magnets, red ends toward stern, were put in the tray and this raised until the heading was due west.

Swung the vessel on the four cardinal points and found residual deviations of less than 1°.

The maximum quadrantal deviation was found by analysis of Table 96 to be 8°, and only this remained. Swung the SCORESBY to head NW. magnetic; put the spheres on

the binnacle and moved them toward the compass until the heading was N. 45° W.

Then swung the vessel on eight equidistant points and found residual deviations less than 1°. In the case of a *ship*, after compensating the semicircular deviation as above, she should be put on the compass heading corresponding to NE., NW., SE., or SW. magnetic, the course noted by another compass, and then the ship held by this while compensating.

SECOND METHOD.—For this the procedure with a *ship* would be as follows: Swing for table of deviations. On one compass course (ζ') obtain the period (T') of ten oscillations of horizontal needle placed instead of compass; for this particular course determine the corresponding magnetic course (ζ) and deviation (δ). Observe time (T) of ten oscillations of horizontal needle on shore. Analyze deviations on Form 10, and by means of formulas in Table III of that form compute exact coefficients and λ. Deduce values of $\lambda.\mathfrak{B}$ and $\lambda.\mathfrak{C}$; then proceed according to Art. 369 to determine the particular magnets to be used, and apply them as directed in Table 97: the ship may be put on any heading for this purpose, as both the strength of the magnets and the distance at which they are to be placed are determined by Art. 369 for all azimuths.

The quadrantal deviation is compensated as in the first method above.

Finally, swing for a table of residual deviations. The principal steps may be indicated by the following example from the SCORESBY: Deviations observed—Table 96; analyzed this and computed $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$. Swung on heading $\zeta' = $ N. 52° E. corresponding to $\zeta = $ N. 24° E., with deviation 28° W.; observed on this heading $T' = 14.23$ seconds, and in field of earth alone $T = 15.98$ seconds, whence

$$\frac{T^2}{T'^2} = \frac{H'}{H} = 1.261. \quad . \quad . \quad . \quad . \quad . \quad (47)$$

Then computed λ by formula (258), p. 944:

$$\lambda=(1.261)\frac{\cos(-28^\circ)}{1+\mathfrak{B}(\cos 24^\circ)-\mathfrak{C}(\sin 24^\circ)+\mathfrak{D}(\cos 48^\circ)-\mathfrak{E}(\sin 48^\circ)}, \quad (48)$$

The particular heading used is marked Q on Fig. 570.

THIRD METHOD.—As this involves observations for deviation and force on only two headings, and offers the twofold advantage of supplying a complete table of deviations as well as data for compensating the compass, it may prove of frequent use, and will therefore be illustrated in full by an experiment with the SCORESBY.

The vessel, still loaded with the same disturbing material in the same position that produced the deviations of Table 96, was swung on two opposite headings marked L and M, Fig. 570, and the following observations were made: [$T=15.98$ seconds, corresponding to shore observations, which had already been made as stated in the *First Method*, p. 1095.]

First heading: L, Fig. 570.		Second heading: M, Fig. 570.	
$\zeta_1'=$ N. 71° 0′ W. $=289°\ 0'$.	(49)	$\zeta_2'=$ S. 66° 0′ E. $=114°\ 0'$.	(55)
$\zeta_1=$ N. 72° 30′ W. $=287°\ 30'$.	(50)	$\zeta_2=$ S. 71° 30′ E. $=108°\ 30'$.	(56)
$\delta_1=1°\ 30'$ W.	(51)	$\delta_2=5°\ 30'$ W.	(57)
$T=15.98$ seconds.	(52)	$T=15.98$ seconds.	(58)
$T_1'=31.63$ seconds.	(53)	$T_2'=12.76$ seconds.	(59)
$R_1=\frac{T^2}{T_1'^2}=\frac{H_1'}{\lambda . H}=0.255$.	(54)	$R_2=\frac{T^2}{T_2'^2}=\frac{H_2'}{\lambda . H}=1.568$.	(60)

Mean magnetic heading, N. 72° W. and S. 72° E.

The mathematical basis of this problem and its solution both by computation and construction have been given in Arts. 325 and 338; the example used there, however, was

for headings not opposite—the general case: here, they differ by 180°, which is particular and more simple.

The solution is by construction on the Dygogram Form—Fig. 571.

The combined force of Ship and Earth R_1 on the first heading ζ_1' is laid off in the direction δ_1; that is, for N. 71° W., $\overline{OR_1} = 0.255$ of scale $\overline{OP}$, and $R_1OP = \delta_1 = 1° \; 30'$ W.: similarly, for $\zeta_2' =$ S. 66° E., $\overline{OR_2} = 1.568$, and $R_2OP = \delta_2 = 5° \; 30'$ W.

Draw $\overline{R_1R_2}$ and bisect it at L; through this draw MLH in the direction of the mean magnetic course—N. 72° W. and S. 72° E.—it may be done by placing parallel rulers on the center O and 72° W. of the arc FPG and moving them parallel to themselves until L is reached.

Through L draw a perpendicular to $\overline{LH}$, and where both this perpendicular and the line $\overline{LH}$ cut $\overline{OX}$, mark the points D and D' respectively. Bisect $\overline{DD'}$ in P', and with radius $\overline{P'D'}$, draw a circle—it should pass through the three points L, D, and D'.

Draw the outline of a ship around L, headed on ζ_2: then STA is the starboard angle α; $\overline{LM}$ the longitudinal component of the total semicircular force, and $\overline{MR_2}$ the transverse component—both negative, since they are toward the stern and to port; $\overline{OP'}$ is the mean directive force, and $\overline{P'D'}$ the force that produces quadrantal deviation.

Both $\mathfrak{A}$ and $\mathfrak{E}$ are considered zero, or negligible, as indeed they are in this case, for by analysis of Table 96, $\mathfrak{A} = -0.0043$ and $\mathfrak{E} = +0.0017$.

Measuring the required quantities on Fig. 571 by the scale $\overline{OP}$, we have:

$$\overline{OP'} = \lambda = 1.02, \quad \ldots \ldots \quad (61)$$

$$\overline{P'D'} = \overline{P'D} = \lambda . \mathfrak{D} = +0.14; \quad \ldots \quad (62)$$

whence

$$\mathfrak{D} = \frac{\lambda . \mathfrak{D}}{\lambda} = \frac{+0.14}{+1.02} = +0.137, \quad \ldots \quad (63)$$

$$\overline{LM} = \lambda . \mathfrak{B} = -0.273; \quad \ldots \quad (64)$$

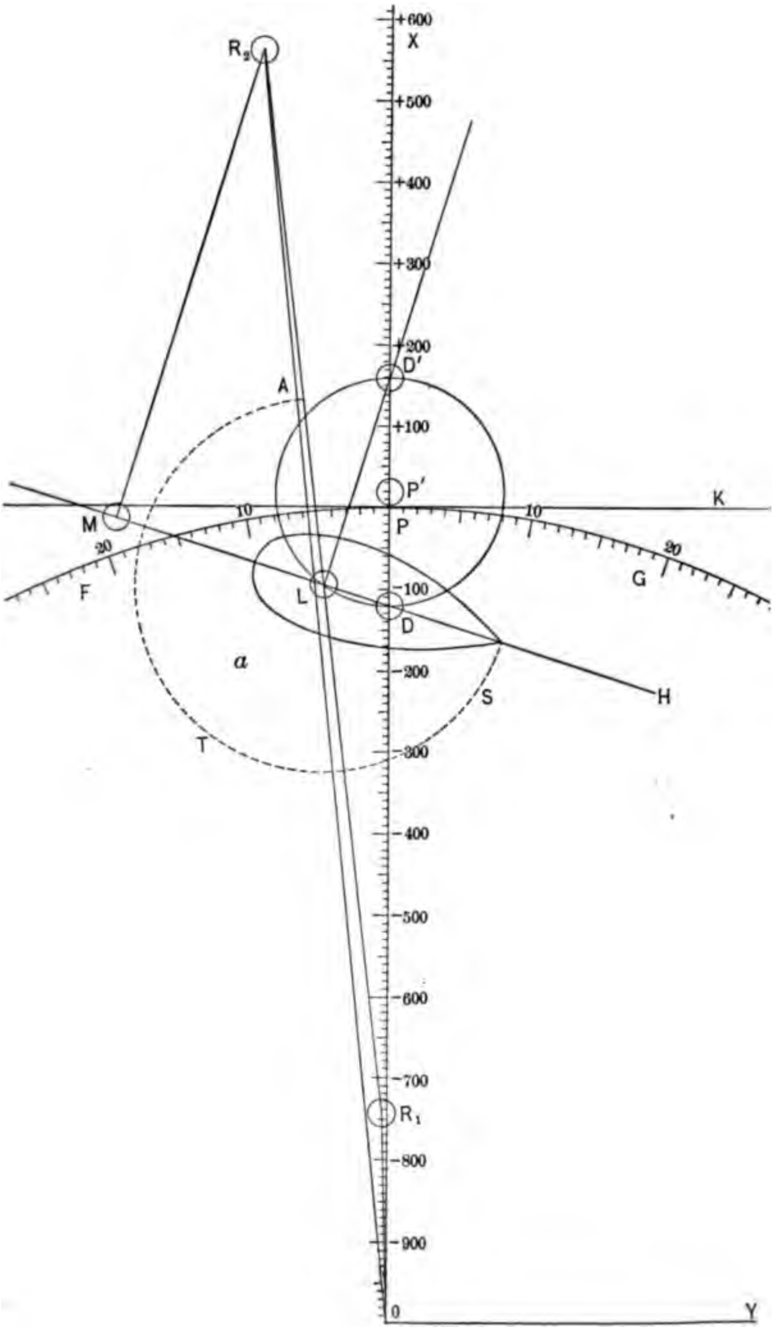
+600
X
R₂
+500
+400
+300
+200
D′
A
+100
P′
K
M
10
10
P
20
20
F
L
-100
G
D
a
-200
S
H
T
-300
-400
-500
-600
-700
R₁
-800
-900
0
Y

FIG. 571.

whence
$$\mathfrak{B} = \frac{-0.273}{+1.02} = -0.267, \quad . \quad . \quad . \quad . \quad (65)$$

$$\overline{MR}_2 = \lambda . \mathfrak{C} = -0.596; \quad . \quad . \quad . \quad . \quad (66)$$

whence
$$\mathfrak{C} = \frac{-0.596}{+1.02} = -0.584. \quad . \quad . \quad . \quad . \quad (67)$$
$$\alpha = 246^\circ.$$

Having $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ from eqs. (63), (65), and (67), we may, by the process of Art. 338, obtain a complete table of deviations and the directive force on each point.

For compensating, from (63), $\mathfrak{D} = +0.137$, which is the natural sine of $7^\circ\ 54'$, and this is the maximum quadrantal deviation, to be corrected on NE., NW., SE., or SW., magnetic as described in the first method; or the particular pair of spheres to be used and the distance at which to place them may be determined as described in Art. 371, and then they can be so placed irrespective of the heading of the ship.

Similarly, the particular magnets to correct $\lambda . \mathfrak{B}$ and $\lambda . \mathfrak{C}$ and the distance at which to place them, may be ascertained by the method of Art. 369, and put in place according to Table 97, with the ship on *any* heading. For the SCORESBY, they were determined in that way: since the strength of a magnet may be represented by the natural tangent of the angle of deflection it produces in the field it is to act, the force $\lambda . \mathfrak{B} = -0.273$ is the natural tangent of $15^\circ\ 18'$, and we must have certain magnets at a definite distance that will produce this deflection; two magnets, broadside on, with centers $15\frac{1}{4}$ inches from compass-pivot, produced it, and were accordingly placed longitudinally at that distance, with their red ends toward the stern. In like manner, $\lambda . \mathfrak{C} = -0.596$ is the natural tangent of $30^\circ\ 48'$, and four magnets, broadside on, centers $13\frac{3}{4}$ inches from compass-pivot, produced this deflection: they were accordingly placed transversely, red ends to port.

Then swung the vessel on eight equidistant points, and found residual deviations of less than 1°.

The compensation thus satisfactorily effected was that of the deviations of Table 96, where the maximum is 54° and the least value of the directive force was 0.255, that of the Earth being unity.

383. Fluctuation of the deviations due to change of geographical position.—The theoretical side of this subject has already been fully stated, and it only remains to cite an actual case in order to press the matter home—that of the U. S. S. MACHIAS. She is a steel cruiser of 1177 tons, with a battery of eight guns and steam propulsion.

When completed, she sailed for the Asiatic station, and at various points on the way was swung for deviations: they are given in Table 98. An inspection of this will show how unstable is the magnetic condition of a *new* vessel. Leaving New York with well-defined deviations, col. (2), whose maximum was 12° 30′, they were reduced by half, col. (3), within the period of a month; near Aden, col. (4), there is a further reduction on some points, an increase on others, and an irregular mixture of all; off the Island of Ceylon, col. (5), the irregularity continues, but nearly all the deviations increase; finally, in the Gulf of Pechili, col. (6), the regularity at New York is restored, but in amounts so small as hardly to merit the name of deviations.

There is scarcely a doubt that it was chiefly the *sur-*charge of construction that thus faded away during the passage. The irregularity in the Gulf of Aden and Bay of Bengal is explicable by the variable terrestrial magnetism experienced by the vessel in those regions—she crossed the *Magnetic* Equator (as will be seen by the data given at the bottom of Table 98), and the vertical induction, from having produced one kind of polarity near Aden, decreased to zero soon afterward, and gave rise to opposite polarity in the Bay of Bengal. Thus the reversal of the polarity or transient

of p; the needle will incline, and to restore level a vertical *steel magnet* must be placed centrally beneath the compass: then move the bead from its last position by the amount of q; the needle will incline again, but this time level must be restored by moving the *Flinders' bar*. Thus the latter may be used for correcting the vertical effect of soft iron in both the semicircular deviation and the heeling error.

Should the ship at any time be on the Magnetic Equator, where there is no *vertical* induction, the part of the heeling error due to *hard* iron can be corrected as follows: head the ship E. or W. magnetic and have her upright; replace the compass by the balanced needle, *without the bead;* it will dip under the influence of the vertical component of the ship's permanent magnetism, and must be restored to level by a vertical *steel magnet* placed centrally beneath the compass. Subsequently, upon arrival in a locality where the Dip is large, the Earth's vertical component will induce polarity in the vertical soft iron and cause the balanced needle—placed as before—to incline: it must be brought to level by moving the *wrought-iron* corrector—the Flinders' bar; and thus the heeling error has been completely and appropriately corrected.

Aside from all the foregoing, change is likely to occur in the permanent magnetism of the ship and of the correctors —it is but the fate of all magnetic bodies, even the hardest steel; again, new masses of soft iron may be brought near the compass to influence it; or, finally, one or several of the many causes enumerated in Art. 292 may arise to break the adjustment: the result in all such cases is a new series of deviations.

If it is decided to remove the correctors and determine the deviations due to both the primary and secondary fields and compensate them *de novo*, the process is that which has already been explained: but if it be preferred to leave the correctors in place, and simply neutralize the new secondary

field, the procedure remains to be illustrated; and this will now be done by an experiment with the Scoresby.

It will be recalled that Fig. 553—*without the masses marked E and F, however*—represents the Scoresby loaded with the disturbing material that produced the primary field that caused the deviations of Table 93: these having been compensated, the residuals of Table 95 remained. Then produced a secondary disturbing field by placing at *E*, Fig. 553, two bars of soft iron, each 18 inches long and about 2 inches cross-section; they were horizontal, in the mid-ship line, on a level with the compass, and nearest ends 10 inches from pivot: also placed at *F*, parallel to *D*, eleven cylindrical steel magnets, each 10½ inches long, with their nearest ends 32 inches from pivot.

Table 99.

Scoresby's Head by Compass No. 24025.	Deviations Produced by Secondary Field: Masses *E* and *F*, Fig. 553.	Residuals After Compensating Deviations of Col. (2).	Scoresby's Head by Compass No. 24025.	Deviations Produced by Secondary Field: Masses *E* and *F*, Fig. 553.	Residuals After Compensating Deviations of Col (2).
(1)	(2)	(3)	(1)	(2)	(3)
N.	8° 30′ E.	0° 30′ E.	S.	7° 30′ W.	0° 30′ E.
N. NE.	11 30 E.	0 0	S. SW.	4 30 W.	0 30 E.
NE.	10 30 E.	0 0	SW.	3 30 W.	0 0
E. NE.	7 0 E.	0 0	W. SW.	2 30 W.	1 0 W.
E.	1 0 E.	1 0 W.	W.	3 0 W.	0 30 W.
E. SE.	4 30 W.	1 30 W.	W. NW.	2 30 W.	0 0
SE.	9 0 W.	1 0 W.	NW.	1 0 W.	0 30 E.
S. SE.	9 30 W.	0 0	N. NW.	3 30 E.	0 30 E.

Then swung ship and found the deviations of col. (2), Table 99: analyzed this, and the resulting coefficients are given in cols. (5) and (6), Table 100.

The algebraic sum of the values of $\mathfrak{B}$ and $\mathfrak{C}$ in cols. (4) and (6) gives $\mathfrak{B}'$ and $\mathfrak{C}'$ col. (8); then $\tan\alpha = \frac{\mathfrak{C}'}{\mathfrak{B}'} = \frac{+.5564}{+.3232}$, whence $\alpha = 59° \ 51'$; and this is the new starboard angle to which the tray (for the resultant method) must be turned.

By c
heading
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MAGNETI
AND C

Values in Cols. (3) and (5).
(1)
A;
B;
C;
D;
E;
Starboard

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to $\alpha = 5$
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CHAPTER XXX.

386. Scope, character, and aim of this Treatise. Much of the course at the U. S. Naval Academy is occupied with mathematics—and justly so; for aside from its mental training, it adequately equips the midshipman for the performance of many duties in his subsequent career.

Pursued through the sequence of nautical astronomy and navigation, the course produces the navigator—to cite only one part of the profession—who comprehends the dependence to be placed on every method for finding the Ship's position at sea—when to take advantage of favorable conditions—when to avoid those fraught with inaccuracy—when the problem is entirely practicable—and when it is wholly impossible.

And this is the educated navigator that the costly sea-structures of to-day *should* have.

An essential part of Navigation is the subject-matter of this Treatise; indeed it would seem a most important part; for whereas many means exist for finding the ship's position, only one is known for guiding her—the Compass.

But that this instrument itself is beset by much to lead it astray is amply shown in the foregoing pages; and hence it is incumbent on the navigator to inform himself thoroughly on its every source of error—to gain that intelligent grasp of the conditions midst which it is placed that the course otherwise in Navigation affords for finding the ship's position.

It is quite true that the mere mechanical compensation of the compass can be taught in a short time—and the question arises, Wherefore such a lengthy Treatise as this?

It is also true, I believe, that much the lawyer has to deal with may be settled by reference to statute enactments, without knowledge of those general principles of jurisprudence that form the foundation of all law.

Even to find the ship's position, the clear, concise rules of old Bowditch suffice, without years spent on geometry, trigonometry, and nautical astronomy: the time-sight, meridian altitude, azimuth, and dead-reckoning are the strict necessities for navigating the ship, and these can be practically learned in a few days.

But such a navigator or such a lawyer literally gropes in the dark of his profession—he is but the animated part of a mechanism. *To proceed in like manner with compass adjustment is positively dangerous:* the conditions are so variable that rigid rules cannot safely be prescribed.

The subject of Compass Deviations is considered of such importance in the Navy that not only are the theory and practice taught at the Naval Academy, but in after years when an officer is ordered as navigator of a ship, he takes a preliminary course of practical work with the SCORESBY in the Compass Office at Washington.

During this course the principal problems he will encounter are represented by magnets and soft iron variously disposed on the little vessel; she is swung—the observations are made—resulting deviations analyzed—the compass compensated—and the reasons given for every step: surely these are extraordinary means taken to ensure thorough knowledge of the duties to be performed.

Incidentally, the theoretical acquisitions of academic days are refreshed; and thus equipped, the navigator goes to his ship prepared to attack with confidence and intelligence any magnetic conditions surrounding the compasses.

Placing the correctors is a mere detail that may readily be performed according to any method after the conditions have been investigated.

It is the aim of this Treatise to give a view of the subject in all its essential bearings—to state the theory—to supply the formulas—and to give numerical examples of the principal cases.

The transition from one formula to another is made with such detail as to facilitate comprehension of the subject without having to work out intermediate steps: to afford knowledge—not supply mathematical tangles for experts to unravel—was the object in view.

The work is necessarily a series of magnetic principles bound to a mathematical framework; but in order that such a structure may not repel by its natural harshness, the endeavor has been made to surround it with ample explanation in as non-technical language as the subject will admit.

A retrospect of the field thus covered will now be taken.

The writer is of opinion that when—if ever—the real nature of Light becomes known, so will that of Electricity and Magnetism—they are so intimately connected. At present facts point to a motion of a medium—specific movements of the ether of space—as the source of all these forces which have been called *Radiant*.

Instead, therefore, of repeating the customary explanation of magnetism that it is a dual fluid—which was only a device for representing observed phenomena, without ever a fact for its existence—the endeavor has been made to establish magnetism among the radiant forces.

Chapters I to IV, then, in connection with XII, are devoted to this view of the subject: in addition, Chapters III and IV contain much relative to wave motion in general, which renders more intelligible the reciprocal action of magnetic waves in the ether sent out by movements of the

Ship and Compass; these magnetic waves are treated subsequently in Part Fourth.

In Chapter VI the universality of electric and magnetic phenomena is dealt with, and these phenomena are treated jointly, with typical illustrations of each: the instances adduced are more to make clear a condition than convey information on particular points.

Chapter VII intimately concerns the navigator: it relates to the great magnet—the Earth—to the surgings of its enveloping ether, as Variation, Dip, and Intensity.

In Chapter VIII are given the principal methods for determining the Magnetic Elements, both afloat and ashore.

Those diverse but related phenomena of which the magnetic storm seems the concentrated embodiment—and during which magnetic needles are in a frenzy the world over—are described in Chapter IX: the subject is of interest as showing into what remote regions of space the influence extends that affects the compass on a ship.

Chapter X treats of the magnet in general and the region about it—the field; physically and analytically—its laws—possible nature—and affecting circumstances: deeming it among the most important parts of the subject, it has been dealt with from every point of view.

In Chapter XI such observed facts are set forth as indicate a probable theory of terrestrial and molecular magnetism; and the chapter closes with an investigation of the Earth's Magnetic Potential, upon which formulas are based for computing the Magnetic Elements.

The subject of Chapter XII has already been mentioned, and this completes PART FIRST and Volume I.

In PART SECOND, Volume II, Chapters XIII, XIV, and XV treat of the Compass—its history; magnetic and mechanical principles; manufacture; magnetic moment; and use. The importance to the seaman of this part of the subject need not be extolled—it cannot be exaggerated.

In PART THIRD, covering Chapters XVI to XX, inclusive, the Ship—after having been shown to be a magnet—is dissected, so to speak, and every disturbing element laid bare to view. This Part affords an insight into the Theory of the Deviations as far as it can be conveyed without mathematics; it was specially prepared for those who may prefer such treatment of the subject.

PART FOURTH, covering Chapters XXI to XXVI, is taken up with the deduction, transformation, and arrangement of all the formulas upon which the Theory rests, and which are used for the numerical computation of the deviations by the various methods employed on board ship.

It is in this Part that the endeavor has been made to facilitate comprehension of the subject by filling up those gaps in the sequence of the formulas that often yawn forbiddingly; and although the skilled mathematician may prefer the curtailed treatment, still it is thought that many others would rather have the easy transition than elegant brevity.

The remaining three chapters of the book constitute PART FIFTH, in which the essential and practicable methods of Swinging Ship and Compensating the Compass are explained and illustrated. The principles involved, the appliances to be used, and numerical examples are all given with sufficient fullness to enable any one to perform the work intelligently.

It will be seen that the Treatise covers a wide range: but all the matters dealt with affect the Compass either proximately or remotely, and their study as a whole should give an enlightened view of the conditions midst which it is placed.

PART SIXTH.

VARIOUS MATTERS BEARING ON THE MAIN SUBJECT.

Section One : Notes on Divers Things.

387. Nature and Object of Part Sixth.—The controlling idea throughout this Treatise was to give with due fullness whatever is intimately connected with the Compass and its Deviations, and to treat subsidiary matters only to such extent as would conduce to an intelligent knowledge of the main subject; but many things bear more or less upon this: to deal with these with the detail they require in order to be well understood, would extend the work far beyond its proper sphere; and it has occurred to the writer that, for those who may wish to pursue such parts further than they are given here, it would be useful to have reference to works that have been found clear and explicit upon the different subjects.

This Part, therefore, contains reference to such books as will give the information indicated by the headlines of Articles 388 to 400.

388. Constants of instruments and of formulas: matters referred to in Part First for explanation here.—An absolute instrument is one whose scales give values in C.G.S. units; a small instrument, portable and convenient for work, generally has an arbitrary scale: the same quantity having been measured several times by a small instrument and an absolute one, affords one of the Constants of the small one, and the other Constants are found in a similar way.

Those quantities in an equation which retain a fixed value are the Constants, and such, for example, are the Magnetic Coefficients in the deviation formula—the Course is the variable. For determining the constants of formulas, see *Laboratory Manual of Physics*

and Applied Electricity, by E. L. Nichols, vol. i, page 13; and *Physical Measurements, by F. Kohlrausch, page 11.*

In Part First, reference was made to Part Sixth for explanation of certain matters as follows: Page 257—Variation of gravity with latitude: see *Mechanics, by Aug. W. Smith, page 179*; and *Traité de Physique, par P. A. Daguin, tome i, page 126.*

Pages 258 and 260—The turning moment of two magnets: this will be found in Art. 399, below. Pages 293 and 296—formulas: these, also, are referred to in Art. 399. Page 514—equation (1): deduced in *Daguin's Physics, vol. i, page 533.*

389. Scales: Earth's Curvature: Duration of light flashes. For converting linear scale divisions into angular measure, see *Kohlrausch, page 116.* The curvature of the Earth is about eight inches to the mile. An impression of light takes about one-seventh of a second to be conveyed by the optic nerve: a red-hot stick whirling during this length of time leaves the impression of a red circle, and for a longer time only a portion of a circle. The portion of time the flash lasts, indicates the portion of the impression made: it is the principle of the moving pictures.

390. Materials absorbent of Moisture, of Heat, and of Light.—Chloride of calcium, or pumice stone, saturated with strong sulphuric acid, will absorb moisture. A solution of alum intercepts heat waves but allows those of light to pass; while a solution of iodine will intercept light waves, but allow those of heat to pass. A thin slab of pure rock salt allows heat waves from every source to pass freely through it, as transparent glass does white light.

Section Two: Units and Standards of Measure.

391. Fundamental and Derived Units.—All measurements depend upon Length, Mass, and Time, and these quantities are therefore called Fundamental: upon them are based measures of Area, Volume, Density, Force, Velocity, Acceleration, Work, Energy, and Momentum, which are hence said to be Derived.

Nations differ as to the specific length and mass they establish for standard units: the English have the foot-grain-second system, and the French the centimeter-gramme-second (C.G.S.) system; this is also the system of many other nations in the daily dealings of life, and it is almost exclusively that of scientific investigation and literature the world over.

Materially, the unit of length—the centimeter—is the one-hundredth part of the distance at 0° Centigrade between two parallel lines engraved on a certain bar of platinum-iridium alloy, deposited in a vault in the Laboratory of the *Comité Internationale des Poids et Mesures* at Sèvres, near Paris. This bar is the *Mètre Prototype*. The gramme is the one-thousandth part of a platinum-iridium piece known as the *Kilogramme Prototype*, deposited as above. (*Webster, Theory of Elec. and Mag.*)

Regarding the C.G.S. system, see: *Units and Physical Constants—J. D. Everett; Absolute Measurements—Andrew Gray, vol. i, chapter iii, and vol. ii, part ii, chapter ix; Electricity and Magnetism—Silvanus Thompson, page 263; Electricity and Magnetism—J. E. H. Gordon, vol. i, page 47; Theory of Physics—J. S. Ames, pages 7, 75, and 101; and Electricity and Magnetism—J. C. Maxwell, vol. i, page 3.*

392. Electrical and Magnetic Units.—There are two systems of these units—one based on the force exerted between two quantities of electricity, and this set are called *electrostatic* units: the other based on the force between two magnet-poles, and these are termed *electromagnetic* units. There is a most important link connecting them, which is explained on pages 521 to 543 of this Treatise.

Both systems are derived from the fundamental quantities Length, Mass, Time, and become specific by using for these the C.G.S. units: this, however, introduces magnitudes of inconvenient size—some too small, others too large—so that certain decimal multiples and submultiples of the basic absolute units have been agreed upon for *practical purposes*, and these have received the names of Volt, Ampere, Ohm, Coulomb, Farad, Watt, Joule, and Henry.

The subject is treated in the following works: *Elec. and Mag., Sil. Thompson, pages 130, 192, 265, and 344; Ames' Physics, pages 273, 299, 363, and 367; Gordon's Elec. and Mag., vol. i, page 257, and vol. ii, page 213; Gray's Abs. Meas., vol. i, chapter iii, and vol. ii, part ii, chapter ix; Everett—Physical Units; Principles of Dynamo-Electric Machines—Carl Hering.*

393. Various kinds of angular measure.—These are such as are reckoned in degrees; or in an arc equal to the radius; or by trigonometrical functions; or by solid angles: except the last, they are explained in Art. 296, and all are briefly dealt with by

Thompson, *El. and Mag., pages 133–136.* "The arc in degrees equal to the radius is approximately $57°.3 = 206,265''$. A ball one foot in actual [linear] diameter would have an apparent [angular] diameter of one second ($1''$) at the distance of 206,265 feet. If its apparent [angular] diameter were $10''$, its distance would be, of course, only one-tenth as great." (Prof. C. A. Young.)

Section Three: Regarding Heat.

394. The nature of Heat.—The prominent features of the theory of heat are these: matter is composed of molecules—infinitesimal particles of varied size and shape, and possessed of different motions; they do not join one to another in an unbroken mass, but are separated by interstices, permeated by the ether of space.

If the body be a solid—ice, for instance—and we apply heat, the inherent motion of the particles increases, and eventually the mass becomes liquid; more heat converts this into vapor; and still more into gas—oxygen and hydrogen: the vibratory and other motions of these elementary particles create waves—heat waves—in the ether, which pass on and on until absorbed by other matter, or are dissipated in the medium itself, as waves of air and water are. The molecules of all matter being thus in motion, they are interchanging *rates* of motion—some imparting more than they receive, others less: differences of temperature, pressure, expansion, and energy may be explained on the basis of this motion; that is, that heat is both a result and a cause of motion—vibratory, oscillatory, and translatory.

The theory of heat is stated in the following works: *Heat—John Tyndall, page 43, 55, 292, 360; Ames' Physics, page 251; The New Chemistry—J. P. Cooke, pages 42 and 192; Matter, Ether, and Motion—Dolbear, pages 132* and *136; Ganot's Physics, pages 309 and 320.*

395. Absolute Temperature.—It follows from the preceding hypothesis on the nature of heat, that if the molecules of a gas have no motion of any kind, they will fall to the bottom of the containing vessel, devoid of every attribute dependent on heat: from this condition a scale of absolute temperature may be devised; reasoning on the related conditions of pressure and volume of gases, absolute zero has been placed at $-273°$ Centigrade.

The following works afford information on this point: *Cooke's Chemistry—pages 14 and 46; Ames' Physics—page 212; Tyndall's Heat—pages 61 and 72.*

The capacity for heat varies with every substance, and the quantity requisite to produce the same effect—say a rise of 1° of temperature—in equal weights of substances is called their specific heat: *Dolbear, page 130.*

396. Conservation of Energy and Mechanical Equivalent of Heat.—"So far as experiment and experience have led us, the antecedents of every physical phenomenon are themselves physical, and more than that, all reactions are quantitative; that is, the product is proportional to the antecedent, and this is sometimes embodied in what is called the doctrine of the Conservation of Energy. The exchange relations between the different forms of energy—mechanical, thermal, chemical, electrical, and magnetical—being quantitative, are therefore mathematical" (Dolbear). See also *Ames' Physics, page 75.*

The Mechanical Equivalent of Heat is but the application of the principle of the conservation of energy to heat effects. It asserts that whenever energy is spent in producing heat effects the amount of work done exactly equals the sum of the internal and external work: *Ames, page 262; Dolbear, page 109; Tyndall's Heat, pages 6, 11, 38, and 61; Tyndall's Sound, page 56;* and *Ganot's Physics, page 380.*

Section Four: Subjects Connected with the Determination of Terrestrial Magnetism.

397. Pendulum-motion.—The oscillation of a magnetic needle is really that of a pendulum.

In the *Traité de Physique par P. A. Daguin, tome i, pages 109–132,* pendulum-motion is treated both descriptively and analytically: the matter is well presented and information on the points usually sought is supplied.

A complete investigation of the subject, however, is contained in the *Cours de Mécanique par Ch. Sturm, tome i, pages 201–215 and 221–223,* in connection with *pages 156–160.* This is purely analytical; the formulas are deduced from the differential equa-

Thompson, *El. and Mag., pages 133–136.* "The arc in degrees equal to the radius is approximately $57^\circ.3 = 206{,}265''$. A ball one foot in actual [linear] diameter would have an apparent [angular] diameter of one second ($1''$) at the distance of 206,265 feet. If its apparent [angular] diameter were $10''$, its distance would be, of course, only one-tenth as great." (Prof. C. A. Young.)

Section Three: Regarding Heat.

394. The nature of Heat.—The prominent features of the theory of heat are these: matter is composed of molecules—infinitesimal particles of varied size and shape, and possessed of different motions; they do not join one to another in an unbroken mass, but are separated by interstices, permeated by the ether of space.

If the body be a solid—ice, for instance—and we apply heat, the inherent motion of the particles increases, and eventually the mass becomes liquid; more heat converts this into vapor; and still more into gas—oxygen and hydrogen: the vibratory and other motions of these elementary particles create waves—heat waves—in the ether, which pass on and on until absorbed by other matter, or are dissipated in the medium itself, as waves of air and water are. The molecules of all matter being thus in motion, they are interchanging *rates* of motion—some imparting more than they receive, others less: differences of temperature, pressure, expansion, and energy may be explained on the basis of this motion; that is, that heat is both a result and a cause of motion—vibratory, oscillatory, and translatory.

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tion of motion—they are integrated—and the subject is treated with full detail. These formulas involve differential equations of the second order, and the general method of integrating these will be found in *Sturm's Cours d'Analyse, tome ii, page 95;* also in *De Morgan's Calculus, page 154.*

398. Harmonic Analysis: Fourier's Theorem: Potential.—From Pendulum-motion to Simple Harmonic Analysis and Fourier's Theorem is but a step, for in truth all have close relationship.

Pendulum-oscillation is wave-motion; harmonic analysis took its name from the investigation of vibrating strings—their fundamental tone and its harmonics—represented by wave-outlines; and Fourier's series is the analytical expression of wave-form in general.

Mechanically, the compounding of two oscillatory motions may be done as shown in Art. 35; *pictorially*, the combination of many such movements may be effected by Lissajous' tuning-forks, Art. 36; and analytically, the superposition of an infinity of waves may be expressed by Fourier's series, eq. (149), p. 886. All deal with *periodic motion*, which abounds in Nature.

"Rhythm or the graduated and alternate action and reaction with which a vibration begins and ends, is a universal law in the manifestation and movements of all natural phenomena; a law which is revealed on a grand scale in the recurring periods of Nature, whether astral, telluric, or meteorological, as well as in the form and manifold phases of organisms and their modes of reproduction. This universal law also applies to the whole mental and organic system of animals and men whenever they become conscious of their own existence. The same universal rhythm constitutes the fundamental form of sound in the vibration of metallic bars, or of strings, and becomes perceptible to the external senses by means of our organ of hearing." (Tito Vignoli.)

"It would be interesting to trace the analogy between the mathematical and mechanical methods of harmonic analysis and the dynamical processes which go on when a compound ray of light is analyzed into its simple vibrations by a prism; when a particular overtone is selected from a complex tone by a resonator; and when the enormously complicated sound-wave of an orchestra, or even the discordant clamors of a crowd, are interpreted into intelligible music or language by the attentive listener, armed with the harp of three thousand strings, the resonance of which, as it hangs in the gateway of his ear, discriminates the multifold

components of the waves of the aerial ocean." (James Clerk Maxwell.)

The distinction between the wave-method and the harmonic-method of analysis is stated by Maxwell in the *Encyc. Brit., vol. xi, page 481;* and an exposition of simple harmonic analysis is given by *Thomson and Tait, Natural Phil.*, and also in the *Encyc. Brit., vol. xv, page 685.*

Some of the formulas occurring in the analytical treatment of periodic motion are deduced in *De Morgan's Calculus, pages 124 and 239.*

Fourier's Theorem will be found in many books—variously stated—but greatly abridged: the most satisfactory presentation of the subject I have found, is by the author himself in *La Théorie Analytique de la Chaleur;* or its translation by Alexander Freeman.

The word *Potential* indicates generally the condition to which it is applied—the power to do work: the region immediately surrounding a central force—electrical, magnetical, or otherwise—is the seat of this power. It may be considered along a line, over a surface, or throughout a volume.

When two or more electrodes are placed at different points of a sheet of tin-foil, or in a square box containing an electrolytic liquid, the electricity in its flow will indicate equipotential lines on the surface of the tin-foil, and equipotential surfaces in the liquid: in both cases they may be calculated mathematically from the distribution of electricity; and in both the accuracy of the calculation has been verified by experiment—the agreement is very close.

Potential is described in Art. 194, and its employment in the investigation of terrestrial magnetism is there illustrated.

The principal features of Potential are described by *Prof. Thompson, El. and Mag., pages 427, 327, and 339.*

Indeed, Potential is treated in almost every work on Electricity and Magnetism; analytically, the several steps are laid down with some variety by each writer, but all lead up to one characteristic equation:

$$\frac{d^2V}{dx^2}+\frac{d^2V}{dy^2}+\frac{d^2V}{dz^2}=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (A)$$

This expresses mathematically the fact that the function V changes by the smallest amounts possible in the directions of the coördinate axes; it is therefore exactly applicable to the potential condition.

A spherical harmonic is a homogeneous function of x, y, z,

Having, then, the *ratio* and the *product* of the two quantities, each is readily separated from the other.

Following are the formulas used in the calculations, with explanations of the symbols entering them, and an account of the corrections to be applied to the observed quantities:

[1.] $2.M.M'\left\{\frac{1}{r^3}+\frac{P}{r^5}+\frac{Q}{r^7}+\frac{R}{r^9}+\text{etc.}\right\}$ = the *moment*, or effort of the deflecting magnet (which may be designated as 𝔅), to turn the suspended magnet 𝔄 from the meridian: M' = magnetic moment of 𝔄.

[2.] $M'.H.\sin\delta$ = the *moment*, or directive force of the Earth's magnetism to keep the magnet 𝔄 in the meridian. When equilibrium occurs between these moments we have

[3.] $$2.M.M'\left\{\frac{1}{r^3}+\frac{P}{r^5}+\frac{Q}{r^7}+\frac{R}{r^9}+\text{etc.}\right\} = M'.H.\sin\delta.$$

The series within brackets is a function of the half length of the deflect*ing* magnet and of the distance r: to consider the length of a magnet is to take into account the magnetism pervading it; so that the numerators P, Q, R, etc., are said to depend on the distribution of magnetism in the bar. As it is only the half length that enters, and as the distance r (compared therewith) is generally quite large, the successive terms, it is evident, become rapidly very small, so that the first two generally suffice: retaining only these, we have from [3]

[4.] $2M\left\{\frac{1}{r^3}+\frac{P}{r^5}\right\} = H.\sin\delta\cdot$ from which

[5.] $\frac{M}{H}2\frac{1}{r^3}\left\{1+\frac{P}{r^2}\right\} = \sin\delta$, and

[6.] $$\frac{M}{H} = \frac{\frac{1}{2}.r^3.\sin\delta}{\left\{1+\frac{P}{r^2}\right\}} = \frac{1}{2}.r^3.\sin\delta\left\{1-\frac{P}{r^2}\right\}$$

$$= \left[\frac{M_1}{H_1}\right]\left\{1-\frac{P}{r^2}\right\}$$: see [22].

To determine P, let observations be made at the distances r and r_1 with corresponding deflections δ and δ_1: two members

similar to the third one of [6] may thus be formed—both equal to $\frac{M}{H}$; whence, equating them and deducing the value of P, it is

$$[7.] \qquad P = \frac{r\ .\sin\delta - r_2^3.\sin\delta_2}{r.\sin\delta - r_2.\sin\delta_2}.$$

(See also on this point *Gray's Abs. Meas., vol. ii, part i, page 92.*)

The *Ratio* required is now determined by [6] and the *Product* by [8]:

$$[8.] \qquad M.H = \frac{\pi^2.K}{T^2}.$$

Whence, multiplying [6] and [8] member by member, we have

$$[9.] \qquad M.H \times \frac{M}{H} = M^2 = \left(\frac{\pi^2.K}{T^2}\left[\frac{M_1}{H_1}\right]\left\{1 - \frac{P}{r^2}\right\}\right).$$

Extracting square root,

$$[10.] \qquad M = \left(\frac{\pi^2.K}{T^2}\left[\frac{M_1}{H_1}\right]\left\{1 - \frac{P}{r^2}\right\}\right)^{\frac{1}{2}}.$$

Dividing [8] by [6], member by member, we have

$$[11.] \qquad \frac{M.H}{\frac{M}{H_1}} = H^2 = \frac{\frac{\pi^2.K}{T^2}}{\left[\frac{M_1}{H_1}\right]\left\{1 - \frac{P}{r^2}\right\}}.$$

Extracting square root

$$[12.] \qquad H = \left(\frac{\frac{\pi^2.K}{T^4}}{\left[\frac{M_1}{H_1}\right]\left\{1 - \frac{P}{r^2}\right\}}\right)^{\frac{1}{2}}.$$

Equations [6], [7], [8], [10], and [12] are the formulas for computing M and H from the observed quantities: these must first be corrected, however, in the manner now to be stated in connection with the explanations of the symbols entering the formulas.

[13.] H = horizontal component of the Earth's magnetic force in C.G.S. units during the deflection and oscillation experiments. Both the Variation (Declination) and Intensity will fluctuate during the period of observation: to correct for change in the Declination, observe the axis of the suspended magnet before and after each deflection; to correct for change in Intensity, observe the period of oscillation of a magnet kept suspended

Having, then, the *ratio* and
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corrections to be applied

[1.] $2.M.M' \left\{ \frac{1}{r^3} + \right.$

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[4.] $2M$

[5.] $\frac{M}{H} 2 \frac{1}{r}$

[6.] $\frac{M}{H} =$

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r and r_2 with

...ing the torsion circle 90°, that is, giving a twist ...ne fiber. $(M.H)$ = magnetic directive force acting ...hen suspended. $\frac{F}{(M.H)}$ = ratio of these forces. It ...n on page 443, vol. i, that the force of torsion is pro-... angle of torsion; on the other hand, the angle to ...nded magnet may be deflected, is inversely as the ... acting on it: therefore where equilibrium occurs ... torsional force and the directive magnetic force, we

$$... (90^\circ - \phi) = M.H : \frac{1}{\phi}; \text{ hence } \frac{F}{(M.H)} = \frac{\phi}{(90^\circ - \phi)}.$$

...n is treated analytically and experimentally in these *Gauss, pages 21 and 83; Lloyd's Magnetism, page 76; Physics, vol. i, page 486; and El. and Mag., Masc. and ...ol. ii, page 61.*

C. = Centigrade scale of temperature.

t = 0° C., and t_1 = *any* observed temperature, Centigrade.

...garding change in magnetic moment due to fluctuation of ...magnet's temperature, see page 292, vol. i, this Treatise.

It has been found by experiment that the following formula ...resses the change: $[q(t_1 - t) + q'(t_1 - t)^2]$; in this, q and q' are ...nstants dependent on the qualities of each particular magnet; ...ey are determined by observing the deflections the magnet ...oduces at different values of t_1.

There are two cases in which the above formula may be used—either to find the magnetic moment M_0 at any observed temperature t_1 (knowing the moment M at t), or the converse, that is, to reduce the former to the latter. As an example:

$$M[q(t_1 - t) + q'(t_1 - t)^2] = \left\{ \begin{array}{l} \text{the change that takes place in the mag-} \\ \text{netic moment of } \mathfrak{B} \text{ when its temperature} \\ \text{is raised from } t \text{ to } t_1; \end{array} \right.$$

and since the moment *de*creases with *in*crease of temperature, we have

$$M - M[q(t_1 - t) + q'(t_1 - t)^2] = \text{moment of } \mathfrak{B} \text{ at } t_1;$$

that is,

$$M_0 = M - M[q(t_1 - t) + q'(t_1 - t)^2] = M\{1 - q(t_1 - t) - q'(t_1 - t)^2\}.$$

The converse proceeding is obvious.

The important experiments of Coulomb on the effect of heat on magnets will be found in *Biot's Physics, vol. iii, page 106.*

[21.] μ = the increase in the magnetic moment of 𝔅 produced by the inducing action of a magnetic field equal to unity in the C.G.S. system. For methods of determining μ, see *Gray's Abs. Meas., vol. ii, part i, pages 78 and 86.*

[22.] $\frac{M_2}{H_2} = \frac{1}{2}r^3 . \sin \delta$ = approximate value of $\frac{M}{H}$; found by using observed values, without applying corrections for induction and temperature. $\left[\frac{M_1}{H_1}\right]$ = value of $\frac{M_2}{H_2}$ with corrections applied, except $\left\{1 - \frac{P}{r^2}\right\}$; then $\left[\frac{M_1}{H_1}\right] = \frac{M_2}{H_2}\left\{1 + \frac{2\mu}{r^3} + q(t_1 - t) + q'(t_1 - t)^2\right\}$. The quantities within brackets are the corrections to be applied to $\frac{M_2}{H_2}$ to obtain $\left[\frac{M_1}{H_1}\right]$; and this latter is then the value used in [6], [9], [10], [11], and [12].

[23.] $\pi = 3.1416$.

[24.] ϵ = linear coefficient of expansionof brass = 0.000018.

[25.] ϵ' = linear coefficient of expansion of steel = 0.000012.

[26.] r_1 = first distance observed on the brass bar from center of deflecting magnet 𝔅 to center of suspended magnet 𝔄: this distance is usually made 30 centimeters. r = distance r_1 corrected for error of graduation of brass bar and for its reduction to temperature 0° C. r_3 = second distance observed on brass bar between centers of magnets 𝔄 and 𝔅: this should be 40 centimeters if $r_1 = 30$ cms. r_2 = distance r_3 corrected for error of graduation and reduction to 0° C. Whatever the first distance be made, the second should be 1.32 times its amount, or $r_3 = 1.32 r_1$. See *Airy's Mag., page 67.* Both r_1 and r_3 to be corrected for ϵ; see [24].

[27.] δ = observed angle of deflection corresponding to r. δ_2 = observed angle of deflection corresponding to r_2. The construction of the instrument is such that the deflecting magnet is turned in azimuth until its axis is at right angles to that of the suspended magnet when the latter comes to rest at the angles δ and δ_2, respectively, from the magnetic meridian; this requires the *sine*—not the tangent—of the angle of deflection to be used in the computations. It would seem best to begin the series of observations with deflections; follow with the oscillations; and close with a repetition of the deflections at the same distances as at first, in order to detect any change in the strength of the deflecting magnet in the interval.

[28.] K = moment of inertia of magnet 𝔅, including its stirrup and other appendages, at 0° C. K_1 = moment of inertia of the regular body used in oscillation with 𝔅, to be computed from its weight and dimensions. Both K and K_1 to be corrected for ϵ'; see [25].

[29.] τ = the time of an oscillation of 𝔅 *without* the inertia mass.

[30.] τ' = the time of an oscillation of 𝔅 *with* the inertia mass. Both τ and τ' must be corrected, before using, for the five quantities enumerated in [15] above.

The complete analytical investigation of the moment of inertia will be found in the *Traité de Mécanique par S. D. Poisson, tome ii, chapter ii, and Sturm's Mécanique, tome ii, page 123.* For partial exposition illustrated by practical examples, see *Smith's Mechanics, chapter vi; Kohlrausch; Nichol's Laboratory Manual, page 57; and El. and Mag.*, by *Masc. and Joub., vol. ii, page 54.*

The particular formula suitable for calculating the moment of inertia of either the magnet itself or the added mass (regular body) will depend on its form and axis of rotation; and the requisite formulas will be found in the works cited above.

The various corrections indicated by the foregoing quantities are explained in Art. 137, vol. i: some of them are tabulated on printed forms which exist for convenience of computation.

400. Works consulted.—Throughout the pages of this Treatise the names of various authors appear: such of their works as bear upon its subject-matter have been consulted; nearly all the books cited by title in this Part Sixth have also been consulted; so that it only remains to quote those not specifically mentioned in either way. They are: *Azuni—Dissertation sur l'orignie de la Boussole; Admiralty Manual of Scientific Enquiry; Diehl, S. W. B., and Southerland, W. H. H.—Practical Problems and Compensation of the Compass; Faye, M. H.—Cours d'Astronomie Nautique; Hoogewerff, J. A.—Observations at the Magnetic Observatory, Washington, D. C.; Harris, W. S.—Magnetism; Lecky, S. T. S.—Wrinkles in Navigation; Maycock, W. P.—The Alternating Current; Marsh, C. C.—Magnetic Observations at the Naval Observatory, Washington, D. C.; Preble, G. H.—Mariner's Compass, in the United Service Magazine; Taylor, Sedley—Sound and Music; Schott, Chas. A.—Magnetic Observations and Essays published by the U. S. Coast and Geodetic Survey; Schellen, H.—Spectrum Analysis, trans. by the Misses Lassell; Spottiswoode, William—Polarization of Light.*

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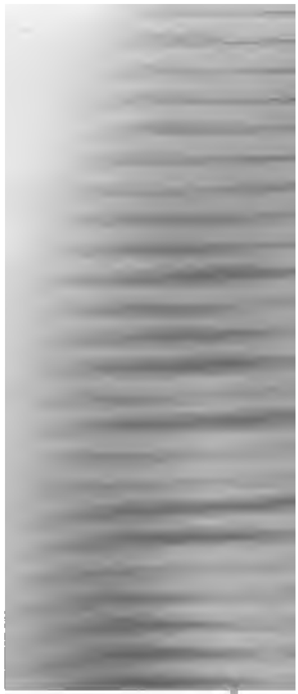

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